2017 Kansas Mathematics Standards

Flip Book

7th Grade

This project used work created by the Departments of Education in Ohio, North Carolina, Georgia and resources created by Achieve the Core, EngageNY, Illustrative Mathematics, and NCTM.

About the Flip Books

This project attempts to organize some of the most valuable resources that help develop the intent, understanding and implementation of the 2017 Kansas Mathematics Standards. These documents provide a starting point for teachers and administrators to begin discussion and exploration into the standards. It is not the only resource to support implementation of the 2017 Kansas Mathematics Standards.

This project is built on the previous work started in the summer of 2012 from Melisa Hancock (Manhattan, KS), Debbie Thompson (Wichita, KS) and Patricia Hart (Wichita, KS) who provided the initial development of the "flip books." The "flip books" are based on a model that Kansas had for earlier standards; however, this edition specifically targets the Kansas Mathematics Standards that were adopted in the summer of 2017. These flip books incorporate the resources from other state departments of education, the mathematics learning progressions, and other reliable sources including The National Council of Teachers of Mathematics and the National Supervisors of Mathematics. In addition, mathematics educators across the country have suggested changes/additions that could or should be made to further enhance its effectiveness. The document is posted on the KSDE Mathematics website at http://community.ksde.org/Default.aspx?tabid=5646 and will continue to undergo changes periodically. When significant changes/additions are implemented, the modifications will be posted and dated.

Planning Advice - Focus on the Clusters

The (mathematics standards) call for a greater focus. Rather than racing to cover topics in today's mile-wide, inch-deep curriculum, we need to use the power of the eraser and significantly narrow and deepen how time and energy is spent in the mathematics classroom. There is a necessity to focus deeply on the major work of each grade to enable students to gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems both in and out of the mathematics classroom. (www.achievethecore.org)

Not all standards should have the same instructional emphasis. Some groups of standards require a greater emphasis than others. In order to be intentional and systematic, priorities need to be set for planning, instruction, and assessment. "Not everything in the Standards should have equal priority" (Zimba, 2011). Therefore, there is a need to elevate the content of some standards over that of others throughout the K-12 curriculum.

When the Standards were developed the following were considerations in the identification of priorities: 1) the need to be qualitative and well-articulated; 2) the understanding that some content will become more important than other; 3) the creation of a focus means that some essential content will get a greater share of the time and resources "while the remaining content is limited in scope." 4) a "lower" priority does not imply exclusion of content, but is usually intended to be taught in conjunction with or in support of one of the major clusters.

"The Standards are built on the progressions, so priorities have to be chosen with an eye to the arc of big ideas in the Standards. A prioritization scheme that respects progressions in the Standards will strike a balance between the journey and the endpoint. If the endpoint is everything, few will have enough wisdom to walk the path, if the endpoint is nothing, few will understand where the journey is headed. Beginnings and the endings both need particular care. … It would also be a mistake to identify such standard as a locus of emphasis. (Zimba, 2011)

The important question in planning instruction is: "What is the mathematics you want the student to walk away with?" In order to accomplish this, educators need to think about "grain size" when planning instruction. Grain size corresponds to the knowledge you want the student to know. Mathematics is simplest at the right grain size. According to Phil Daro (*Teaching Chapters, Not Lessons—Grain Size of Mathematics*), strands are too vague and too large a grain size, while lessons are too small a grain size. Units or chapters produce about the right "grain size". In the planning process educators should attend to the clusters, and think of the standards as the ingredients of a cluster. Coherence of mathematical ideas and concepts exists at the cluster level across grades.

A caution--Grain size is important but can result in conversations that do not advance the intent of this structure. Extended discussions among teachers where it is argued for "2 days" instead of "3 days" on a topic because it is a lower priority can detract from the overall intent of suggested priorities. The reverse is also true. As Daro indicates, focusing on lessons can provide too narrow a view which compromises the coherence value of closely related standards.

The video clip Teaching Chapters, Not Lessons—Grain Size of Mathematics presents Phil Daro further explaining grain size and the importance of it in the planning process. (Click on photo to view video.)

Along with "grain size", clusters have been given **priorities** which have important implications for instruction. These priorities should help guide the focus for teachers as they determine allocation of time for both planning and instruction. The priorities provided help guide the focus for teachers as they determine distribution of time for both planning and instruction, helping to assure that students really understand mathematics before moving on. Each cluster has been given a priority level. As professional educators begin planning, developing and writing units, these priorities provide guidance in assigning time for instruction and formative assessment within the classroom.

Each cluster within the standards has been given a priority level influenced by the work of Jason Zimba. The three levels are referred to as — **Major, Supporting** and **Additional**. Zimba suggests that about 70% of instruction should relate to the **Major** clusters. The lower two priorities (**Supporting** and **Additional**) can work together by supporting the **Major** priorities. You can find the grade Level Focus Documents for the 2017 Kansas Math Standards at: http://community.ksde.org/Default.aspx?tabid=6340.

Recommendations for Cluster Level Priorities

Appropriate Use:

- Use the priorities as guidance to inform instructional decisions regarding time and resources spent on clusters by varying the degrees of emphasis.
- Focus should be on the major work of the grade in order to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through sense-making, reasoning, arguing and critiquing, modeling, etc.
- Evaluate instructional materials by taking the cluster level priorities into account. The major work of the grade must be presented with the highest possible quality; the additional work of the grade should support the major priorities and not detract from them.
- Set priorities for other implementation efforts such as staff development, new curriculum development, and revision of existing formative or summative testing at the state, district or school level.

Things to Avoid:

- Neglecting any of the material in the standards. Seeing Supporting and Additional clusters as optional.
- Sorting clusters (from Major to Supporting to Additional) and then teaching the clusters in order. This would remove the coherence of mathematical ideas and create missed opportunities to enhance the major work of the grade with the other clusters.
- Using the cluster headings as a replacement for the actual standards. All features of the standards matter—from the practices to surrounding text, including the particular wording of the individual content standards. Guidance for priorities is given at the cluster level as a way of thinking about the content with the necessary specificity yet without going so far into detail as to comprise the coherence of the standards (grain size).

Mathematics Teaching Practices

(High Leverage Teacher Actions)

[National Council of Teachers of Mathematics. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: National Council of Teachers of Mathematics.]

The eight Mathematics Teaching Practices should be the foundation for mathematics instruction and learning. This framework was informed by over twenty years of research and presented in *Principles to Actions* by the National Council of Teachers of Mathematics (NCTM). If teachers are guided by this framework, they can move "toward improved instructional practice" and support "one another in becoming skilled at teaching in ways that matter for ensuring successful mathematics learning for all students" (NCTM, 2014, p. 12).

1. **Establish mathematics goals to focus learning.**
   Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
2. **Implement tasks that promote reasoning and problem solving.**
   Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
3. **Use and connect mathematical representations.**
   Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
4. **Facilitate meaningful mathematical discourse.**
   Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
5. **Pose purposeful questions.**
   Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
6. **Build procedural fluency from conceptual understanding.**
   Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
7. **Support productive struggle in learning mathematics.**
   Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
8. **Elicit and use evidence of student thinking.**
   Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Standards for Mathematical Practice in Grade 7

The Common Core State Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that Grade 7 students complete.

Practice	Explanation and Example
1) Make sense of problems and persevere in solving them.	Mathematically proficient students in Grade 7 start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They solve real world problems involving ratios and rates and discuss how they solved them. They see the meaning of a problem and look for efficient ways to represent and solve it. They check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?" They understand the approaches of others to solving complex problems and identify correspondences between the different approaches. Example: Seventh graders should navigate among tables, graphs, and equations representing linear relationships to gain insights into the role played by constant rate of change.
2) Reason abstractly and quantitatively.	Mathematically proficient students in Grade 7 make sense of quantities and their relationships in problem situations. They represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. They *contextualize* to understand the meaning of the number or variable as related to the problem. They *decontextualize* to manipulate symbolic representations by applying properties of operations. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. Examples: 1)They apply ratio reasoning to convert measurement units and proportional relationships to solve percent problems, 2) they solve problems involving unit rates by representing the situations in equation form, and 3) they use properties of operation to generate equivalent expressions and use the number line to understand multiplication and division of rational numbers.
3) Construct viable arguments and critique the reasoning of others.	Mathematically proficient students in Grade 7 understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plot, dot plots, histograms, etc.) Example: Use of numerical counterexamples to identify common errors in algebraic manipulation, such as thinking that 5 – 2x is equivalent to 3x. Proficient MS students progress from arguing exclusively through concrete referents such as physical objects and pictorial referents, to also including symbolic representations such as expressions and equations.
4) Model with mathematics.	Mathematically proficient students Grade 7 can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. They analyze relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the mode if it has not served its purpose. Examples: Seventh grade students might apply proportional reasoning to plan a school event or analyze a problem in the community, or they can roughly fit a line to a scatter plot to make predictions and gather experimental data to approximate a probability.

5) Use appropriate tools strategically.	Mathematically proficient students in Grade 7 consider the available tools when solving a mathematical problem. These tools might include pencil/paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, a statistical package, or dynamic geometry software. They are sufficiently familiar with tools appropriate for their grade to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They are able to use technological tools to explore and deepen their understanding of concepts. Examples: Use graphs to model functions, algebra tiles to see how properties of operations apply to equations, and dynamic geometry software to discover properties of parallelograms. They might use a computer applet demonstrating Archimedes' procedure for approximating the value of π.
6) Attend to precision.	Mathematically proficient students in Grade 7 try to communicate precisely to others. They try to use clear definitions in discussions with others and in their *own* reasoning. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. Examples: 1) Seventh grade students can use the definition of rational numbers to explain why a number is irrational, and describe congruence and similarity in terms of transformations in the plane and 2) they accurately apply scientific notation to large numbers and use measures of center to describe data sets.
7) Look for and make use of structure.	Mathematically proficient students in Grade 7 look for and notice patterns and then articulate what they *see*. They can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5-3(x-y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*. Examples: 1) Seventh grade students might use the structure of the number line to demonstrate that the distance between two rational numbers is the absolute value of their difference, ascertain the relationship between slopes and solution sets of systems of linear equations, and see the equation $3x = 2y$ represents a proportional relationship with a unit rate of $\frac{3}{2} = 1.5$, 2) they might recognize how the Pythagorean theorem is used to find distances between points in the coordinate plane and identify right triangles that can be used to find the length of a diagonal in a rectangular prism.
8) Look for and express regularity in repeated reasoning.	Mathematically proficient students in Grade 7 notice if calculations are repeated and look both for general methods and for shortcuts. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation $\frac{(y-2)}{(x-1)} = 3$. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. Examples: 1)By working with tables of equivalent ratios, seventh graders can deduce the corresponding multiplicative relationships and connections to unit rates, 2) they notice the regularity with which interior angle sums increase with the number of sides in a polygon leads to a general formula for the interior angle sum of an *n*-gon, 3) Seventh graders learn to see subtraction as addition of opposite, and use this in a general purpose tool for collecting like terms in linear expressions.

Implementing Standards for Mathematical Practice

This guide was created to help educators implement these standards into their classroom instruction. These are the practices for the **students,** and the teacher can assist students in using them efficiently and effectively.

#1 – Make sense of problems and persevere in solving them.

Summary of this Practice:

- Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem.
- Plan a solution pathway instead of jumping to a solution.
- Monitor their progress and change the approach if necessary.
- See relationships between various representations.
- Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another.
- Continually ask themselves, “Does this make sense?”
- Understand various approaches to solutions.

Student Actions	Teacher Actions
• Actively engage in solving problems and thinking is visible (doing mathematics vs. following steps or procedures with no understanding). • Relate current “situation” to concepts or skills previously learned, and checking answers using different methods. • Monitor and evaluate their own progress and change course when necessary. • Always ask, “Does this make sense?” as they are solving problems.	• Allow students time to initiate a plan; using question prompts as needed to assist students in developing a pathway. • Constantly ask students if their plans and solutions make sense. • Question students to see connections to previous solution attempts and/or tasks to make sense of the current problem. • Consistently ask students to defend and justify their solution(s) by comparing solution paths.

What questions develop this Practice?

- How would you describe the problem in your own words? How would you describe what you are trying to find?
- What do you notice about...?
- What information is given in the problem? Describe the relationship between the quantities.
- Describe what you have already tried. What might you change? Talk me through the steps you’ve used to this point.
- What steps in the process are you most confident about? What are some other strategies you might try?
- What are some other problems that are similar to this one?
- How might you use one of your previous problems to help you begin? How else might you organize...represent...show...?

What are the characteristics of a good math task for this Practice?

- Requires students to engage with conceptual ideas that underlie the procedures to complete the task and develop understanding.
- Requires cognitive effort - while procedures may be followed, the approach or pathway is not explicitly suggested by the task, or task instructions and multiple entry points are available.
- Encourages multiple representations, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations to develop meaning.
- Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.

#2 – Reason abstractly and quantitatively.

Summary of this Practice:

- Make sense of quantities and their relationships.
- Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships.
- Understand the meaning of quantities and are flexible in the use of operations and their properties.
- Create a logical representation of the problem.
- Attend to the meaning of quantities, not just how to compute them.

Student Actions	Teacher Actions
• Use varied representations and approaches when solving problems. • Represent situations symbolically and manipulating those symbols easily. • Give meaning to quantities (not just computing them) and making sense of the relationships within problems.	• Ask students to explain the meaning of the symbols in the problem and in their solution. • Expect students to give meaning to all quantities in the task. • Question students so that understanding of the relationships between the quantities and/or the symbols in the problem and the solution are fully understood.

What questions develop this Practice?

- What do the numbers used in the problem represent? What is the relationship of the quantities?
- How is_____related to______?
- What is the relationship between _____ and _____?
- What does ______ mean to you? (e.g. symbol, quantity, diagram)
- What properties might you use to find a solution?
- How did you decide that you needed to use _____? Could we have used another operation or property to solve this task? Why or why not?

What are the characteristics of a good math task for this Practice?

- Includes questions that require students to attend to the meaning of quantities and their relationships, not just how to compute them.
- Consistently expects students to convert situations into symbols in order to solve the problem; and then requires students to explain the solution within a meaningful situation.
- Contains relevant, realistic content.

#3 – Construct viable arguments and critique the reasoning of others.

Summary of this Practice:

- Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments.
- Justify conclusions with mathematical ideas.
- Listen to the arguments of others and ask useful questions to determine if an argument makes sense.
- Ask clarifying questions or suggest ideas to improve/revise the argument.
- Compare two arguments and determine correct or flawed logic.

Student Actions	Teacher Actions
• Make conjectures and exploring the truth of those conjectures. • Recognize and use counter examples. • Justify and defend all conclusions and using data within those conclusions. • Recognize and explain flaws in arguments, which may need to be demonstrated using objects, pictures, diagrams, or actions.	• Encourage students to use proven mathematical understandings, (definitions, properties, conventions, theorems etc.), to support their reasoning. • Question students so they can tell the difference between assumptions and logical conjectures. • Ask questions that require students to justify their solution and their solution pathway. • Prompt students to respectfully evaluate peer arguments when solutions are shared. • Ask students to compare and contrast various solution methods • Create various instructional opportunities for students to engage in mathematical discussions (whole group, small group, partners, etc.)

What questions develop this Practice?

- What mathematical evidence would support your solution? How can we be sure that...? How could you prove that...?
- Will it still work if...?
- What were you considering when...? How did you decide to try that strategy?
- How did you test whether your approach worked?
- How did you decide what the problem was asking you to find? (What was unknown?)
- Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not?
- What is the same and what is different about...? How could you demonstrate a counter-example?

What are the characteristics of a good math task for this Practice?

- Structured to bring out multiple representations, approaches, or error analysis.
- Embeds discussion and communication of reasoning and justification with others.
- Requires students to provide evidence to explain their thinking beyond merely using computational skills to find a solution.
- Expects students to give feedback and ask questions of others' solutions.

#4 – Model with mathematics.

Summary of this Practice:

- Understand reasoning quantitatively and abstractly (able to decontextualize and contextualize).
- Apply the math they know to solve problems in everyday life.
- Simplify a complex problem and identify important quantities to look at relationships.
- Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation.
- Reflect on whether the results make sense, possibly improving/revising the model.
- Ask themselves, "How can I represent this mathematically?"

Student Actions	Teacher Actions
• Apply mathematics to everyday life. • Write equations to describe situations. • Illustrate mathematical relationships using diagrams, data displays, and/or formulas. • Identify important quantities and analyzing relationships to draw conclusions.	• Demonstrate and provide students experiences with the use of various mathematical models. • Question students to justify their choice of model and the thinking behind the model. • Ask students about the appropriateness of the model chosen. • Assist students in seeing and making connections among models.

What questions develop this Practice?

- What number model could you construct to represent the problem?
- How can you represent the quantities?
- What is an equation or expression that matches the diagram..., number line..., chart..., table...?
- Where did you see one of the quantities in the task in your equation or expression?
- What math do you know that you could use to represent this situation?
- What assumptions do you have to make to solve the problem?
- What formula might apply in this situation?

What are the characteristics of a good math task for this Practice?

- Structures represent the problem situation and their solution symbolically, graphically, and/or pictorially (may include technological tools) appropriate to the context of the problem.
- Invites students to create a context (real-world situation) that explains numerical/symbolic representations.
- Asks students to take complex mathematics and make it simpler by creating a model that will represent the relationship between the quantities.

#5 – Use appropriate tools strategically.

Summary of this Practice:
- Use available tools recognizing the strengths and limitations of each.
- Use estimation and other mathematical knowledge to detect possible errors.
- Identify relevant external mathematical resources to pose and solve problems.
- Use technological tools to deepen their understanding of mathematics.
- Use mathematical models for visualize and analyze information

Student Actions	Teacher Actions
• Choose tools that are appropriate for the task. • Know when to use estimates and exact answers. • Use tools to pose or solve problems to be most effective and efficient.	• Demonstrate and provide students experiences with the use of various math tools. A variety of tools are within the environment and readily available. • Question students as to why they chose the tools they used to solve the problem. • Consistently model how and when to estimate effectively, and requiring students to use estimation strategies in a variety of situations. • Ask student to explain their mathematical thinking with the chosen tool. • Ask students to explore other options when some tools are not available.

What questions develop this practice?
- What mathematical tools could we use to visualize and represent the situation?
- What information do you have?
- What do you know that is not stated in the problem? What approach are you considering trying first?
- What estimate did you make for the solution?
- In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? Why was it helpful to use...?
- What can using a_____show us that _____may not?
- In what situations might it be more informative or helpful to use...?

What are the characteristics of a good math task for this Practice?
- Lends itself to multiple learning tools. (Tools may include; concrete models, measurement tools, graphs, diagrams, spreadsheets, statistical software, etc.)
- Requires students to determine and use appropriate tools to solve problems.
- Asks students to estimate in a variety of situations:
    - a task when there is no need to have an exact answer
    - a task when there is not enough information to get an exact answer
    - a task to check if the answer from a calculation is reasonable

#6 – Attend to precision.

Summary of this Practice:

- Communicate precisely with others and try to use clear mathematical language when discussing their reasoning.
- Understand meanings of symbols used in mathematics and can label quantities appropriately.
- Express numerical answers with a degree of precision appropriate for the problem context.
- Calculate efficiently and accurately.

Student Actions	Teacher Actions
• Use mathematical terms, both orally and in written form, appropriately. • Use and understanding the meanings of math symbols that are used in tasks. • Calculate accurately and efficiently. • Understand the importance of the unit in quantities.	• Consistently use and model correct content terminology. • Expect students to use precise mathematical vocabulary during mathematical conversations. • Question students to identify symbols, quantities and units in a clear manner.

What questions develop this Practice?

- What mathematical terms apply in this situation? How did you know your solution was reasonable?
- Explain how you might show that your solution answers the problem.
- Is there a more efficient strategy?
- How are you showing the meaning of the quantities?
- What symbols or mathematical notations are important in this problem?
- What mathematical language..., definitions..., properties can you use to explain...?
- How could you test your solution to see if it answers the problem?

What are the characteristics of a good math task for this Practice?

- Requires students to use precise vocabulary (in written and verbal responses) when communicating mathematical ideas.
- Expects students to use symbols appropriately.
- Embeds expectations of how precise the solution needs to be (some may more appropriately be estimates).

#7 – Look for and make use of structure.

Summary of this Practice:

- Apply general mathematical rules to specific situations.
- Look for the overall structure and patterns in mathematics.
- See complicated things as single objects or as being composed of several objects.

Student Actions	Teacher Actions
• Look closely at patterns in numbers and their relationships to solve problems. • Associate patterns with the properties of operations and their relationships. • Compose and decompose numbers and number sentences/expressions.	• Encourage students to look for something they recognize and having students apply the information in identifying solution paths (i.e. compose/decompose numbers and geometric figures, identify properties, operations, etc.) • Expect students to explain the overall structure of the problem and the big math idea used to solve the problem.

What questions develop this Practice?

- What observations do you make about...? What do you notice when...?
- What parts of the problem might you eliminate..., simplify...?
- What patterns do you find in...?
- How do you know if something is a pattern?
- What ideas that we have learned before were useful in solving this problem?
- What are some other problems that are similar to this one? How does this relate to...?
- In what ways does this problem connect to other mathematical concepts?

What are the characteristics of a good math task for this Practice?

- Requires students to look for the structure within mathematics in order to solve the problem. (i.e. – decomposing numbers by place value; working with properties; etc.)
- Asks students to take a complex idea and then identify and use the component parts to solve problems. i.e. Building on the structure of equal sharing, students connect the understanding to the traditional division algorithm. When "unit size" cannot be equally distributed, it is necessary to break down into a smaller "unit size". (example below)

$$\begin{array}{r} 4\,\overline{)351} \\ \underline{-32} \\ 31 \\ \underline{-28} \\ 3 \end{array}$$

3 *hundreds* units cannot be distributed into 4 equal groups. Therefore, they must be broken down into *tens* units.

There are now 35 *tens* units to distribute into 4 groups. Each group gets 8 sets of tens, leaving 3 extra *tens* units that need to become *ones* units.

This leaves 31 *ones* units to distribute into 4 groups. Each group gets 7 *ones* units, with 3 *ones* units remaining. The quotient means that each group has 87 with 3 left.

- Expects students to recognize and identify structures from previous experience(s) and apply this understanding in a new situation. i.e. $7 \times 8 = (7 \times 5) + (7 \times 3)$ OR $7 \times 8 = (7 \times 4) + (7 \times 4)$ new situations could be, distributive property, area of composite figures, multiplication fact strategies.

#8 – Look for and express regularity in repeated reasoning.

Summary of this Practice:

- See repeated calculations and look for generalizations and shortcuts.
- See the overall process of the problem and still attend to the details.
- Understand the broader application of patterns and see the structure in similar situations.
- Continually evaluate the reasonableness of their intermediate results.

Student Actions	Teacher Actions
• Notice if processes are repeated and look for both general methods and shortcuts. • Evaluate the reasonableness of intermediate results while solving. • Make generalizations based on discoveries and constructing formulas when appropriate.	• Ask what math relationships or patterns can be used to assist in making sense of the problem. • Ask for predictions about solutions at midpoints throughout the solution process. • Question students to assist them in creating generalizations based on repetition in thinking and procedures.

What questions develop this Practice?

- Will the same strategy work in other situations?
- Is this always true, sometimes true or never true? How would we prove that...?
- What do you notice about...?
- What is happening in this situation? What would happen if...?
- Is there a mathematical rule for...?
- What predictions or generalizations can this pattern support? What mathematical consistencies do you notice?

What are the characteristics of a good math task for this Practice?

- Present several opportunities to reveal patterns or repetition in thinking, so students can make a generalization or rule.
- Requires students to see patterns or relationships in order to develop a mathematical rule.
- Expects students to discover the underlying structure of the problem and come to a generalization.
- Connects to a previous task to extend learning of a mathematical concept.

Critical Areas for Mathematics in 7th Grade

In Grade 7, instructional time should focus on **five** critical areas:

1. **Developing understanding of and applying proportional relationships;**
   Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line. They distinguish proportional relationships from other relationships.

2. **Developing understanding of operations with rational numbers.**
   Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percent as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division.

3. **Working with expressions and linear equations.**
   Students refine their work by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

4. **Solving problems involving scale drawings and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume.**
   Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle. Students will explore and generalize formulas for volume and surface area of right prisms and cylinders. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings. Students work with the relationships between three-dimensional figures and two- dimensional figures by examining cross- sections of three-dimensional figures and shapes created by rotating a two-dimensional shape around an edge. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, right prisms, and cylinders. This sets the stage for studying cones and pyramids in Grade 8.

5. **Drawing inferences about populations based on samples.**
   Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Dynamic Learning Maps (DLM) and Essential Elements

The Dynamic Learning Maps and Essential Elements are knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities.

For more information please visit the Dynamic Learning Maps and Essential Elements website.

Growth Mindset

The term "growth mindset" comes from the groundbreaking work of Carol Dweck. She identified that everyone holds ideas about their own potential. Some people believe that their intelligence is more or less fixed in math – that you can do math or you can't, while others believe they can learn anything and that their intelligence can grow.

In a fixed mindset, people believe their basic qualities, like their intelligence or talent, are simply fixed traits. They spend their time documenting their intelligence or talent instead of developing it. They also believe that talent alone creates success—without effort. Students with a fixed mindset are those who are more likely to give up easily.

In a **growth mindset**, people believe that their most basic abilities can be developed through dedication and hard work—brains and talent are just the starting point. This view creates a love of learning and a resilience that is essential for great accomplishment. Students with a growth mindset are those who keep going even when work is hard, and who are persistent.

It is possible to change mindsets and to shift students' mindsets from fixed to growth and cause higher mathematics achievement and success in life. Watch this short video to get a better understanding of what Growth Mindset is and the benefits it can bring our students.

You can find a variety of resources related to **Growth Mindset** at: http://community.ksde.org/Default.aspx?tabid=6383.

youcubed at Stanford University

Building a Mathematical Mindset Community

MATHEMATICAL MINDSETS

Teachers and students believe *everyone* can learn maths at HIGH LEVELS.
- Students are not tracked or grouped by achievement
- All students are offered high level work
- "I know you can do this" "I believe in you"
- Praise effort and ideas, not the person
- Students vocalize self-belief and confidence

Communication and *connections* are valued.
- Students work in groups sharing ideas and visuals.
- Students relate ideas to previous lessons or topics
- Students connect their ideas to their peers' ideas, visuals, and representations.
- Teachers create opportunities for students to see connections.
- Students relate ideas to events in their lives and the world.

The maths is VISUAL.
- Teachers ask students to draw their ideas
- Tasks are posed with a visual component
- Students draw for each other when they explain
- Students gesture to illustrate their thinking

The maths is OPEN.
- Students are invited to see maths differently
- Students are encouraged to use and share different ideas, methods, and perspectives
- Creativity is valued and modeled.
- Students' work looks different from each other
- Students use ownership words - "my method", "my idea"

The environment is filled with *WONDER* and *CURIOSITY*.
- Students extend their work and investigate
- Teacher invites curiosity when posing tasks
- Students see maths as an unexplored puzzle
- Students freely ask and pose questions
- Students seek important information
- "I've never thought of it like that before."

The classroom is a risk-taking, *MISTAKE VALUING* environment
- Students share ideas even when they are wrong
- Peers seek to understand rather than correct
- Students feel comfortable when they are stuck or wrong
- Teachers and students work together when stuck
- Tasks are low floor/high ceiling
- Students disagree with each other and the teacher

Developed by Jo Boaler/Youcubed.org and Tulare County Office of Education

Grade 7 Content Standards Overview

Ratios and Proportional Relationships (7.RP)

A. Analyze proportional relationships and use them to solve real-world and mathematical problems.
   7.RP.1 7.RP.2 7.RP.3

The Number System (7.NS)

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
  7.NS.1 7.NS.2 7.NS.3

Expressions and Equations (7.EE)

A. Use properties of operations to generate equivalent expressions.
   7.EE.1 7.EE.2
B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
   7.EE.3 7.EE.4

Geometry (7.G)

A. Draw, construct, and describe geometrical figures and describe the relationships between them.
   7.G.1 7.G.2 7.G.3
B. Solve real-life and mathematical problems involving area, surface area, and volume.
   7.G.4 7.G.5 7.G.6

Statistics and Probability (7.SP)

A. Use random sampling to draw inferences about a population.
   7.SP.1 7.SP.2
B. Draw informal comparative inferences about two populations.
   7.SP.3 7.SP.4
C. Investigate chance processes and develop, use, and evaluate probability models.
   7.SP.5 7.SP.6 7.SP.7 7.SP.8

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Domain: Ratios and Proportional Relationships (RP)

▶ **Cluster:** *Analyze proportional relationships and use them to solve real-world and mathematical problems.*

Standard: Grade 7.RP.1

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the* ***complex fraction*** $\frac{\frac{1}{2}}{\frac{1}{4}}$ *miles per hour (interpreting a complex fraction as division of fractions), equivalently 2 miles per hour.* **(7.RP.1)**

Suggested Standards for Mathematical Practice (MP):

- ✓ *MP.1 Make sense of problems and persevere in solving them.*
- ✓ *MP.2 Reason abstractly and quantitatively.*
- ✓ *MP.3 Construct viable arguments and critique the reasoning of others.*
- ✓ *MP.4 Model with mathematics.*
- ✓ *MP.5 Use appropriate tools strategically.*
- ✓ *MP.6 Attend to precision.*
- ✓ *MP.7 Look for and make use of structure.*
- ✓ *MP.8 Look for and express regularity in repeated reasoning.*

Connections:

This cluster is connected to:

- ▶Grade 7 Critical Area of Focus #1: **Developing understanding of and applying proportional relationships** and
- ▶Critical Area of Focus #2**: Developing understanding of operations with rational numbers and working with expressions and linear equations.**
- This cluster grows out of Ratio and Proportional Relationships (Grade 6) and the Number System (Grade 6)
- Relates to Expressions and Equations (Grade 7). Cross curricular connections - economics, personal finance, reading strategies.

Explanations and Examples:

Students continue to work with unit rates from 6th grade; however, the comparison now includes fractions compared to fractions. For example, if $\frac{1}{2}$ gallon of paint covers $\frac{1}{6}$ of a wall, then the amount of paint needed for the one wall (the unit rate) can be computed by rewriting the ratio, $\frac{\frac{1}{2}\,gal}{\frac{1}{6}\,wall}$, into an equivalent form with a denominator of 1. This calculation gives 3 $\frac{gallons}{wall}$. This standard requires only the use of ratios as quotients. Fractions may be proper or improper.

Instructional Strategies:

Building from the development of rate and unit concepts in Grade 6, applications now need to focus on solving unit-rate problems with more sophisticated numbers: fractions per fractions.

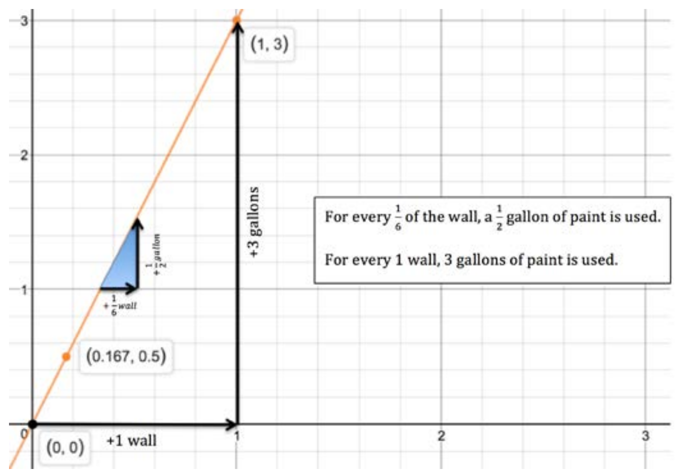
Proportional relationships are further developed through the analysis of graphs, tables, equations and diagrams. Ratio tables serve a valuable purpose in the solution of proportional problems. This is the time to push for a deep understanding of what a representation of a proportional relationship looks like and what the characteristics are: a straight line through the origin on a graph, a "rule" that applies for all ordered pairs, an equivalent ratio or an expression that describes the situation, etc. Use the ratio table to compute the unit rate. Students will develop an understanding that, in order to find the unit rate, one can compute $\frac{y}{x}$ with any pair of values from the relationship.

The previous example could be explored using multiple representations to compute the unit rate, i.e. the amount of paint required to cover *one* wall.

Portion of Wall	Amount of paint (gallon)
$\frac{1}{6}$	$\frac{1}{2}$
$\frac{2}{6}$	1
$\frac{3}{6}$	$\frac{3}{2}$
$\frac{4}{6}$	2
$\frac{5}{6}$	$\frac{5}{2}$
1	3

$\frac{\frac{1}{2}\,gal}{\frac{1}{6}\,wall}=3$

$\frac{3\ gal}{1\ wall} = 3$



This is not the time for students to learn to cross multiply to solve problems.

Although algorithms provide efficient means for finding solutions, *the cross-product algorithm* commonly used for solving proportions ***will not*** aid in the development of proportional reasoning. Delaying the introduction of rules and algorithms will encourage thinking about multiplicative situations instead of indiscriminately applying rules.

Because percents have been introduced as rates in Grade 6, the work with percents should continue to follow the thinking involved with rates and proportions. Solutions to problems can be found by using the same strategies for solving rates, such as looking for equivalent ratios or based upon understandings of decimals. Previously, percents have focused on "out of 100"; now percents above 100 are encountered.

Providing opportunities to solve problems based within contexts that are relevant to seventh graders will connect meaning to rates, ratios and proportions.

Examples include: researching newspaper ads and constructing their own question(s), keeping a log of prices (particularly sales) and determining savings by purchasing items on sale, timing students as they walk a lap on the track and figuring their rates, creating open-ended problem scenarios with and without numbers to give students the opportunity to demonstrate conceptual understanding, inviting students to create a similar problem to a given problem and explain their reasoning.

Tools/Resources:

For detailed information see Ratios and Proportional Reasoning Learning Progression.

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.RP.A
  - Track Practice
  - Stock Swaps, Variation 2
  - Stock Swaps, Variation 3
  - Sale!
  - Thunder and Lightning
  - Climbing the steps of El Castillo
  - Dueling Candidates
  - Track Practice
- 7.RP.A.1
  - Cooking with the Whole Cup
  - Molly's Run
  - Molly's Run, Assessment Variation

Georgia Department of Education website:

- "See Saw Nickels"-. Students focus on extending their conceptual understanding of proportional relationships and direct variation to include inverse relationships. Students will use manipulatives, completed charts, and graphs to further their understanding.

Common Misconceptions:

Students may confuse the significance of the numerator compared to the denominator.

Students may believe that the denominator with a greater digit automatically has a greater value than a fraction with a lesser denominator, e.g. , $\frac{1}{8} > \frac{1}{3}$.

Students may rely on one configuration for setting up proportions without realizing that other configurations may also be correct (*within ratios* and *between ratios*).

Students may have difficulty calculating unit rate, recognizing unit rate when it is graphed on a coordinate plane, and realizing that unit rate is also the slope of a line.

Students may misinterpret or not have mastery of the precise meanings and appropriate use of ratio and proportion vocabulary.

Students may miscomprehend the difference between additive reasoning versus multiplicative reasoning.

Students may compute the unit rate as $\frac{x}{y}$ instead of $\frac{y}{x}$. Discuss the different meanings between $\frac{1}{3}$ wall/gallon and 3 gallons/wall. Students should write the correct unit rate for the ratio table.

Domain: Ratios and Proportional Relationships (RP)

▶ ***Cluster**: Analyze proportional relationships and use them to solve real-world and mathematical problems*

Standard: Grade 7.RP.2

Recognize and represent proportional relationships between quantities:

7.RP.2a. Determine whether two quantities are in a proportional relationship*, e.g. by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.* **(7.RP.2a)**

7.RP.2b. Analyze a table or graph and recognize that, in a proportional relationship, every pair of numbers has the same unit rate (referred to as the "m"). **(7.RP.2b)**

7.RP.2c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$*.* **(7.RP.2c)**

7.RP.2d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. **(7.RP.2d)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 7.RP. 1

Explanations and Examples:

Students' understanding of the multiplicative reasoning used with proportions continues from 6th grade.
Students determine if two quantities are in a proportional relationship from a table.
For example, the table below gives the price for different number of books.
Do the numbers in the table represent a proportional relationship?

Number of Books	Price
1	3
3	9
4	12
7	18

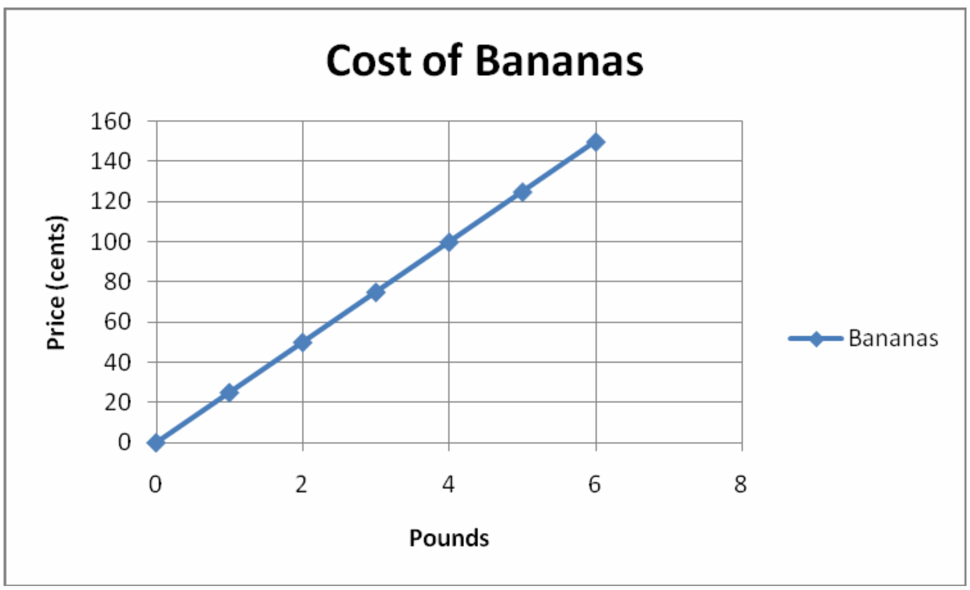
Students can examine the numbers to see that 1 book at 3 dollars is equivalent to 4 books for 12 dollars since both sides of the tables can be multiplied by 4. However, the 7 and 18 are not proportional since 1 book multiplied by 7 and 3 dollars multiplied by 7 will not give 7 books for 18 dollars. Seven books for $18 is not proportional to the other amounts in the table; it does not have the same unit rate.

Students graph relationships to determine if two quantities are in a proportional relationship and interpret the ordered pairs. If the amounts from the table above are graphed (number of books, price), the pairs (1, 3), (3, 9), and (4, 12) will form a straight line through the origin (0 books cost 0 dollars), indicating that these pairs are in a proportional relationship. The ordered pair (4, 12) means that 4 books cost $12. However, the ordered pair (7, 18) would not be on the line, indicating that it is not proportional to the other pairs.

The ordered pair (1, 3) indicates that 1 book is $3, which is the unit rate. The *y*-coordinate when $x = 1$ will be the unit rate. Students identify this amount from tables (see example above), graphs, equations and verbal descriptions of proportional relationships.

The graph below represents the price of the bananas at one store. What is the unit rate? From the graph, it can be determined that 4 pounds of bananas is $1.00; therefore, 1 pound of bananas is $0.25, the unit rate for the graph.

Note: Any point on the graph will yield this unit rate.



The cost of bananas at another store can be determined by the equation: $P = \$0.35n$, where P is the price and n is the number of pounds. What is the unit rate?
Students write equations from context and identify the coefficient as the unit rate.

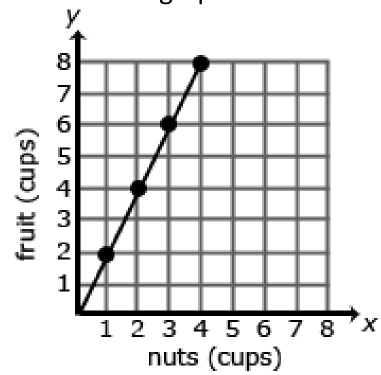
Note: This standard focuses on the representations of proportions. Solving proportions is addressed in **7.RP.3**.

Students may use a content web site and/or interactive white board to create tables and graphs of proportional or non-proportional relationships. Graphing proportional relationships represented in a table helps students recognize that the graph is a line through the origin (0,0) with a unit rate equal to the slope of the line.

Examples:

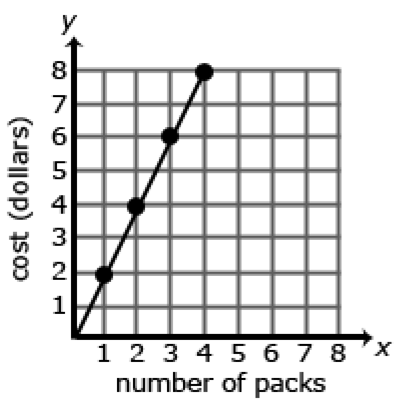
A student is making trail mix. Create a graph to determine if the quantities of nuts and fruit are proportional for each serving size listed in the table. If the quantities are proportional, what is the unit rate that defines the relationship? Explain how you determined the unit rate and how it relates to both the table and graph.

Serving Size	**1**	**2**	**3**	**4**
Cups of Nuts (*x*)	1	2	3	4
Cups of Fruit (*y*)	2	4	6	8



The relationship is proportional. For each of the other serving sizes there are 2 cups of fruit for every 1 cup of nuts (2:1). The unit rate is shown in the first column of the table and by the slope of the line on the graph.

The graph below represents the cost of gum packs as a unit rate of $2 dollars for every pack of gum. The unit rate is represented as $2 per pack. Represent the relationship using a table and an equation.



Solution:

Table:

Number of Packs of Gum	**Cost in Dollars**
0	0
1	2
2	4
3	6
4	8

Equation: $d = 2g$, where *d* is the cost in dollars and g is the packs of gum.

A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using x and y. Constructing verbal models can also be helpful. A student might describe the situation as “the number of packs of gum times the cost for each pack is the total cost in dollars”. They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table.

The checking process helps student revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost $g \times 2 = d$.

Tools/Resources:

For detailed information see Progressions for the Common Core State Standards in Mathematics: 3-5 Number and Operations – Fractions.

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.RP.A.2
  - Music Companies, Variation 1
  - Art Class, Variation 1
  - Art Class, Variation 2
  - Buying Coffee
  - Sore Throats, Variation 1
  - Robot Races
  - Robot Races, Assessment Variation
  - Art Class, Assessment Variation
  - Buying Bananas, Assessment Version
  - Walk-a-thon 2
- 7.RP.A.2.c
  - Proportionality

Georgia Department of Education website:

- "Walking to Scoops"-. Students use a real-world scenario to explore the walking rate on time and distance traveled.
- "The Final Challenge"- Students construct plane figures to create a regular octagon using tools and construction techniques.
- "Similar Triangles"- Students use an object perpendicular to the ground and measurement tool and their shadow to determine height of objects.

NCTM Illuminations – NCTM has many great resources available to educators, some of these resources (i.e. interactives) are open to any educator while others (i.e. lessons) require an individual or institutional membership. If you find that a resource referenced in the flip books requires membership access, check with your school/district to see if they have an institutional membership which would grant you access all NCTM documents. If they do not have a membership, this would be a valuable resource to request.

- "Feeding Frenzy", Illuminations Lesson-students multiply and divide a recipe to feed groups of various sizes. Students will use unit rates and proportions and think critically about real world applications of a backing problem.

Domain: Ratios and Proportional Relationships (RP)

▶**Cluster**: *Analyze proportional relationships and use them to solve real-world and mathematical problems.*

Standard: Grade 7.RP.3

Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error*. **(7.RP.3)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 7.RP.1

Explanations and Examples:

In 6th grade, students used ratio tables and unit rates to solve problems. Students expand their understanding of proportional reasoning to solve problems that are easier to solve with cross-multiplication. Students understand the mathematical foundation for cross-multiplication.

For example, a recipe calls for $\frac{3}{4}$ teaspoon of butter for every 2 cups of milk.
If you increase the recipe to use 3 cups of milk, how many teaspoons of butter are needed?
Using these numbers to find the unit rate may not be the most efficient method. Students can set up the following proportion to show the relationship between butter and milk.

$$\frac{\frac{3}{4}}{2} = \frac{x}{3}$$

The use of proportional relationships is also extended to solve percent problems involving tax, markups and markdowns simple interest ($I = prt, I = interest, p = principle, r = rate, and\ t = time$), gratuities and commissions, fees, percent increase and decrease, and percent error.

For example, Games Unlimited buys video games for $10. The store increases the price 300%? What is the price of the video game?

> Using proportional reasoning, if $10 is 100% then what amount would be 300%? Since 300% is 3 times 100%, $30 would be $10 times 3. Thirty dollars represents the amount of increase from $10 so the new price of the video game would be $40.

Finding the percent error is the process of expressing the size of the error (or deviation) between two measurements. To calculate the percent error, students determine the absolute deviation (positive difference) between an actual measurement and the accepted value and then divide by the accepted value. Multiplying by 100 will give the percent error.

$$\%\ error = \frac{|your\ result - accepted\ value|}{accepted\ value} \times 100\%$$

For example, you need to purchase a countertop for your kitchen. You measured the countertop as 5 ft. The actual measurement is 4.5 ft. What is the percent error?

$$\%\ error = \frac{|5ft - 4.5ft|}{4.5} \times 100$$

$$\%\ error = \frac{|0.5ft|}{4.5} \times 100$$

Students should be able to explain or show their work using a representation (numbers, words, pictures, physical objects, or equations) and verify that their answer is reasonable. Models help students to identify the parts of the problem and how the values are related. For percent increase and decrease, students identify the starting value, determine the difference, and compare the difference in the two values to the starting value.

Examples:
Gas prices are projected to increase 124% by April 2015. A gallon of gas currently costs \$4.17. What is the projected cost of a gallon of gas for April 2015?

A student might say: "The original cost of a gallon of gas is \$4.17. An increase of 100% means that the cost will double. I will also need to add another 24% to figure out the final projected cost of a gallon of gas. Since 25% of \$4.17 is about \$1.04, the projected cost of a gallon of gas should be around \$9.40."

$$\$4.17 + \$4.17 + (0.24 \cdot 4.17) = 2.24 \times 4.17$$

100%	100%	24%
\$4.17	\$4.17	?

A sweater is marked down 33%. Its original price was \$37.50.
What is the price of the sweater before sales tax?

\$37.50 Original Price of Sweater	
33% of \$37.50	67% of \$37.50 Sale Price of sweater

The discount is 33% times 37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or Sale Price = 0.67 x Original Price.

A shirt is on sale for 40% off. The sale price is $12. What was the original price?
What was the amount of the discount?

Discount 40% of Original Price	Sale Price - $12 60% of original price
Original Price (p)	

At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs and is going to give all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify your solution.

A salesperson set a goal to earn $2,000 in May. He receives a base salary of $500 as well as a 10% commission for all sales. How much merchandise will he have to sell to meet his goal?

After eating at a restaurant, your bill before tax is $52.50. The sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount. How much is the tip you leave for the waiter? How much will the total bill be, including tax and tip? Express your solution as a multiple of the bill.

$$\textit{The Amount Paid} = 0.20 \times \$52.50 + 0.08 \times \$52.50 = 0.28 \times \$52.50$$

Instructional Strategies: See 7.RP.1

Tools & Resources:

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.RP.A
  - Stock Swaps, Variation 2
  - Sale!
- 7.RP.A.3
  - Buying Protein Bars and Magazines
  - Chess Club
  - Comparing Years
  - Friends meeting on bikes
  - Music Companies, Variation 2
  - Selling Computers
  - Tax and Tip
  - Sand Under the Swing Set

Domain: The Number System (NS)

▶**Cluster:** *Apply and extend previous understandings of operations with positive rational numbers to add, subtract, multiply, and divide all rational numbers.*

Standard: Grade 7.NS.1

Represent addition and subtraction on a horizontal or vertical number line diagram.

7.NS.1a. Describe situations in which opposite quantities combine to make 0. Show that a number and its opposite have a sum of 0 (are additive inverses). *For example, show zero-pairs with two-color counters.* **(7.NS.1a)**

7.NS.1b. Show $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. **(7.NS.1b)**

7.NS.1c. Model subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. **(7.NS.1c)**

7.NS.1d. Model subtraction as the distance between two rational numbers on the number line where the distance is the absolute value of their difference. **(7.NS.1c)**

7.NS.1e. Apply properties of operations as strategies to add and subtract rational numbers. **(7.NS.1d)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.6 Attend to precision
- ✓ MP.7 Look for and make use of structure

Connections:

This cluster is connected to:

- 6.NS.5- Students are introduced to the sign signifying a direction on a number line and in the coordinate plane and the definition for opposite numbers.
- 6.NS.6b- Students learn how to position integers on a horizontal and vertical number line.
- 6.NS.7c- Students' understand that the absolute value of a number is its distance from zero on the number line.
- ▶Grade 7 Critical Area of Focus #2: Developing understanding of operations with rational numbers and working with expressions and linear equations.

Explanations and Examples:

Students add and subtract rational numbers using a number line. For example, to add $-5 + 7$, students would find -5 on the number line and move 7 in a positive direction (to the right). The stopping point of 2 is the sum of this expression. *Students also add negative fractions and decimals and interpret solutions in given contexts.*

Visual representations may be helpful as students begin this work; they become less necessary as students become more fluent with the operations.

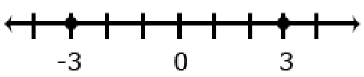
Examples:

Use a number line to illustrate:

$$p - q$$
$$p + (-q)$$

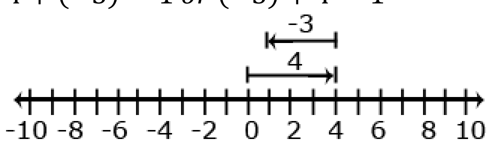
Is this equation true: $p - q = p + (-q)$

-3 and 3 are shown to be opposites on the number line because they are equal distance from zero and therefore have the same absolute value and the sum of the number and it's opposite is zero.



You have \$4 and you need to pay a friend \$3. What will you have after paying your friend?

$4 + (-3) = 1 \; or \; (-3) + 4 = 1$



Name a number that makes each statement true. Justify your solution by showing a model or providing an explanation.

- $-4.8 + ? = a\ positive\ number$
- $? - \frac{3}{2} = a\ negaitve\ number$
- $-2.15 - ? = a\ negative\ number$

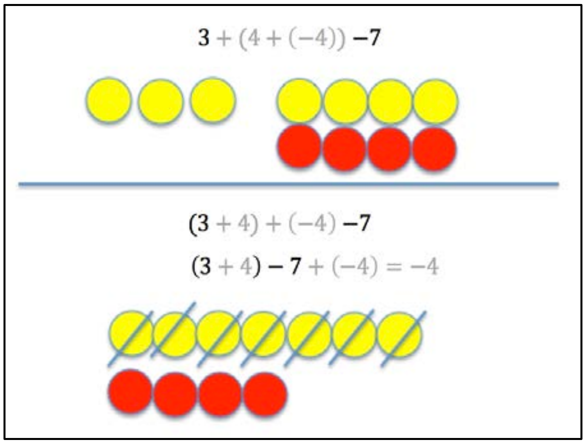
Instructional Strategies:

This cluster (7.NS.1-3) builds upon the understandings of rational numbers in Grade 6:

- quantities can be shown using + or – as having opposite directions or values,
- points on a number line show distance and direction,
- opposite signs of numbers indicate locations on opposite sides of 0 on the number line,
- the opposite of an opposite is the number itself,
- the absolute value of a rational number is its distance from 0 on the number line,
- the absolute value is the magnitude for a positive or negative quantity, and
- locating and comparing locations on a coordinate grid by using negative and positive numbers.

Learning now moves to exploring and ultimately formalizing rules for operations (addition, subtraction, multiplication and division) with integers. Using both contextual and numerical problems, students should explore what happens when negatives and positives are combined.

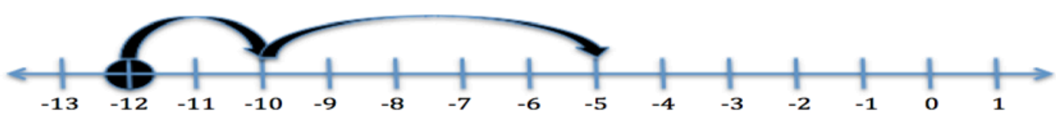
Two-color counters or colored chips can be used as a physical and kinesthetic model for adding and subtracting integers. With one color designated to represent positives and a second color for negatives, addition/subtraction can be represented by placing the appropriate numbers of chips for the addends and their signs on a board. Using the notion of opposites, the board is simplified by removing pairs of opposite colored chips. The answer is the total of the remaining chips with the sign representing the appropriate color. Repeated opportunities over time will allow students to compare the results of adding and subtracting pairs of numbers, leading to the generalization of the rules.



Additionally, it is important for students to understand that adding zero pairs to the modeled situation does not change the value. For example, $3-7=$ can be modeled to reinforce the concept of subtraction as "taking away" by adding zero pairs until you have enough positive counters to allow removal of 7 positive counters.

Number lines present a visual image for students to explore and record addition and subtraction results. Fractional rational numbers and whole numbers should be used in computations and explorations and a number line provides a more flexible representation than counters.

Applying properties of operations as strategies can be practiced using well-structured number talks. As the student explains the strategy used, the teacher highlights the property. For example, $-12+7=$ can be modeled with a number line as below.



Math Representation	Student Explanation	Teacher Restatement
-12+7= -12+(2+5)= (-12+2)+5= -10+5=-5	I knew I could jump -10 by adding 2 so I decomposed 7 into 2+5. Then I added -10 and 5 to get -5.	Great job. First you decomposed 7 into 2+5. Then you used the associative property to regroup and solve -12+2. This created an easier problem to solve mentally -10+5

Math Representation	Student Explanation	Teacher Restatement
-12+7= (-10+-2)+(2+5)= -10+(-2+2)+5= -10+5=-5	I saw that I could take a -2 from -12 and a positive 2 from 7 to make a zero pair. This made an easier problem for me to solve, -10+5.	That is a great observation! You decomposed each of the numbers to make an easier problem. First, you decomposed -12 to make (-10+-2). Then you decomposed 7 to make (2+5). Regrouping using the associative property, you found a set of zero pairs. Now you are at the same place as the other student.

Students should be able to give contextual examples of integer operations, write and solve equations for real- world problems and explain how the properties of operations apply. Real-world situations could include: profit/loss, money, weight, sea level, debit/credit, football yardage, etc.

Resources/Tools:

For detailed information, see Progressions for the Common Core State Standards in Mathematics: Number System 6-8.

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.NS.A.1
  - Comparing Freezing Points
  - Operations on the number line
  - Distances on the Number Line 2
  - Bookstore Account
  - Rounding and Subtracting
  - Distances Between Houses
  - Differences and Distances
  - Differences of Integers

Domain: The Number System (NS)

▶**Cluster:** *Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.*

Standard: Grade 7.NS.2

Apply and extend previous understandings of multiplication and division of positive rational numbers to multiply and divide all rational numbers.

7.NS.2a. Describe how multiplication is extended from positive rational numbers to all rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. **(7.NS.2a)**

7.NS.2b. Explain that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. Leading to situations such that if *p* and *q* are integers, then $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$. **(7.NS.2b)**

7.NS.2c. Apply properties of operations as strategies to multiply and divide rational numbers. **(7.NS.2c)**

7.NS.2d. Convert a rational number in the form of a fraction to its decimal equivalent using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **(7.NS.2d)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.6 Attend to precision

Connections: See 7.NS.1

Explanations and Examples:

Students recognize that when division of rational numbers is represented with a fraction bar, each number can have a negative sign. Using long division from elementary school, students understand the difference between terminating and repeating decimals. This understanding is foundational for work with rational and irrational numbers in 8th grade. For example, identify which fractions will terminate (the denominator of the fraction in reduced form only has factors of 2 and/or 5.)

Multiplication and division of integers is an extension of multiplication and division of whole numbers.

Using what students already know about positive and negative whole numbers and multiplication with its relationship to division, students should generalize rules for multiplying and dividing rational numbers. Multiply or divide the same as for positive numbers, then designate the sign according to the number of negative factors. Students should analyze and solve problems leading to the generalization of the rules for operations with integers. For example, beginning with known facts, students predict the answers for related facts, keeping in mind that the properties of operations apply (See Tables 1, 2 and 3 below).

Table 1	**Table 2**	**Table 3**
4 x 4 = 16	4 x 4 = 16	-4 x -4 = 16
4 x 3 = 12	4 x 3 = 12	-4 x -3 = 12
4 x 2 = 8	4 x 2 = 8	-4 x -2 = 8
4 x 1 = 4	4 x 1 = 4	-4 x -1 = 4
4 x 0 = 0	4 x 0 = 0	-4 x 0 = 0
4 x -1 =	-4 x 1 =	-1 x - 4 =
4 x - 2 =	-4 x 2 =	-2 x - 4 =
4 x - 3 =	-4 x 3 =	-3 x - 4 =

Using the language of "the opposite of" helps some students understand the multiplication of negatively signed numbers ($-4 \times -4 = 16$, the opposite of 4 groups of -4). Discussion about the tables should address the patterns in the products, the role of the signs in the products and commutativity of multiplication. Then students should be asked to answer these questions and prove their responses:

- Is it always true that multiplying a negative factor by a positive factor results in a negative product?
- Does a positive factor times a positive factor always result in a positive product?
- What is the sign of the product of two negative factors?
- When three factors are multiplied, how is the sign of the product determined?
- How is the numerical value of the product of any two numbers found?

Students can use number lines with arrows and hops, groups of colored chips or logic to explain their reasoning. When using number lines, establishing which factor will represent the length, number and direction of the hops will facilitate understanding. Through discussion, generalization of the rules for multiplying integers would result.

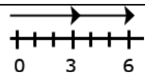
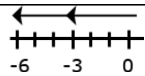
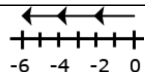
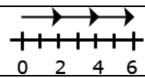
Division of integers is best understood by relating division to multiplication and applying the rules. In time, students will transfer the rules to division situations. (Note: In 2b, this algebraic language $\left(-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}\right)$ is written for the teacher's information, not as an expectation for students.)

Ultimately, students should solve other mathematical and real-world problems requiring the application of these rules with fractions and decimals.

In Grade 7, the awareness of rational and irrational numbers is initiated by observing the result of changing fractions to decimals. Students should be provided with families of fractions, such as, sevenths, ninths, thirds, etc. to convert to decimals using long division. The equivalents can be grouped and named (terminating or repeating). Students should begin to see why these patterns occur. Knowing the formal vocabulary *rational* and *irrational* is not expected

Examples

Examine the family of equations. What pattern do you see?
Create a model and context for each of the products.
Write and model the family of equations related to $2 \times 3 = 6$.

Equation	Number Line Model	Context
$2 \times 3 = 6$		Selling two packages of apples at $3.00 per pack
$2 \times -3 = -6$		Spending 3 dollars each on 2 packages of apples
$-2 \times 3 = -6$		Owing 2 dollars to each of your three friends
$-2 \times -3 = 6$		Forgiving 3 debts of $2.00 each

Instructional Strategies:

Instruction needs to focus on developing understanding of operations with rational numbers. Students need multiple opportunities to develop a unified understanding of number, recognizing fractions, decimals (that have a finite or repeating decimal representation), and percent as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g. amounts owed or temperatures below zero), students explain and interpret the rules for "operating" with negative numbers.

In Grade 7 the awareness of rational and irrational numbers is initiated by observing the result of changing fractions to decimals. Students should be provided with families of fractions, such as, sevenths, ninths, thirds, etc. to convert to decimals using long division. The equivalents can be grouped and named (terminating or repeating). Students should begin to see why these patterns occur. Knowing the formal vocabulary *rational* and *irrational* is not expected.

Tools/Resources:

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.NS.A.2.d
  - Equivalent fractions approach to non-repeating decimals
  - Repeating decimal as approximation
  - Decimal Expansions of Fractions

Common Misconceptions:

- Students may incorrectly use integer rules
- Students may confuse the absolute value symbol with the number one
- Students may incorrectly use the additive inverse when working with operations of integers
- Students may have confusion and misapplication of a complex fractions
- Students may think that a number divided by zero is zero rather than undefined

Domain: The Number System (NS)

Cluster: *Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.*

Standard: Grade 7.NS.3

Solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) **(7.NS.3)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 7.NS.2

Explanations and Examples:

Students use order of operations from sixth grade to write and solve problems with all rational numbers.

Examples:

Your cell phone bill is automatically deducting $32 from your bank account every month.
How much will the deductions total for the year?

$$-32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 + -32 = 12(-32)$$

It took a submarine 20 seconds to drop to 100 feet below sea level from the surface.
What was the rate of the descent?

$$\frac{-100\ feet}{20\ seconds} = \frac{-5\ feet}{1\ second} = -5\ ft/sec$$

The three seventh grade classes at Ft. Riley Middle School collected the most box tops for a school fundraiser, and so they won a $600 prize to share between them. Mrs. Molt's class collected 3,760 box tops, Mrs. Johnson's class collected 2,301, and Mr. Handlos' class collected 1,855. How should they divide the money so that each class gets the same fraction of the prize money as the fraction of the box tops that they collected?

A teacher might start out by asking questions like, "Which class should get the most prize money?
Should Mrs. Molt's class get more or less than half of the money?
Mrs. Molt's class collected about twice as many box tops as Mr. Handlos' class - does that mean that
Mrs. Molt's class will get about twice as much prize money as Mr. Canyon's class?"

This task represents an opportunity for students to engage in Standard MP.5 *Use appropriate tools strategically.* There is little benefit in students doing the computations by hand (few adults would), and so provides an opportunity to discuss the value of having a calculator and when it is (and is not) appropriate to use it.

***Sample Solution*:**
All together, the students collected $3{,}750 + 2{,}301 + 1{,}855 = 7{,}916$ box tops.
Mr. Molt's class collected $\frac{3760}{7916}$of the box tops.
The amount for Mrs. Molt's class is $\left(\frac{3760}{7916}\right) 600 \approx 284.99$
Mrs. Johnson's class collected $\frac{2301}{7916}$of the box tops.
The amount for Mrs. Johnson's class is $\left(\frac{2301}{7916}\right) 600 \approx 174.41$
Mr. Handlos' class collected $\frac{1855}{7916}$of the box tops.
The amount for Mr. Handlos' class is $\left(\frac{1855}{7916}\right) 600 \approx 140.60$
\$284.99 should go to Mrs. Molt's class, \$174.41 should go to Mrs. Johnson's class, and \$140.60 should go to Mr. Handlos' class.

Instructional Strategies: See 7.NS. 1-2
See Also: Number Systems (Grade 6-8) and Number High School

Illustrative Mathematics tasks:
- "Sharing Prize Money":

For detailed information, see Progressions for the Common Core State Standards in Mathematics: Number System 6-8.

Domain: Expressions and Equations (EE)

▶**Cluster***:* *Use properties of operations to generate equivalent expressions.*

Standard: **Grade 7.EE.1**

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with integer coefficients. *For example: apply the distributive property to the expression* $24x + 18y$ *to produce the equivalent expression* $6(4x + 3y)$. **(7.EE.1)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure

Connections: See 7.EE.1-2

This cluster is connected to:

- ▶Grade 7 Critical Area of Focus #2: **Developing understanding of operations with rational numbers and working with expressions and linear equations**.
- 6.EE.3-4 Students used properties of operations to write equivalent expressions.
- The concepts in this cluster build from Operations and Algebraic Thinking -write and interpret numerical expressions from Grade 5 and provides the foundation for equation work in Grade 8.
- It also assists in building the foundational work for writing equivalent non-linear expressions in the High School Conceptual Category - Algebra.

Explanations and Examples:

This is a continuation of work from 6^{th} grade using properties of operations and combining like terms. Students apply properties of operations and work with rational numbers (integers and positive / negative fractions and decimals) to write equivalent expressions.

Examples:

Write an equivalent expression for $3(x + 5) - 2$.

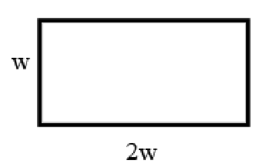
Suzanne thinks the two expressions $2(3a - 2) + 4a$ and $10a - 2$ are equivalent? Is she correct? Explain why or why not?

Write equivalent expressions for: $3a + 12$.

Possible solutions might include factoring as in $3(a + 4)$, or other expressions such as $a + 2a + 7 + 5$.

- A rectangle is twice as long as wide. One way to write an expression to find the perimeter would be $w + w + 2w + 2w$. Write the expression in two other ways.

Solution: $6w$ or $2(w) + 2(2w)$.



An equilateral triangle has a perimeter of $6x + 15$. What is the length of each of the sides of the triangle?

Solution: $3(2x + 5)$, therefore each side is $2x + 5$ units long.

For numbers 1a–1e, select Yes or No to indicate whether each of these expressions is equivalent to $2(2x + 1)$.

1a.	$4x + 2$	Yes	No
1b.	$2(1 + 2x)$	Yes	No
1c.	$2(2x) + 1$	Yes	No
1d.	$2x + 1 + 2x + 1$	Yes	No
1e.	$x + x + x + x + 1 + 1$	Yes	No

Solution:
1a. Y - Equivalent by distributive property
1b. Y - Equivalent by commutative property
1c. N - Not equivalent by misapplying distributive property
1d. Y - Equivalent by understanding 2 as a factor
1e. Y - Equivalent by understanding 2 as a factor and distributive property $2x = (x + x)$

Instructional Strategies:
Have students build on their understanding of order of operations and use the properties of operations to rewrite equivalent numerical expressions that were developed in Grade 6. Students continue to use properties that were initially used with whole numbers and now develop the understanding that properties hold for integers, rational and real numbers.

Provide opportunities to build upon this experience of writing expressions using variables to represent situations and use the properties of operations to generate equivalent expressions. These expressions may look different and use different numbers, but the values of the expressions are the same.

One method that students can use to become convinced that expressions are equivalent is by substituting a numerical value for the variable and evaluating the expression. For example $5(3 + 2x)$is equal to $5 \cdot 3 + 5 \cdot 2x$ $Let\ x = 6$ and substitute 6 for *x* in both equations.

5(3 + 2•6)	5•3 + 5•2•6
5(3 + 12)	15 + 60
5(15)	75
75	

Another method students can use to become convinced that expressions are equivalent is to justify each step of simplification of an expression with an operation property. These include: the commutative, associative, identity, and inverse properties of addition and of multiplication, and the zero property of multiplication.

Tools/Resources:

For detailed information, see Learning Progressions for Expressions and Equations.

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.EE.A
    - Miles to Kilometers
    - Equivalent Expressions?
- 7.EE.A.1
    - Writing Expressions

Common Misconceptions:

As students begin to build and work with expressions containing more than two operations, students tend to set aside the order of operations.

For example, having a student simplify an expression like $8 + 4(2x - 5) + 3x$ can bring to light several misconceptions.

- Do the students immediately add the 8 and 4 before distributing the 4?
- Do they only multiply the 4 and the 2x and not distribute the 4 to both terms in the parenthesis?
- Do they collect all like terms $8 + 4 – 5$, and $2x + 3x$?

Each of these show gaps in students' understanding of how to simplify numerical expressions with multiple operations.

Domain: Expressions and Equations (EE)

▶**Cluster:** *Use properties of operations to generate equivalent expressions.*

Standard: Grade 7.EE.2

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* **(7.EE.2)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See 7.EE.1

Explanations and Examples:

Students understand the reason for rewriting an expression in terms of a contextual situation. For example, students understand that a 20% discount is the same as finding 80% of the cost (.80c).

All varieties of a brand of cookies are \$3.50. A person buys 2 peanut butter, 3 sugar and 1 chocolate. Instead of multiplying $2 \times \$3.50$ to get the cost of the peanut butter cookies, $3 \times \$3.50$to get the cost of the sugar cookies and $1 \times \$3.50$ for the chocolate cookies and then adding those totals together. Students recognize that multiplying \$3.50 times 6 will give the same total.

Examples:

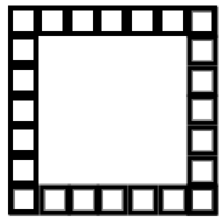
Jamie and Ted both get paid an equal hourly wage of \$9 per hour. This week, Ted made an additional \$27 dollars in overtime. Write an expression that represents the weekly wages of both if J = the number of hours that Jamie worked this week and T = the number of hours Ted worked this week? Can you write the expression in another way?

Students may create several different expressions depending upon how they group the quantities in the problem.
Possible student responses:

- To find the total wage, I would first multiply the number of hours Jamie worked by 9. Then I would multiply the number of hours Ted worked by 9. I would add these two values with the \$27 overtime to find the total wages for the week. The student would write the expression $9J + 9T + 27$.
- To find the total wages, I would add the number of hours that Ted and Jamie worked. I would multiply the total number of hours worked by 9. I would then add the overtime to that value to get the total wages for the week. The student would write the expression $9(J + T) + 27$.
- To find the total wages, I would need to figure out how much Jamie made and add that to how much Ted made for the week. To figure out Jamie's wages, I would multiply the number of hours she worked by 9. To figure out Ted's wages, I would multiply the number of hours he worked by 9 and then add the \$27 he earned in overtime. My final step would be to add Jamie and Ted wages for the week to find their combined total wages. The student would write the expression $(9J) + (9T + 27)$.

Given a square pool as shown in the picture, write four different expressions to find the total number of tiles in the border. Explain how each of the expressions relates to the diagram and demonstrate that the expressions are equivalent.

Which expression do you think is most useful? Explain your thinking.



Instructional Strategies: 7.EE.3**;** (See 7.EE.1)

Provide opportunities for students to experience expressions for amounts of increase and decrease. In Standard 2, the expression is rewritten and the variable has a different coefficient. In context, the coefficient aids in the understanding of the situation. Another example is this situation which represents a 10% decrease: $b - 0.10b = 1.00b - 0.10b$ which equals $0.90b \; or \; 90\%$ of the amount.

Resources/Tools:

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.EE.A.2
  - Ticket to Ride

Common Misconceptions: See 7.EE.1

Domain: Expressions and Equations (EE)

▶***Cluster****:* *Solve real-life and mathematical problems using numerical and algebraic expressions and equations.*

Standard: Grade 7.EE.3

Solve multi-step real-life and mathematical problems with rational numbers. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional* $\frac{1}{10}$ *of her salary an hour, or $2.50, for a new salary of $27.50.* **(7.EE.3)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

- ▶Grade 7 Critical Area of Focus #2: **Developing understanding of operations with rational numbers and working with expressions and linear equations**
- ● Critical Area of Focus #3: **solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume**.
- 7.NS.3- This standards connect naturally with the work done solving real-world and mathematical problems involving the four operations.

Explanations and Examples:

Students solve contextual problems using rational numbers. Students convert between fractions, decimals, and percents as needed to solve the problem. Students use estimation to justify the reasonableness of answers.

Estimation strategies for calculations with fractions and decimals extend from students' work with whole number operations.

Estimation strategies include, but are not limited to:

- front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts),
- clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate),
- rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values),
- using friendly or compatible numbers such as factors (students seek to fit numbers together - i.e., rounding to factors and grouping numbers together that have round sums like 100 or 1000),
- using benchmark numbers that are easy to compute (student's select close whole numbers for fractions or decimals to determine an estimate).

Examples:

The youth group is going on a trip to the state fair. The trip costs $52. Included in that price is $11 for a concert ticket and the cost of 2 passes, one for the rides and one for the game booths.

Each of the passes cost the same price. Write an equation representing the cost of the trip and determine the price of one pass.

x	x	11
52		

$$2x + 11 = 52$$
$$2x = 41$$
$$x = \$20.50$$

Renee, Susan, and Martha will share the cost to rent a vacation house for a week.

- Renee will pay 40% of the cost.
- Susan will pay 0.35 of the cost.
- Martha will pay the remainder of the cost.

Part A

Martha thinks she will pay $\frac{1}{3}$ of the cost. Is Martha correct?

Use mathematics to justify your answer.

Part B

The cost to rent a vacation house for a week is $850. How much will Renee, Susan, and Martha each pay to rent this house for a week?

Sample Response:

Part A: Martha is incorrect. She will pay $\mathbf{\frac{1}{4}}$ of the cost.

$$1 - (40\% + 0.35)$$
$$1 - (0.40 + 0.35)$$
$$1 - 0.75 = 0.25 = \frac{25}{100} = \frac{1}{4}$$

Part B

Renee: $0.40 \times \$850 = \$340$

Susan: $0.35 \times \$850 = \$297.50$

Martha: $0.25 \times \$850 = \$212.50$

When working on a report for class, Catrina read that a woman over the age of 40 can lose approximately 0.06 centimeters of height per year.

a. Catrina's aunt Nancy is 40 years old and is 5 feet 7 inches tall. Assuming her height decreases at this rate after the age of 40, about how tall will she be at age 65? (Remember that 1 inch = 2.54 centimeters.)
b. Catrina's 90-year-old grandmother is 5 feet 1 inch tall. Assuming her grandmother's height has also decreased at this rate, about how tall was she at age 40? Explain your reasoning.

Solution:
Convert the rate of shrinkage to inches per year.
Note that there is a significant amount of rounding in the final answer. This is because people almost never report their heights more precisely than the closest half-inch. If we assume that the heights reported in the task stem are rounded to the nearest half-inch, then we should report the heights given in the solution at the same level of precision.

If a person loses an average of 0.06 cm per year after age 40 and $1\ inch = 2.54\ cm$, after age 40 they lose, on average $0.06 \div 2.54 = 0.024\ inches\ per\ year$.

a. In the 25 years from age 40 to age 65, Nancy could be expected to lose approximately $25 \times 0.024 = 0.6\ inches$. Subtracting this from Nancy's current height, Nancy's height at age 65 could be expected to be approximately $5\ feet, 6\frac{1}{2}\ inches$.
b. In the 55 years from age 40 to age 90, Catrina's grandmother could be expected to lose approximately twice Nancy's loss in height, or 1.2 inches. Adding this to Catrina's grandmother's current height, Catrina's grandmother could be expected to have been approximately 5 *feet,* 2 *inches* tall at age 40.

Instructional Strategies:

To assist students' assessment of the reasonableness of answers, especially problem situations involving fractional or decimal numbers, use whole-number approximations for the computation and then compare to the actual computation. Connections between performing the inverse operation and undoing the operations are appropriate here. It is appropriate to expect students to show the steps in their work. Students should be able to explain their thinking using the correct terminology for the properties and operations.

Continue to build on students' understanding and application of writing and solving one-step equations from a problem situation to multi-step problem situations. This is also the context for students to practice using rational numbers including: integers, and positive and negative fractions and decimals. As students analyze a situation, they need to identify what operation should be completed first, then the values for that computation. Each set of the needed operation and values is determined in order. Finally an equation matching the order of operations is written. *For example, Bonnie goes out to eat and buys a meal that costs $12.50 that includes a tax of $0.75. She only wants to leave a tip based on the cost of the food*. In this situation, students need to realize that the tax must be subtracted from the total cost before being multiplied by the percent of tip and then added back to obtain the final cost.
$C = (12.50 - 0.75)(1 + T) + 0.75 = 11.75(1 + T) + 0.75$ where C = cost and T = tip.

Provide multiple opportunities for students to work with multi-step problem situations that have multiple solutions and therefore can be represented by an inequality. Students need to be aware that values can satisfy an inequality but not be appropriate for the situation, therefore limiting the solutions for that particular problem.

Tools/ Resources:

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.EE.B.3
    - Shrinking
    - Discounted Books
    - Gotham City Taxis
    - Anna in D.C.
    - Who is the better batter?
    - Stained Glass

Common Misconceptions:

As students begin to build and work with expressions containing more than two operations, students tend to set aside the order of operations.

For example having a student simplify an expression like $8 + 4(2x - 5)$ can bring to light several misconceptions.

- Do the students immediately add the 8 and 4 before distributing the 4?
- Do they only multiply the 4 and the 2x and not distribute the 4 to both terms in the parenthesis?
- Do they collect all like terms $8 + 4 - 5$, and $2x + 3x$?

Each of these show gaps in students' understanding of how to simplify numerical expressions with multiple operations.

Domain: Expressions and Equations (EE)

▶**Cluster:** *Solve real-life and mathematical problems using numerical and algebraic expressions and equations.*

Standard: Grade 7.EE.4

Use variables to represent quantities in a real-world or mathematical problem, and construct two-step equations and inequalities to solve problems by reasoning about the quantities.

7.EE.4a. Solve word problems leading to equations of the form $px + q = r$, and $p(x + q) = r$ where *p, q,* and *r* are specific rational numbers. Solve equations of these forms fluently (efficiently, accurately, and flexibly). Compare an algebraic solution to an arithmetic solution, identifying the **sequence** of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* **(7.EE.4a)**

7.EE.4b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$ where *p*, *q*, and *r* are specific rational numbers and $p > 0$. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.* **(7.EE.4b)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 7.EE.3

Explanations and Examples:

Students solve multi-step equations and inequalities derived from word problems. Students use the arithmetic from the problem to generalize an algebraic solution. Students graph inequalities and make sense of the inequality in context. Inequalities may have negative coefficients. Problems can be used to find a maximum or minimum value when in context.

Examples:

Amie had $26 dollars to spend on school supplies. After buying 10 pens, she had $14.30 left.
How much did each pen cost?

The sum of three consecutive even numbers is 48. What is the smallest of these numbers?

Solve: $\frac{5}{4}n + 5 = 20$

Florencia has at most $60 to spend on clothes. She wants to buy a pair of jeans for $22 dollars and spend the rest on t-shirts. Each t-shirt costs $8. Write an inequality for the number of t-shirts she can purchase.

Steven has $25 dollars. He spent $10.81, including tax, to buy a new DVD. He needs to set aside $10.00 to pay for his lunch next week. If peanuts cost $0.38 per package including tax, what is the maximum number of packages that Steven can buy?

Write an equation or inequality to model the situation. Explain how you determined whether to write an equation or inequality and the properties of the real number system that you used to find a solution.

- Solve $\frac{1}{2}x + 3 > 2$ and graph your solution on a number line.

Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can only carry 1200 pounds of people and gear for safety reasons. Assume the average weight of a person is 150 pounds. Each group will require 200 lbs. of gear for the boat plus 10 lbs. of gear for each person.

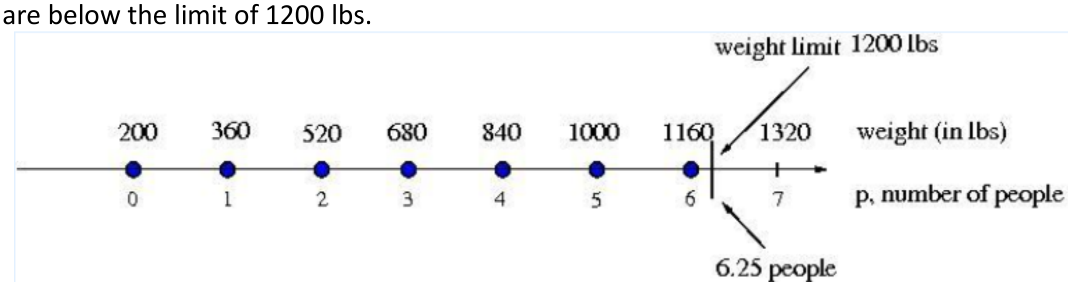
a. Create an inequality describing the restrictions on the number of people possible in a rented boat. Graph the solution set.
b. Several groups of people wish to rent a boat. Group 1 has 4 people. Group 2 has 5 people. Group 3 has 8 people. Which of the groups, if any, can safely rent a boat? What is the maximum number of people that may rent a boat?

Sample Solution:

a. Let *p* be the number of people in a group that wishes to rent a boat. Then *150p* represents the total weight of the people in the boat, in pounds. Also, *10p* represents the weight of the gear that is needed for each person on the boat. So the total weight in the boat that is contributed solely by the people is $150p + 10p = 160p$.

   Because each group requires 200 pounds of gear regardless of how many people there are, we add this to the above amount. We also know that the total weight cannot exceed 1,200 pounds.
   So we arrive at the following inequality: $160p + 200 < 1200$

   One possible graph illustrating the solutions is shown below. We observe that our solutions are values of *p* listed below the number line and shown by the dots, so that the corresponding weights 160*p*+200, listed above the line, are below the limit of 1200 lbs.



b. We can find out which of the groups, if any, can safely rent a boat by substituting the number of people in each group for p in our inequality. We see that:

For Group 1: $160(4) + 200 = 840 < 1200$
For Group 2: $160(5) + 200 = 1000 < 1200$
For Group 3: $160(8) + 200 = 1480 < 1200$

We find that both Group 1 and Group 2 can safely rent a boat, but that Group 3 exceeds the weight limit, and so cannot rent a boat. To find the maximum number of people that may rent a boat, we solve our inequality for *p*.

$$160p + 200 < 1200$$
$$160p < 100$$
$$p < 6.25$$

As we cannot have 0.25 person, we see that 6 is the largest number of people that may rent a boat at once. This also matches our graph; since only integer values of *p* make sense, 6 is the largest value of *p* whose corresponding weight value lies below the limit of 1200 lbs.

Instructional Strategies:
Continue to build on students' understanding and application of writing and solving one-step equations from a problem situation to multi-step problem situations. This is also the context for students to practice using rational numbers including: integers, and positive and negative fractions and decimals. As students analyze a situation, they need to identify what operation should be completed first, then the values for that computation. Each set of the needed operation and values is determined in order. Finally an equation matching the order of operations is written. *For example, Bonnie goes out to eat and buys a meal that costs $12.50 that includes a tax of $.75. She only wants to leave a tip based on the cost of the food*. In this situation, students need to realize that the tax must be subtracted from the total cost before being multiplied by the percent of tip and then added back to obtain the final cost.
$C = (12.50 - 0.75)(1 + T) + 0.75 = 11.75(1 + T) + 0.75$ where C = cost and T = tip.

Provide multiple opportunities for students to work with multi-step problem situations that have multiple solutions and therefore can be represented by an inequality. Students need to be aware that values can satisfy an inequality but not be appropriate for the situation, therefore limiting the solutions for that particular problem.

Tools/ Resources:
See engageNY Modules.

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.EE.B.4
  - Fishing Adventures 2
  - Gotham City Taxis
  - Bookstore Account
- 7.EE.B.4.b
  - Sports Equipment Set

Common Misconceptions: See 7.EE.3

Domain: Geometry (G)

● **Cluster:** *Draw, construct, and describe geometrical figures and describe the relationships between them.*

Standard: Grade 7.G.1

Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **(7.G.1)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

- ● Grade 7 Critical Area of Focus #3: **Solving problems involving scale drawings and informal geometric constructions, and working with two dimensional shapes to solve problems involving area, surface area, and volume**.
- Connections should be made between this cluster and the ● Grade 7 Geometry Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. (● Grade 7.G.4-6)
- ▶ Grade 7 Ratios and Proportional Relationships.

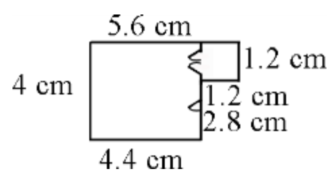
This cluster leads to the development of the triangle congruence criteria in Grade 8.

Explanations and Examples:

Students determine the dimensions of figures when given a scale and identify the impact of a scale on actual length (one-dimension) and area (two-dimensions). Students identify the scale factor given two figures. Using a given scale drawing, students reproduce the drawing at a different scale. Students understand that the lengths will change by a factor equal to the product of the magnitude of the two size transformations.

Examples:

Julie showed you the scale drawing of her room. If each 2 cm on the scale drawing equals 5 ft., what are the actual dimensions of Julie's room? Reproduce the drawing at 3 times its current size.



A company designed two rectangular maps of the same region. These maps are described below.

- *Map 1*: The dimensions are 8 inches by 10 inches. The scale is $\frac{3}{4}$ mile to 1 inch.
- *Map 2*: The dimensions are 4 inches by 5 inches.

Write a ratio that represents the scale on Map 2.

Solution:

$$\frac{3}{4}\ mile\ to\ \frac{1}{2}\ inch$$

Instructional Strategies:

This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometric problems.

Scale drawings of geometric figures connect understandings of proportionality to geometry and lead to future work in similarity and congruence. As an introduction to scale drawings in geometry, students should be given the opportunity to explore scale factor as the number of time you multiple the measure of one object to obtain the measure of a similar object. It is important that students first experience this concept concretely progressing to abstract contextual situations. Pattern blocks (not the hexagon) provide a convenient means of developing the foundation of scale. Choosing one of the pattern blocks as an original shape, students can then create the next-size shape using only those same-shaped blocks. Questions about the relationship of the original block to the created shape should be asked and recorded. A sample of a recording sheet is shown.

Shape	Original Side Length	Created Side Length	Scale Relationship of Created to Original
Square	1 unit		
Triangle	1 unit		
Rhombus	1 unit		

This can be repeated for multiple iterations of each shape by comparing each side length to the original's side length. An extension would be for students to compare the later iterations to the previous. Students should also be expected to use side lengths equal to fractional and decimal parts. For example, if the original side can be stated to represent 2.5 inches, what would be the new lengths and what would be the scale?

Shape	Original Side Length	Created Side Length	Scale Relationship of Created to Original
Square	2.5 inches		
Parallelogram	3.25 centimeters		
Trapezoid	(Actual measurements)	Length 1 Length 2	

Provide opportunities for students to use scale drawings of geometric figures with a given scale that requires them to draw and label the dimensions of the new shape. Initially, measurements should be in whole numbers, progressing to measurements expressed with rational numbers. This will challenge students to apply their understanding of fractions and decimals.

After students have explored multiple iterations with a couple of shapes, ask them to choose and replicate a shape with given scales to find the new side lengths, as well as both the perimeters and areas. Starting with simple shapes and whole-number side lengths allows all students access to discover and understand the relationships. An interesting discovery is the relationship of the scale of the side lengths to the scale of the respective perimeters (same scale) and areas (scale squared). A sample recording sheet is shown.

Shape	Side Length	Scale	Original Perimeter	Scaled Perimeter	Perimeter Scale	Original Area	Scaled Area	Area Scale
Rectangle	2 x 3 in.	2	10 inches	20 inches	2	6 sq. in.	24 sq. in.	4
Triangle	1.5 inches	2	4.5 inches	9 inches	2	2.25 sq. in.	9 sq. in.	4

Students should move on to drawing scaled figures on grid paper with proper figure labels, scale and dimensions. Provide word problems that require finding missing side lengths, perimeters or areas. For example, if a 4 by 4.5 cm rectangle is enlarged by a scale of 3, what will be the new perimeter?

What is the new area? or If the scale is 6, what will the new side length look like? or Suppose the area of one triangle is 16 sq. units and the scale factor between this triangle and a new triangle is 2.5. What is the area of the new triangle?

Reading scales on maps and determining the actual distance (length) is an appropriate contextual situation.

Constructions facilitate understanding of geometry. Provide opportunities for students to physically construct triangles with straws, sticks, or geometry apps prior to using rulers and protractors to discover and justify the side and angle conditions that will form triangles.

Explorations should involve giving students: three side measures, three angle measures, two side measures and an included angle measure, and two angles and an included side measure to determine if a unique triangle, no triangle or an infinite set of triangles results. Through discussion of their exploration results, students should conclude that triangles cannot be formed by any three arbitrary side or angle measures. They may realize that for a triangle to result the sum of any two side lengths must be greater than the third side length, or the sum of the three angles must equal 180 degrees. Students should be able to transfer from these explorations to reviewing measures of three side lengths or three angle measures and determining if they are from a triangle justifying their conclusions with both sketches and reasoning.

Further construction work can be replicated with quadrilaterals, determining the angle sum, noticing the variety of polygons that can be created with the same side lengths but different angle measures, and ultimately generalizing a method for finding the angle sums for regular polygons and the measures of individual angles.

For example, subdividing a polygon into triangles using a vertex (N-2)180° or subdividing polygons into triangles using an interior point $180°N - 360°$ where $N = the\ number\ of\ sides\ in\ the\ polygon.$ An extension would be to realize that the two equations are equal.

Slicing three-dimensional figures helps develop three-dimensional visualization skills. Students should have the opportunity to physically create some of the three-dimensional figures, slice them in different ways, and describe in pictures and words what has been found. For example, use clay to form a cube, then pull string through it in different angles and record the shape of the slices found. Challenges can also be given: “See how many different two-dimensional figures can be found by slicing a cube" or "What three-dimensional figure can produce a hexagon slice?" This can be repeated with other three-dimensional figures using a chart to record and sketch the figure, slices and resulting two-dimensional figures.

Tools/Resources:

See engageNY Modules.

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.G.A.1
  - Floor Plan
  - Map distance

Common Misconceptions:

Students often confuse the vocabulary associated with this domain. Teachers should provide experiences for the explicit discovery of these terms to apply meaning through written, pictorial, and experimental means. They continue to misuse units for distance, area, and volume. This too should be explicitly reviewed from the sixth grade domain.

Student's may have misconceptions about correctly setting up proportions, how to read a ruler, doubling side measures and does not double perimeter.

Domain: Geometry (G)

●**Cluster:** *Draw, construct, and describe geometrical figures and describe the relationships between them.*

Standard: **Grade 7.G.2**

Identify three-dimensional objects generated by rotating a two-dimensional (rectangular or triangular) object around one edge. **(G.GMD.4)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP. 7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 7.G.1

- Building a three-dimensional object from a two-dimensional figure connects with 7.G.3, deconstructing a three-dimensional objection into a two-dimensional figure.

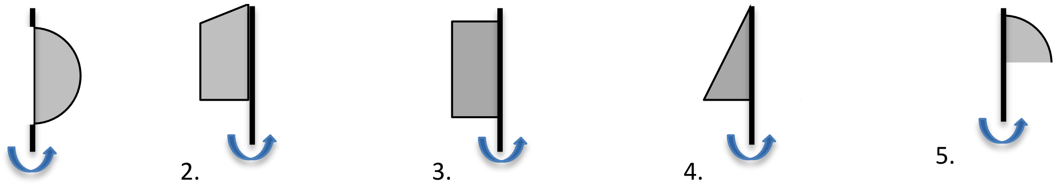
Explanations and Examples:

This standard supports the development of geometric thinking generally and, specifically, helps contribute meaning and vocabulary for the discussion of formulas related to cylinders (7th grade), cones (8th grade), and spheres (8th grade). Generally, "decomposing and rearranging provide a geometric way of both *seeing that* a measurement formula is the right one and *seeing why* it is the right one," according to NCTM's Developing Essential Understanding of Geometry in Grades 6-8. Additionally, this standard will strengthen the student's mental imagery skills. There is a transition and development of imagery skills from elementary, where objects are concrete, to middle school, where students must learn to "see *through* drawn or concrete objects to the underlying geometric one. "Geometric awareness develops through practice in visualizing, diagramming, and constructing."

Beyond the general benefits that come from imagining the creation of a 3-dimensional object through rotation, this standard will help students understand the connection between the height and radius of a cylinder and the rectangle used to generate the cylinder. Once students are in 8th grade, they will be able to visualize the right triangle used to create the cone and connect the hypotenuse to the lateral edge. This connection does not need to be made in 7th grade but questions about where the legs and hypotenuse of the right triangle are in the cone will lay a foundation that will make surface area and volume formulas more understandable.

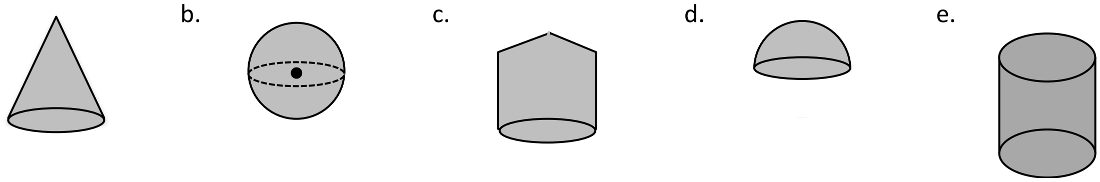
Examples:

Match the solid that would result if you rotated the two-dimensional figure about the axis indicated.



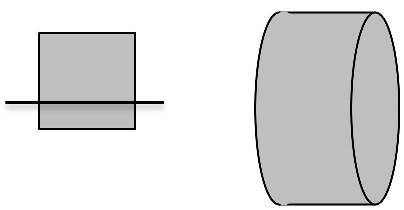
1. 2. 3. 4. 5.

a. b. c. d. e.



Can three-dimensional cube be created by rotating a two-dimensional shape? If so, what two-dimensional shape will create a cube? If not, why? Justify your answer.

A square with an area of $81\ cm^2$ is rotated to form a cylinder. What is the volume of the cylinder?



Instructional Strategies:

Constructions facilitate understanding of geometry. Provide opportunities for students to physically handle three-dimensional solids prior to using geometry apps.

Resources/Tools:

Videos of rotation can be found at http://www.schoolmath.jp/3d/e/student/lesson01/lesson_02.htm

Khan Academy:

- Course-Geometry
- Unit- Solid Geometry
- Topic- 2D vs. 3D Objects
- Lesson- Rotating 2D Shapes in 3D

Domain: Geometry (G)

● **Cluster:** *Draw, construct, and describe geometrical figures and describe the relationships between them.*

Standard: **Grade 7.G.3**

Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right cylinder. **(G.GMD.4)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 7.G.1

Explanations and Examples:

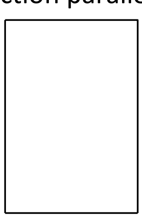
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: **scale drawing, dimensions, scale factor, plane sections, right rectangular prism, right cylinder, parallel, and perpendicular.**

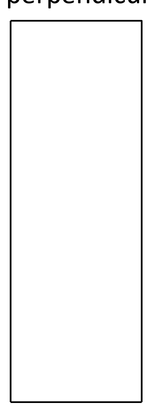
Students need to describe the resulting face shape from cuts made parallel and perpendicular to the bases of right rectangular prisms and cylinders. Cuts made parallel will take the shape of the base; cuts made perpendicular will take the shape of the lateral (side) face. Cuts made at an angle through the right rectangular prism will produce a parallelogram; cuts made at an angle through the right cylinder will also produce an ellipse.

Examples:

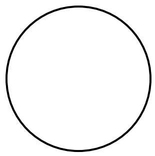
For each of the figures below, sketch a solid that could have the given cross sections.

1. Cross section parallel to a base: Cross section perpendicular to a base:

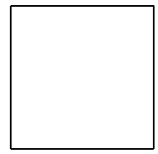




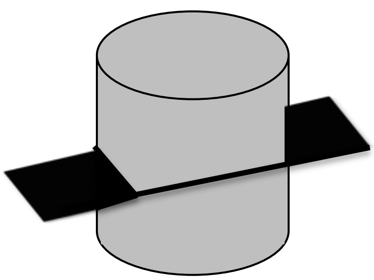
2. Cross section parallel to a base:

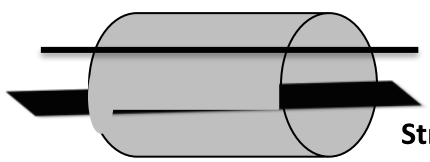


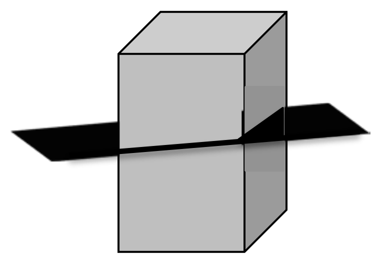
Cross section perpendicular to a base:

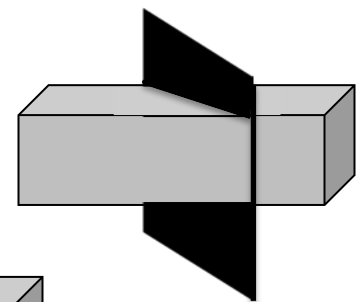


3. Determine the two-dimensional shape that would be created if the three-dimensional shape were sliced as shown.









Instructional Strategies:

This cluster focuses on the importance of visualization in the understanding of Geometry. Being able to visualize and then represent geometric figures on paper is essential to solving geometry problems.

Slicing three-dimensional figures helps develop three-dimensional visualization skills. Students should have the opportunity to physically create some of the three-dimensional figures, slice them in different ways, and describe in pictures and words what has been found. For example, use clay to form a cube, then pull string through it in different angles and record the shape of the slices found.

Challenges can also be given: “See how many different two-dimensional figures can be found by slicing a cube” or “What three-dimensional figure can produce a hexagon slice?” This can be repeated with other three-dimensional figures using a chart to record and sketch the figure, slices and resulting two-dimensional figures.

Resources/Tools:

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.G.A.3
    - Cube Ninjas!

Domain: Geometry (G)

● **Cluster:** *Solve real-life and mathematical problems involving area, surface area, and volume.*

Standard: **Grade 7.G.4**

Use the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **(7.G.4)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

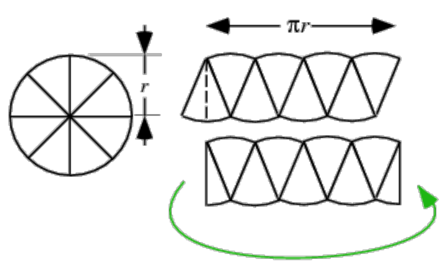
This cluster is connected to:

- ● Grade 7 Critical Area of Focus #3: **Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume**.
- This cluster builds from understandings of Geometry and in Measurement and Data Grades 3-6.
- It also utilizes the scope of the number system experienced thus far and begins the formal use of equations, formulas and variables in representing and solving mathematical situations.

Explanations and Examples:

Students understand the relationship between radius and diameter. Students also understand the ratio of circumference to diameter can be expressed as π. Building on these understandings, students generate the formulas for circumference and area.

The illustration shows the relationship between the circumference and area. If a circle is cut into wedges and laid out as shown below, a parallelogram results. Half of an end wedge can be moved to the other end a parallelogram results. The height of the parallelogram is the same as the radius of the circle. The base length is $\frac{1}{2}$ the circumference$(2\pi r)$. The area of the parallelogram (and therefore the circle) is found by the following calculations:



$$Area\ of\ Parallelogram = Base \times Height$$
$$Area = \frac{1}{2}(2\pi r) \times r$$
$$Area = \pi r^2$$

Explanations and Examples:

Students solve problems (mathematical and real-world) including finding the area of left-over materials when circles are cut from squares and triangles or from cutting squares and triangles from circles. “Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of why the formula works and how the formula relates to the measure (area and circumference) and the figure. This should be an expectation for ALL students.

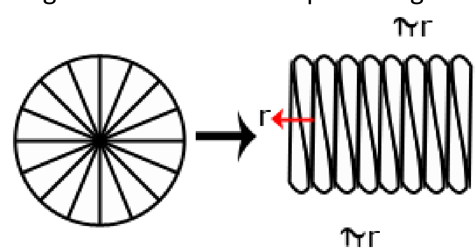
Examples:

The seventh grade class is building a mini golf game for the school carnival. The end of the putting green will be a circle. If the circle is 10 feet in diameter, how many square feet of grass carpet will they need to buy to cover the circle? How might you communicate this information to the salesperson to make sure you receive a piece of carpet that is the correct size?

Students measure the circumference and diameter of several circular objects in the room (clock, trash can, door knob, wheel, etc.). Students organize their information and discover the relationship between circumference and diameter by noticing the pattern in the ratio of the measures.

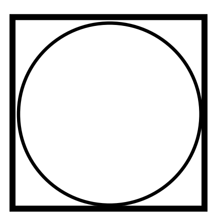
Students write an expression that could be used to find the circumference of a circle with any diameter and check their expression on other circles.

Students will use a circle as a model to make several equal parts as you would in a pie model. The greater number the cuts, the better. The pie pieces are laid out to form a shape similar to a parallelogram. Students will then write an expression for the area of the parallelogram related to the radius (note: the length of the base of the parallelogram is half the circumference, or πr, and the height is r, resulting in an area of πr^2.



If students are given the circumference of a circle, could they write a formula to determine the circle’s area or given the area of a circle, could they write the formula for the circumference?

An artist used silver wire to make a square that has a perimeter of 40 inches. She then used copper wire to make the largest circle that could fin in the square, as shown below.



How many more inches of silver wire did the artist use compared to cooper wire? (Use π = 3.14)
Show all work necessary to justify your response.

Sample Response:
Each side of the square has a length of $40 \times \frac{1}{4} = 10$ inches.
The radius of the circle is $\frac{10}{2} = 5$ inches, so the circumference of the circle is $2 \times \pi \times 5 = 10 \times 3.14 = 31.4\ inches$.
The perimeter of the square minus the circumference of the circle is $40 - 31.4 = 8.6\ inches$.

Instructional Strategies:
This is the students' initial work with circles. Knowing that a circle is created by connecting all the points equidistant from a point (center) is essential to understanding the relationships between radius, diameter, circumference, pi and area.

Students can observe this by folding a paper plate several times, finding the center at the intersection, then measuring the lengths between the center and several points on the circle, the radius. Measuring the folds through the center, or diameters leads to the realization that a diameter is two times a radius.

Given multiple-size circles, students should then explore the relationship between the radius and the length measure of the circle (circumference) finding an approximation of pi and ultimately deriving a formula for circumference. String or yarn laid over the circle and compared to a ruler is an adequate estimate of the circumference. This same process can be followed in finding the relationship between the diameter and the area of a circle by using grid paper to estimate the area.

Another visual for understanding the area of a circle can be modeled by cutting up a paper plate into 16 pieces along diameters and reshaping the pieces into a parallelogram. In figuring area of a circle, the squaring of the radius can also be explained by showing a circle inside a square. Again, the formula is derived and then learned. After explorations, students should then solve problems, set in relevant contexts, using the formulas for area and circumference.

In previous grades, students have studied angles by type according to size: acute, obtuse and right, and their role as an attribute in polygons. Now angles are considered based upon the special relationships that exist among them: supplementary, complementary, vertical and adjacent angles. Provide students the opportunities to explore these relationships first through measuring and finding the patterns among the angles of intersecting lines or within polygons, then utilize the relationships to write and solve equations for multi-step problems.

Real-world and mathematical multi-step problems that require finding area, perimeter, volume, surface area of figures composed of triangles, quadrilaterals, polygons, cubes and right prisms should reflect situations relevant to seventh graders. The computations should make use of formulas and involve whole numbers, fractions, decimals, ratios and various units of measure with same system conversions.

Resources/Tools:

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.G.B
  - The Circumference of a Circle and the Area of the Region it Encloses
  - Drinking the Lake
- 7.G.B.4
  - Eight Circles
  - Measuring the area of a circle
  - Designs
  - Stained Glass

NCTM Illuminations – NCTM has many great resources available to educators, some of these resources (i.e. interactives) are open to any educator while others (i.e. lessons) require an individual or institutional membership. If you find that a resource referenced in the flip books requires membership access, check with your school/district to see if they have an institutional membership which would grant you access all NCTM documents. If they do not have a membership, this would be a valuable resource to request.

- "The Ratio of Circumference to Diameter'
- "Geometry of Circles"
- "Discovering the Area Formula for Circles"
- "The Giant Cookie Dilemma"
- "Tetrahedral Kites"
- Popcorn, Anyone?
- Area Contractor
- "Measuring the Area of a Circle"

Common Misconceptions:

Students may believe that *Pi* is an exact number rather than understanding that 3.14 is just an approximation of *pi*. Many students are confused when dealing with circumference (linear measurement) and area.

This confusion is about an attribute that is measured using linear units (surrounding) vs. an attribute that is measured using area units (covering).

Domain: Geometry (G)

● **Cluster:** *Solve real-life and mathematical problems involving area, surface area, and volume.*

Standard: Grade 7.G.5

Investigate the relationship between three-dimensional geometric shapes;

a. Generalize the volume formula for prisms and cylinders ($V = Bh$ where *B* is the base and *h* is the height). **(2017)**
b. Generalize the surface area formula for prisms and cylinders ($SA = 2B + Ph$ where *B* is the area of the base, *P* is the perimeter of the base, and *h* is the height (in the case of a cylinder, perimeter is replaced by circumference)). **(2017)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 7.G.4 and 5.MD.5, 6.G.4

Explanations and Examples:

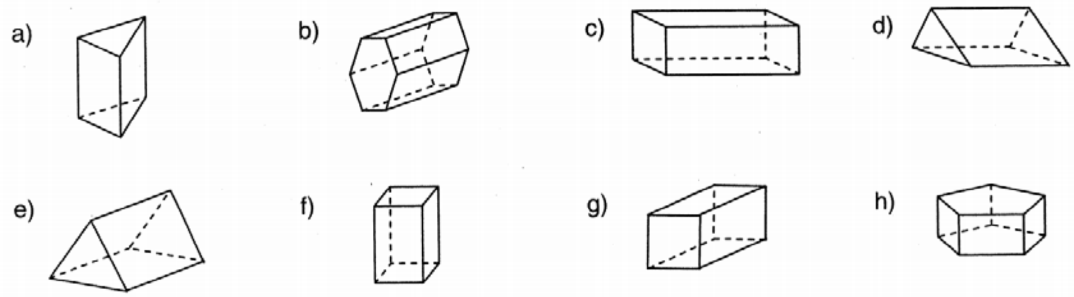
This standard is an excellent opportunity to highlight math practices as students observe the general rule appearing in each shape, use that pattern to develop the general rule, and then justify that the generality will apply to any shaped prism. Even though the standard only requires prisms and cylinders, illustrating the principals with non-standard shapes might help solidify the learning.

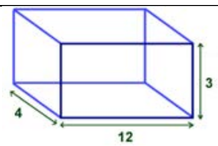
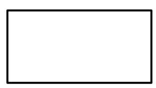
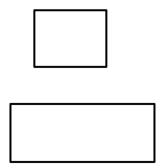
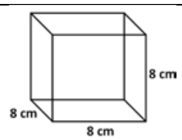
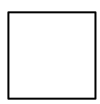
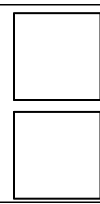
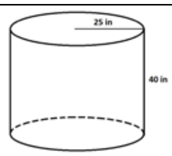
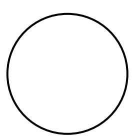
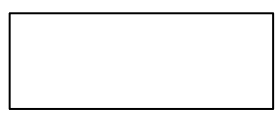
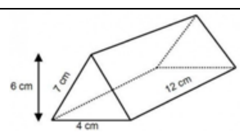
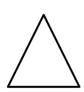
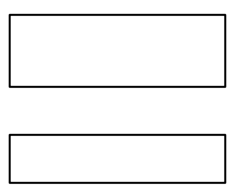
In fifth grade, students find volume by packing a solid figure with unit cubes and compare the results to the product of length by width by height. Students also learned to apply the formulas $V = l \cdot w \cdot h$ and $V = B \cdot h$. This standard reviews and focuses on the reasoning behind the formula, highlighting the relationship between two-dimensional shapes and the three dimensional solid.

Similarly, the second part of this standard furthers the connection between two-dimensional shapes and three-dimensional solids through using nets to explore surface area. Students began this exploration in 6th grade but now the focus is on generalizing the formula to a variety of three-dimensional shapes. This standard provides an opportunity to While teaching this standard, it might be beneficial to explicitly explain that area is additive. Shapes can be decomposed and recomposed into different arrangements but the area remains the same. This understanding is necessary for students to generalize the surface area formula from specific nets to $2B + Ph$.

Examples:

Shade a pair of bases for each prism.
REMEMBER: Unless all its faces are rectangles, a prism has two bases.



	Base: label the dimensions of the base.	**Face:** label the dimensions of the drawn face(s)	**Volume:** Calculate the volume	**Surface Area:** Calculate the surface area
				
				
				
				

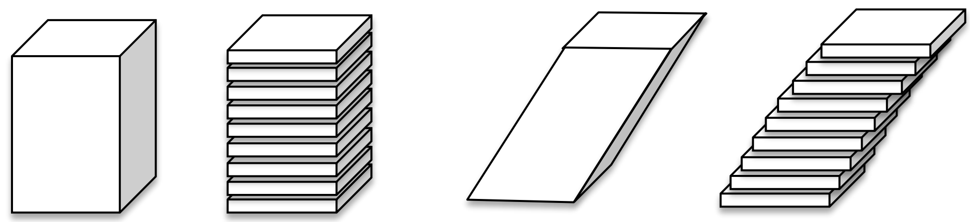
Instructional Strategies:

Clarity of vocabulary will help students identify shapes for which these general formulas apply.

A *prism* is:

- A three-dimensional shape
- With two bases
    - Bases are congruent
    - Bases are parallel
- And faces that are parallelograms
    - Faces are formed by connecting corresponding vertices of the bases

Exploring a variety of prisms with different bases but congruent areas and heights will help students see how "stacking bases" creates the general formula $V = B \cdot h$. Cardboard, folders, or card stock cut into shapes and stacked could be used to develop this understanding with students.



When exploring surface area, student struggle to see why the perimeter of the base times the height is equivalent to the area of the faces. It is helpful for students to physically build the prism from a net. Use a marker to trace the perimeter of the base and one lateral edge after the shape is built. Then, once the shape is flattened again, students can more clearly see the connection between the three-dimensional shape and its two-dimensional net.

Resources/Tools:

MAP formative assessments:

- Designing 3D Products: Candy Cartons
- Using Space Efficiently: Packing a Truck
- Designing a 3D Product in 2D: A Sports Bag
- Estimating Volume: The Money Munchers

Illustrative Mathematics **Grade 6** tasks: **Scroll to the appropriate section to find named tasks.**

- 6.G.A.2
  - Computing Volume Progression 1
  - Computing Volume Progression 2
  - Computing Volume Progression 3
  - Banana Bread
  - Volumes with Fractional Edge Lengths
- 6.G.A.4
  - Nets for Pyramids and Prisms

Illustrative Mathematics **Grade 7** tasks: **Scroll to the appropriate section to find named tasks.**

- 7.G.B.6
  - Sand Under the Swing Set

Open Up resources:

- Area and Surface Area
- Angles, Triangles, and Prisms

NCTM Illuminations – NCTM has many great resources available to educators, some of these resources (i.e. interactives) are open to any educator while others (i.e. lessons) require an individual or institutional membership. If you find that a resource referenced in the flip books requires membership access, check with your school/district to see if they have an institutional membership which would grant you access all NCTM documents. If they do not have a membership, this would be a valuable resource to request.

- Fill 'Er Up
- Popcorn, Anyone?
- Fishing for the Best Prism
- Cubed Cans

Domain: Geometry (G)

● **Cluster**: *Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.*

Standard: Grade 7.G.6

Solve real-world and mathematical problems involving area of two-dimensional objects and volume and surface area of three-dimensional objects including cylinders and right prisms. (Solutions should **not** require students to take square roots or cube roots. For example, given the volume of a cylinder and the area of the base, students would identify the height.) **(7.G.6)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 7.G.4

Explanations and Examples:

Students continue work from 5th and 6th grade to work with area, volume and surface area of two- dimensional and three-dimensional objects (composite shapes).

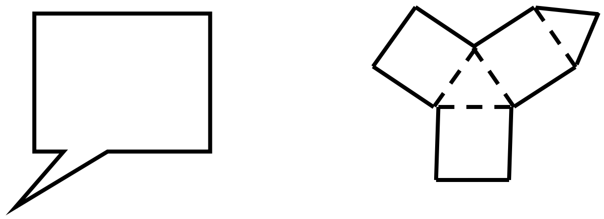
"Know the formula" does not mean memorization of the formula. To "know" means to have an understanding of why the formula works and how the formula relates to the measure (area and volume) and the figure.

Building on work with nets in the 6th grade, students should recognize that finding the area of each face of a three-dimensional figure and adding the areas will give the surface area.

Students understanding of volume can be supported by focusing on the area of base times the height to calculate volume. Students understanding of surface area can be supported by focusing on the sum of the area of the faces. Nets can be used to evaluate surface area calculations.

Examples:

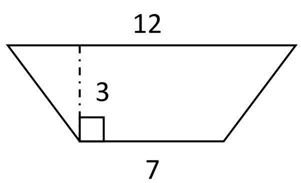
Choose one of the figures shown below and write a step by step procedure for determining the area. Find another person that chose the same figure as you did. How are your procedures the same and different? Do they yield the same result?



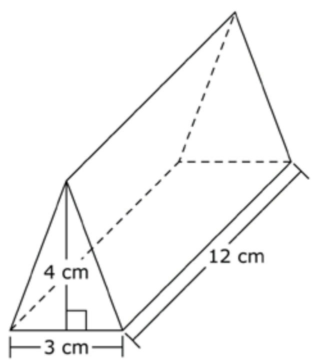
A cereal box is a rectangular prism. What is the volume of the cereal box? What is the surface area of the cereal box? (Hint: Create a net of the cereal box and use the net to calculate the surface area.)

Find the area of a triangle with a base length of three units and a height of four units.

Find the area of the trapezoid shown below using the formulas for rectangles and triangles.

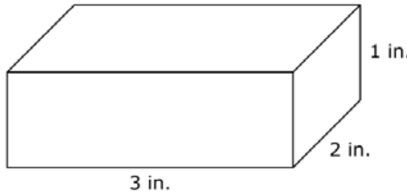


Look at the triangular prism below. Each triangular face of the prism has a base of 3 centimeters (cm) and a height of 4 cm. The length of the prism is 12 cm.



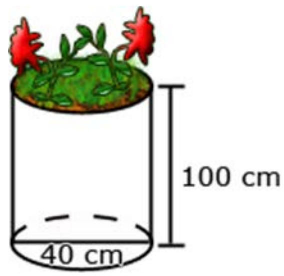
What is the volume of this triangular prism?

Using the rectangular prism shown below, create a new prism with a **surface area** of between 44 square inches and 54 square inches.



Solution: Four prisms should be stacked vertically.

James wanted to plant pansies in his new planter. He wondered how much potting soil he should buy to fill it. Use the measurements in the diagram below to determine the planter's volume.



A cylindrical tank has a height of 10 feet and a radius of 4 feet. Jane fills the tank with water at a rate of 8 cubic feet per minute. At this rate, how many minutes will it take Jane to completely fill the tank without overflowing it? Round your answer to the nearest **minute**.

Solution: 63 minutes

Juan needs a right cylindrical storage tank that holds between 110 and 115 cubic feet of water. Using whole numbers only, provide the radius and height for 3 different tanks that hold between 110 and 115 cubic feet of water.

Tank #1	
radius	ft.
height	ft.

Tank #2	
radius	ft.
height	ft.

Tank #3	
radius	ft.
height	ft.

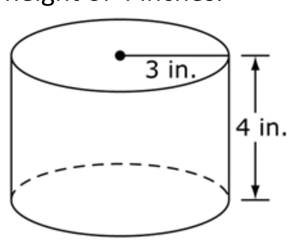
Sample Response:

Tank #1	
radius	2 ft.
height	9 ft.

Tank #2	
radius	3 ft.
height	4 ft.

Tank #3	
radius	6 ft.
height	1 ft.

This right cylinder has a radius of 3 inches and a height of 4 inches.



What is the volume, in cubic inches, of the cylinder?

Solution: 36π cu in. (or any number between 113 and 113.1)

Instructional Strategies:

Real-world and mathematical multi-step problems that require finding area, perimeter, volume, surface area of figures composed of triangles, quadrilaterals, polygons, cubes and right prisms should reflect situations relevant to seventh graders. The computations should make use of formulas and involve whole numbers, fractions, decimals, ratios and various units of measure with same system conversions.

Tools/Resources:

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.G.B.6
    - Sand Under the Swing Set

NCTM Illuminations – NCTM has many great resources available to educators, some of these resources (i.e. interactives) are open to any educator while others (i.e. lessons) require an individual or institutional membership. If you find that a resource referenced in the flip books requires membership access, check with your school/district to see if they have an institutional membership which would grant you access all NCTM documents. If they do not have a membership, this would be a valuable resource to request.

- "Popcorn Anyone?"
- "Area Contractor"

Domain: Statistics and Probability (SP)

◆**Cluster:** *Use random sampling to draw inferences*

Standard: Grade 7.SP.1

Use statistics to gain information about a population by examining a sample of the population;

7.SP.1a. Know that generalizations about a population from a sample are valid only if the sample is representative of that population and generate a valid representative sample of a population. **(7.SP.1)**

7.SP.1b. Identify if a particular random sample would be representative of a population and justify your reasoning. **(7.SP.1)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.6 Attend to precision.

Connections:

This cluster is connected to:

- ◆Grade 7 Critical Area of Focus #4: **Drawing inferences about populations based on samples**.
- Initial understanding of statistics, specifically variability and the measures of center and spread begins in Grade 6.

Explanations and Examples:

Students recognize that it is difficult to gather statistics on an entire population. Instead a random sample can be representative of the total population and will generate valid results. Students use this information to draw inferences from data. A random sample must be used in conjunction with the population to get accuracy. For example, a random sample of elementary students cannot be used to give a survey about the prom.

Examples:

The school food service wants to increase the number of students who eat hot lunch in the cafeteria. The student council has been asked to conduct a survey of the student body to determine the students' preferences for hot lunch. They have determined two ways to do the survey. The two methods are listed below. Identify the type of sampling used in each survey option. Which survey option should the student council use and why?

- *Method 1*: Write all of the students' names on cards and pull them out in a draw to determine who will complete the survey.
- *Method 2*: Survey the first 20 students that enter the lunch room.

Amanda asked a random sampling of 40 students from her school to identify their birth month. There are 300 students in her school. Amanda's data is shown in this table.

Students Birth Months

Birth Month	Number of Students
January	3
February	0
March	3
April	10
May	4
June	3
July	4
August	3
September	2
October	2
November	3
December	3

Which of these statements is supported by the data?

- Exactly 25% of the students in Amanda's school have April as their birth month.
- There are no students in Amanda's school that have a February birth month.
- There are probably more students at Amanda's school with an April birth month than a July birth month.
- There are probably more students at Amanda's school with a July birth month than a June birth month.

Instructional Strategies:

In Grade 6, students used measures of center and variability to describe data. Students continue to use this knowledge in Grade 7 as they use random samples to make predictions about an entire population and judge the possible discrepancies of the predictions. Providing opportunities for students to use real-life situations from science and social studies shows the purpose for using random sampling to make inferences about a population.

Make available to students the tools needed to develop the skills and understandings required to produce a representative sample of the general population. One key element of a representative sample is understanding that a random sampling guarantees that each element of the population has an equal opportunity to be selected in the sample.

Have students compare the random sample to population, asking questions like "Are all the elements of the entire population represented in the sample?" and "Are the elements represented proportionally?" Students can then continue the process of analysis by determining the measures of center and variability to make inferences about the general population based on the analysis.

Provide students with random samples from a population, including the statistical measures.

Ask students guiding questions to help them make inferences from the sample.

Tools/Resources:

For detailed information, see Progressions for Common Core State Standards in Mathematics: 6-8 Statistics and Probability

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.SP.A
  - Estimating the Mean State Area
  - Election Poll, Variation 2
  - Election Poll, Variation 3
  - Election Poll, Variation 1
- 7.SP.A.1
  - Mr. Briggs's Class Likes Math

Georgia Department of Education website:

- "The Eyes Have It" -. Students analyze data and draw conclusions about the data using box-and-whisker plots. Students collect data from experiments on eye blinks.
- "See Saw Nickels"- Students focus on extending their conceptual understanding of proportional relationships and direct variation to include inverse relationships. Students will use manipulatives, completed charts, and graphs to further their understanding.

Common Misconceptions:

Students may believe:

One random sample is not representative of the entire population and that many samples must be taken in order to make an inference that is valid. By comparing the results of one random sample with the results of multiple random samples, students can correct this misconception. Students' understanding that the random sample must be representative of the population is key to supporting valid inferences.

Domain: Statistics and Probability (SP)

◆**Cluster:** *Use random sampling to draw inferences about a population.*

Standard: Grade 7.SP.2

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* **(7.SP.2)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See 7.SP.1

Explanations and Examples:

Students collect and use multiple samples of data to answer question(s) about a population. Issues of variation in the samples should be addressed.

Example:

Below is the data collected from two random samples of 100 students regarding student's school lunch preferences. Make at least two inferences based on the results.

Lunch Preferences				
Student Sample	**Hamburgers**	**Tacos**	**Pizza**	**Total**
#1	12	14	74	100
#2	12	11	77	100

Instructional Strategies:

Make available to students the tools needed to develop the skills and understandings required to produce a representative sample of the general population. One key element of a representative sample is understanding that a random sampling guarantees that each element of the population has an equal opportunity to be selected in the sample.

Have students compare the random sample to population, asking questions like “Are all the elements of the entire population represented in the sample?” and “Are the elements represented proportionally?” Students can then continue the process of analysis by determining the measures of center and variability to make inferences about the general population based on the analysis. Provide students with random samples from a population, including the statistical measures. Ask students guiding questions to help them make inferences from the sample.

Resources/Tools:

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.SP.A.2
  - Valentine Marbles

Common Misconceptions: See 7.SP.1

Domain: Statistics and Probability (SP)

● **Cluster:** *Draw informal comparative inferences about two populations.*

Standard: Grade 7.SP.3

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability (requires introduction of **mean absolute deviation**). *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* **(7.SP.3)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections:

This Cluster is connected to:

- ● Grade 7 Critical Area of Focus #4**: drawing inferences about populations based on samples**. It expands standards 1 and 2 to make inferences between populations.
- Measures of center and variability are developed in Statistics and Probability Grade 6.

Explanations and Examples:

This is the students' first experience with comparing two data sets.

Students understand that:

1. a full understanding of the data requires consideration of the measures of variability as well as mean or median,
2. variability is responsible for the overlap of two data sets and that an increase in variability can increase the overlap, and
3. median is paired with the interquartile range and mean is paired with the mean absolute deviation.

Students can readily find data as described in the example on sports team or college websites.

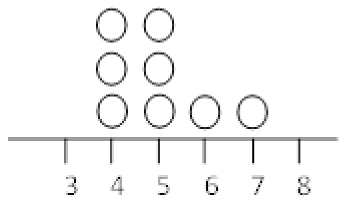
Other sources for data include American Fact Finder (Census Bureau), Fed Stats, Ecology Explorers, USGS, or CIA World Facebook. Researching data sets provides opportunities to connect mathematics to their interests and other academic subjects. Students can utilize statistic functions in graphing calculators or spreadsheets for calculations with larger data sets or to check their computations. Students calculate mean absolute deviations in preparation for later work with standard deviations.

The Mean Absolute Deviation describes the variability of the data set by determining the absolute value deviation (the distance) of each data piece from the mean and then finding the average of these deviations. Both the interquartile range and the Mean Absolute Deviation are represented by a single numerical value. Higher values represent a greater variability in the data.

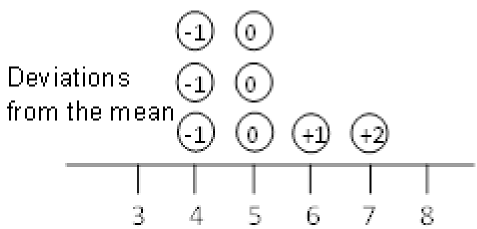
Understanding Mean Absolute Deviation

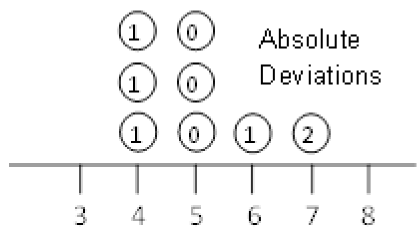
The use of mean absolute deviation in 6th grade is mainly exploratory. The intent is to build a deeper understanding of variability. Students would understand the mean distance between the pieces of data and the mean of the data set expresses the spread of the data set. Students can see that the larger the mean distance, the greater the variability. Comparisons can be made between different data sets.

In the previous data set, the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. There were 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters. This data can be represented on a dot plot. The mean of the data set is 5



To find the mean absolute deviation, students examine each of the data points and its difference from the mean. This analysis can be represented on the dot plot itself or in a table. Each of the names with 4 letters has one fewer letter than the mean, each of the names with 5 letters has zero difference in letters as compared to the mean, each of the names with 6 letters has one more letter than the mean, and each of the names with 7 letters has two more letters than the mean. The absolute deviations are the absolute value of each difference.





Name	Number of letters in a name	Deviation from the Mean	Absolute Deviation from the Mean
John	4	-1	1
Luis	4	-1	1
Mike	4	-1	1
Carol	5	0	0
Maria	5	0	0
Karen	5	0	0
Sierra	6	+1	1
Monique	7	+2	2
Total	40	0	6

The mean of the absolute deviations is found by summing the absolute deviations and dividing by the number of data points. In this case, the mean absolute deviation would be 6 ÷ 8 or $\frac{3}{4}$ or 0.75.

The mean absolute deviation is a small number, indicating that there is little variability in the data set.

Consider a different data set also containing 8 names. If the names were Sue, Joe, Jim, Amy, Sabrina, Monique, Timothy, and Adelita. Summarize the data set and its variability. How does this compare to the first data set?

The mean of this data set is still 5. $\frac{(3+3+3+3+7+7+7)}{8} = \frac{40}{8} = 5$

Name	Number of letters in a name	Deviation from the Mean	Absolute Deviation from the Mean
Sue	3	-2	2
Joe	3	-2	2
Jim	3	-2	2
Amy	3	-2	2
Sabrina	7	+2	2
Timothy	7	+2	2
Adelita	7	+2	2
Monique	7	+2	2
Total	40	0	16

The mean deviation of this data set is 16 ÷ 8 or 2. Although the mean is the same, there is much more variability in this data set.

Example:

Jason wanted to compare the mean height of the players on his favorite basketball and soccer teams. He thinks the mean height of the players on the basketball team will be greater but doesn't know how much greater. He also wonders if the variability of heights of the athletes is related to the sport they play. He thinks that there will be a greater variability in the heights of soccer players as compared to basketball players. He used the rosters and player statistics from the team websites to generate the following lists.

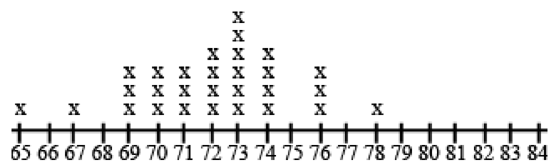
Basketball Team – Height of Players in inches for 2010-2011 Season:

75, 73, 76, 78, 79, 78, 79, 81, 80, 82, 81, 84, 82, 84, 80, 84

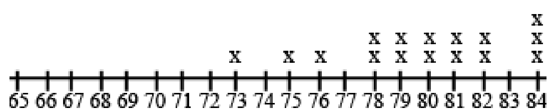
Soccer Team – Height of Players in inches for 2010-2011 Season:

73, 73, 73, 72, 69, 76, 72, 73, 74, 70, 65, 71, 74, 76, 70, 72, 71, 74, 71, 74, 73, 67, 70, 72, 69, 78, 73, 76, 69

To compare the data sets, Jason creates a two dot plots on the same scale. The shortest player is 65 inches and the tallest players are 84 inches.



Height of Soccer Players (in)



Height of Basketball Players (in)

In looking at the distribution of the data, Jason observes that there is some overlap between the two data sets. Some players on both teams have players between 73 and 78 inches tall. Jason decides to use the mean and mean absolute deviation to compare the data sets. Jason sets up a table for each data set to help him with the calculations. (*See next page*)

The mean height of the basketball players is 79.75 inches as compared to the mean height of the soccer players at 72.07 inches, a difference of 7.68 inches.

The mean absolute deviation (MAD) is calculated by taking the mean of the absolute deviations for each data point. The difference between each data point and the mean is recorded in the second column of the table. Jason used rounded values (80 inches for the mean height of basketball players and 72 inches for the mean height of soccer players) to find the differences. The absolute deviation, absolute value of the deviation, is recorded in the third column. The absolute deviations are summed and divided by the number of data points in the set.

The mean absolute deviation is 2.14 inches for the basketball players and 2.53 for the soccer players. These values indicate moderate variation in both data sets. There is slightly more variability in the height of the soccer players. The difference between the heights of the teams is approximately 3 times the variability of the data sets $7.68 \div 2.53 = 3.04$.

Soccer Players (n = 29)		
Height (in)	Deviation from Mean (in)	Absolute Deviation (in)
65	-7	7
67	-5	5
69	-3	3
69	-3	3
69	-3	3
70	-2	2
70	-2	2
70	-2	2
71	-1	1
71	-1	1
71	-1	1
72	0	0
72	0	0
72	0	0
72	0	0
73	+1	1
73	+1	1
73	+1	1
73	+1	1
73	+1	1
73	+1	1
74	+2	2
74	+2	2
74	+2	2
74	+2	2
76	+4	4
76	+4	4
76	+4	4
78	+6	6
Σ = 2090		**Σ = 62**

Mean = 2090 ÷ 29 =72 inches
MAD = 62 ÷ 29 = 2.13 inches

Basketball Players (n = 16)		
Height (in)	Deviation from Mean (in)	Absolute Deviation (in)
73	-7	7
75	-5	5
76	-4	4
78	-2	2
78	-2	2
79	-1	1
79	-1	1
80	0	0
80	0	0
81	1	1
81	1	1
82	2	2
82	2	2
84	4	4
84	4	4
84	4	4
Σ = 1276		**Σ = 40**

Mean = 1276 ÷ 16 =80 inches
MAD = 40 ÷ 16 = 2.5 inches

Instructional Strategies:
In Grade 6, students used measures of center and variability to describe sets of data. In the cluster "Use random sampling to draw inferences about a population" of Statistics and Probability in Grade 7, students learn to draw inferences about one population from a random sampling of that population.

Students continue using these skills to draw informal comparative inferences about two populations. Provide opportunities for students to deal with small populations, determining measures of center and variability for each population. Then have students compare those measures and make inferences.

The use of graphical representations of the same data (Grade 6) provides another method for making comparisons. Students begin to develop understanding of the benefits of each method by analyzing data with both methods.

When students study large populations, random sampling is used as a basis for the population inference. This builds on the skill developed in the Grade 7 cluster "Use random sampling to draw inferences about a population" of Statistics and Probability.

Measures of center and variability are used to make inferences on each of the general populations.

Then students can make comparisons for the two populations based on those inferences.

This is a great opportunity to have students examine how different inferences can be made based on the same two sets of data. Have students investigate how advertising agencies uses data to persuade customers to use their products. Additionally, provide students with two populations and have them use the data to persuade both sides of an argument.

Tools/Resources:

Georgia Department of Education website:

- "The Eyes Have It"-. Students analyze data and draw conclusions about the data using box-and-whisker plots. Students collect data from experiments on eye blinks.

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.SP.B.3
    - Offensive Linemen
    - College Athletes

Domain: Statistics and Probability (SP)

●**Cluster**: *Draw informal comparative inferences about two populations.*

Standard: Grade 7.SP.4

Use measures of center (mean, median and/or mode) and measures of variability (range, **interquartile range** and/or **mean absolute deviation**) for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book. (NOTE: Students should not have to calculate mean absolute deviation but use it to interpret data).* **(7.SP.4)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See 7.SP.3

Explanations and Examples:

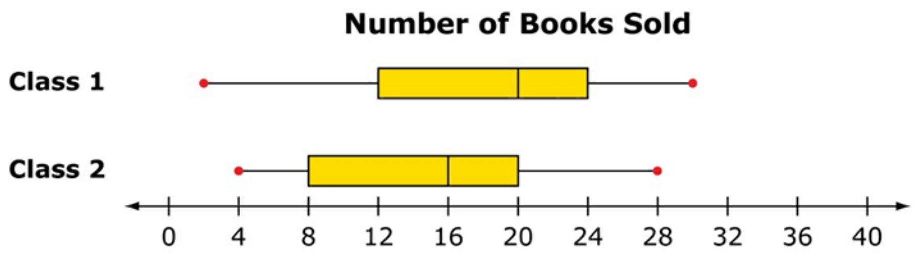
Students are expected to compare two sets of data using measures of center and variability. Measures of center include mean, median, and mode. The measures of variability include range, mean absolute deviation, and interquartile range.

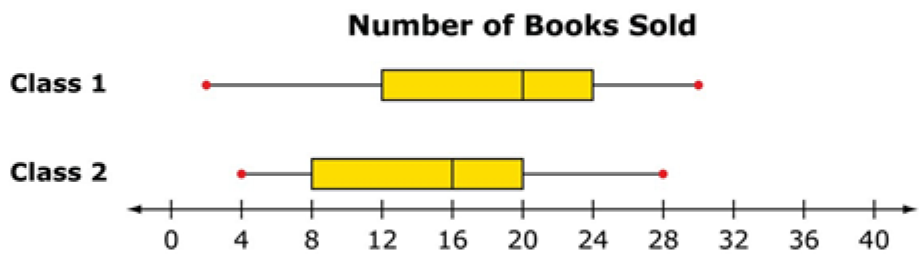
Examples:

The two data sets below depict random samples of the housing prices sold in the King River and Toby Ranch areas of Arizona. Based on the prices below, which measure of center will provide the most accurate estimation of housing prices in Arizona? Explain your reasoning.

- King River: {1.2 million; 242,000; 265,500; 140,000; 281,000; 265,000; 211,000}
- Toby Ranch: {5 million; 154,000; 250,000; 250,000; 200,000; 160,000; 190,000}

The number of books sold by each student in two classes for a fundraiser is summarized by these box plots.





The principal concluded that there was more variability in the number of books sold by Class 1 than Class 2.

Which statement is true about the principal's conclusion? Explain your reasoning.

1. It is valid because the median for Class 1 is greater than the median for Class 2.
2. It is valid because the range for Class 1 is greater than the range for Class 2.
3. It is invalid because the minimum value for Class 1 is less than the minimum value for Class 2.
4. It is invalid because the interquartile range for Class 1 is less than the interquartile range for Class 2.

Sample Response:

1. Not true - statement assumed the median is a measure of variability
2. Correct- supports the principal's statement of more variability as shown by a greater range.
3. Not true- statement assumed the minimum value is a measure of variability
4. Not true – statement did not correctly determine interquartile range

Instructional Strategies: See 7.SP.3

Tools/Resources:

See 7.SP.3

Domain: Statistics and Probability (SP)

◆**Cluster:** *Investigate chance processes and develop, use, and evaluate probability models*

Standard: Grade 7.SP.5

Express the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. (Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.) **(7.SP.5)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6.Attend to precision
- ✓ MP.7 Look for and make use of structure.

Connections:

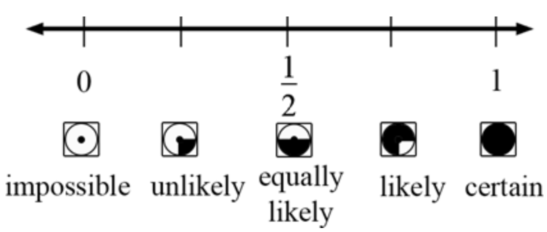
This cluster is connected to:

- This cluster goes beyond the Grade 7 Critical Areas of Focus to address Investigating chance.
- Ratio and Proportional Relationships in Grade 6 is the development of fractions as ratios and percents as ratios. In Grade 7, students write the same number represented as a fraction, decimal or percent.
- Random sampling and simulation are closely connected in Grade 7.SP. Random sampling and simulation is used to determine the experimental probability of event occurring in a population or to describe a population.

Explanations and Examples:

This is students' first formal introduction to probability. Students recognize that all probabilities are between 0 and 1, inclusive, the sum of all possible outcomes is 1. For example, there are three choices of jellybeans – grape, cherry and orange. If the probability of getting a grape is $\frac{3}{10}$ and the probability of getting cherry is $\frac{1}{5}$, what is the probability of getting oranges? The probability of any single event can be recognized as a fraction. The closer the fraction is to 1, the greater the probability the event will occur. Larger numbers indicate greater likelihood. For example, if you have 10 oranges and 3 apples, you have a greater likelihood of getting an orange.

Probability can be expressed in terms such as impossible, unlikely, likely, or certain or as a number between 0 and 1 as illustrated on the number line.



Students can use simulations such as Marble Mania or the Random Drawing Tool on NCTM's Illuminations to generate data and examine patterns.

ScienceNetLinks website :

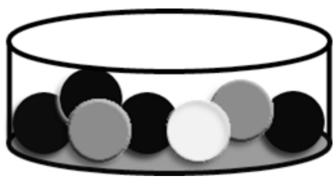
- Marble Mania

NCTM Illuminations – NCTM has many great resources available to educators, some of these resources (i.e. interactives) are open to any educator while others (i.e. lessons) require an individual or institutional membership. If you find that a resource referenced in the flip books requires membership access, check with your school/district to see if they have an institutional membership which would grant you access all NCTM documents. If they do not have a membership, this would be a valuable resource to request.

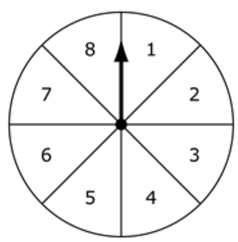
- Random Drawing Tool

Examples:

A container contains 2 gray, 1 white, and 4 black marbles. Without looking, if you choose a marble from the container, will the probability be closer to 0 or to 1 that you will select a white marble? A gray marble? A black marble? Justify each of your predictions.



Carl and Beneta are playing a game using this spinner.



Carl will win the game on his next spin if the arrow lands on a section labeled 6, 7, or 8. Carl claims it is likely, but not certain, that he will win the game on his next spin.

Explain why Carl's claim is **not** correct.

Beneta will win the game on her next spin if the result of the spin satisfies event *X*. Beneta claims it is likely, but not certain, that she will win the game on her next spin.

Describe an event *X* for which Beneta's claim is correct.

Sample Response:

Carl's claim is incorrect. The probability that Carl will spin a 6 or higher is 0.375. This means that it is more likely that Carl will spin a number less than 6 on his next turn.

For Beneta, event *X* could be "the arrow lands on a section labeled with a number greater than 2."

Instructional Strategies:

Grade 7 is the introduction to the formal study of probability. Through multiple experiences, students begin to understand the probability of chance (simple and compound), develop and use sample spaces, compare experimental and theoretical probabilities, develop and use graphical organizers, and use information from simulations for predictions.

Help students understand the probability of chance is using the benchmarks of probability: 0, 1 and $\frac{1}{2}$. Provide students with situations that have clearly defined probability of never happening as zero, always happening as 1 or equally likely to happen as to not happen as $\frac{1}{2}$.

Then advance to situations in which the probability is somewhere between any two of these benchmark values. This builds to the concept of expressing the probability as a number between 0 and 1. Use this to build the understanding that the closer the probability is to 0, the more likely it will not happen, and the closer to 1, the more likely it will happen.

Students learn to make predictions about the relative frequency of an event by using simulations to collect, record, organize and analyze data. Students also develop the understanding that the more the simulation for an event is repeated, the closer the experimental probability approaches the theoretical probability.

Have students develop probability models to be used to find the probability of events. Provide students with models of equal outcomes and models of not equal outcomes are developed to be used in determining the probabilities of events.

Students should begin to expand the knowledge and understanding of the probability of simple events, to find the probabilities of compound events by creating organized lists, tables and tree diagrams. This helps students create a visual representation of the data; i.e., a sample space of the compound event. From each sample space, students determine the probability or fraction of each possible outcome.

Students continue to build on the use of simulations for simple probabilities and now expand the simulation of compound probability. Providing opportunities for students to match situations and sample spaces assists students in visualizing the sample spaces for situations.

Students often struggle making organized lists or trees for a situation in order to determine the theoretical probability. Having students start with simpler situations that have fewer elements enables them to have successful experiences with organizing lists and trees diagrams. Ask guiding questions to help students create methods for creating organized lists and trees for situations with more elements.

Students often see skills of creating organized lists, tree diagrams, etc. as the end product. Provide students with experiences that require the use of these graphic organizers to determine the theoretical probabilities. Have them practice making the connections between the process of creating lists, tree diagrams, etc. and the interpretation of those models.

Additionally, students often struggle when converting forms of probability from fractions to percents and vice versa. To help students with the discussion of probability, don't allow the symbol manipulation/conversions to detract from the conversations. By having students use technology such as a graphing calculator or computer software to simulate a situation and graph the results, the focus is on the interpretation of the data. Students then make predictions about the general population based on these probabilities

Common Misconceptions:

Students may attempt to give probability as a number greater than one rather than representing it as a number between zero and one. For example, if there are 2 blue marbles and 3 red marbles, the probability of picking a blue marble is $\frac{2}{5}$, not 2.

Students often expect the theoretical and experimental probabilities of the same data to match.

By providing multiple opportunities for students to experience simulations of situations in order to find and compare the experimental probability to the theoretical probability, students discover that rarely are those probabilities the same. Students often expect that simulations will result in all of the possibilities. All possibilities may occur in a simulation, but not necessarily. Theoretical probability does use all possibilities.

Note examples in simulations when some possibilities are not shown.

Domain: Statistics and Probability (SP)

◆**Cluster:** *Investigate chance processes and develop, use, and evaluate probability models*

Standard: Grade 7.SP.6

Collect data from a chance process (probability experiment). Approximate the probability by observing its long-run relative frequency. Recognize that as the number of trials increase, the experimental probability approaches the theoretical probability. Conversely, predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* **(7.SP.6)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics
- ✓ MP.5 Use appropriate tools strategically.

Connections: See 7.SP.5

Explanations and Examples:

Students collect data from a probability experiment, recognizing that as the number of trials increase, the experimental probability approaches the theoretical probability. The focus of this standard is relative frequency -- The relative frequency is the observed number of successful events for a finite sample of trials. Relative frequency is the observed proportion of successful events.

Students can collect data using physical objects or graphing calculator or web-based simulations. Students can perform experiments multiple times, pool data with other groups, or increase the number of trials in a simulation to look at the long-run relative frequencies.

Examples:

Each group receives a bag that contains 4 green marbles, 6 red marbles, and 10 blue marbles. Each group performs 50 pulls, recording the color of marble drawn and replacing the marble into the bag before the next draw. Students compile their data as a group and then as a class. They summarize their data as experimental probabilities and make conjectures about theoretical probabilities

How many green draws would you expect if you were to conduct 1000 pulls? 10,000 pulls?

Students create another scenario with a different ratio of marbles in the bag and make a conjecture about the outcome of 50 marble pulls with replacement. An example would be 3 green marbles, 6 blue marbles, and 3 blue marbles.

Students try the experiment and compare their predictions to the experimental outcomes to continue to explore and refine conjectures about theoretical probability.

Instructional Strategies: See 7.SP.5

Students learn to make predictions about the relative frequency of an event by using simulations to collect, record, organize and analyze data. Students also develop the understanding that the more the simulation for an event is repeated, the closer the experimental probability approaches the theoretical probability.

Have students develop probability models to be used to find the probability of events. Provide students with models of equal outcomes and models of not equal outcomes are developed to be used in determining the probabilities of events.

Tools /Resources:

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.SP.C.6
  - Rolling Dice
  - Tossing Cylinders
  - Heads or Tails

"Odd and Even", Great Tasks for Mathematics Grades 6-12, NCSM, (2013). Students explore experimental probabilities and calculate theoretical probabilities of odd and even sums of random numbers.

Common Misconceptions:

Students may have trouble understanding the difference between the probability that should happen in theory and the outcomes of an actual event. Students often expect the theoretical and experimental probabilities of the same data to match. By providing multiple opportunities for students to experience simulations of situations in order to find and compare the experimental probability to the theoretical probability, students discover that rarely are those probabilities the same.

Students may confuse finding the probability of event A or event B occurring (either one could occur) vs. the probability of even A and even B both occurring (compound even).

Domain: Statistics and Probability (SP)

◆**Cluster:** *Investigate chance processes and develop, use, and evaluate probability models.*

Standard: Grade 7.SP.7

Develop a **probability model** and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

7.SP.7a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* **(7.SP.7a)**

7.SP.7b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* **(7.SP.7b)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning

Connections: See 7.SP.5

Explanations and Examples:

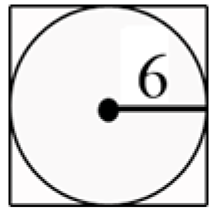
Probabilities are useful for predicting what will happen over the long run. Using theoretical probability, students predict frequencies of outcomes. Students recognize an appropriate design to conduct an experiment with simple probability events, understanding that the experimental data give realistic estimates of the probability of an event but are affected by sample size.

Students need multiple opportunities to perform probability experiments and compare these results to theoretical probabilities. Critical components of the experiment process are making predictions about the outcomes by applying the principles of theoretical probability, comparing the predictions to the outcomes of the experiments, and replicating the experiment to compare results. Experiments can be replicated by the same group or by compiling class data.

Experiments can be conducted using various random generation devices including, but not limited to, bag pulls, spinners, number cubes, coin toss, and colored chips. Students can collect data using physical objects or graphing calculator or web- based simulations. Students can also develop models for geometric probability (i.e. a target).

Example:

If you choose a point in the square, what is the probability that it is not in the circle?



Instructional Strategies: See 7.SP.5

Have students develop probability models to be used to find the probability of events. Provide students with models of equal outcomes and models of not equal outcomes are developed to be used in determining the probabilities of events.

Resources/Tools:

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.SP.C.7.a
  - How Many Buttons?

Georgia Department of Education website:

- Is it Fair?"-Students play the game "Is It Fair?" and record their information using probability to determine whether they feel the game is fair or not. Predictions are made before the game begins. Based on their trials, students determine all outcomes, create tree diagrams and determine the theoretical chance of winning for each player.

Common Misconceptions:

Students often expect the theoretical and experimental probabilities of the same data to match.

By providing multiple opportunities for students to experience simulations of situations in order to find and compare the experimental probability to the theoretical probability, students discover that rarely are those probabilities the same.

Students often expect that simulations will result in all of the possibilities. All possibilities may occur in a simulation, but not necessarily. Theoretical probability does use all possibilities.

Note examples in simulations when some possibilities are not shown.

Domain: Statistics and Probability (SP)

◆**Cluster:** *Investigate chance processes and develop, use, and evaluate probability.*

Standard: Grade 7.SP.8

Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

7.SP.8a. Know that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. **(7.SP.8a)**

7.SP.8b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language *(e.g. "rolling double sixes")*, identify the outcomes in the sample space which compose the event. **(7.SP.8b)**

7.SP.8c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?* **(7.SP.8c)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.4 Model with mathematics
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 7.SP.5

Explanations and Examples:

Students use tree diagrams, frequency tables, and organized lists, and simulations to determine the probability of compound events.

Probabilities are useful for predicting what will happen over the long run. Using theoretical probability, students predict frequencies of outcomes. Students recognize an appropriate design to conduct an experiment with simple probability events, understanding that the experimental data give realistic estimates of the probability of an event but are affected by sample size.

Examples:

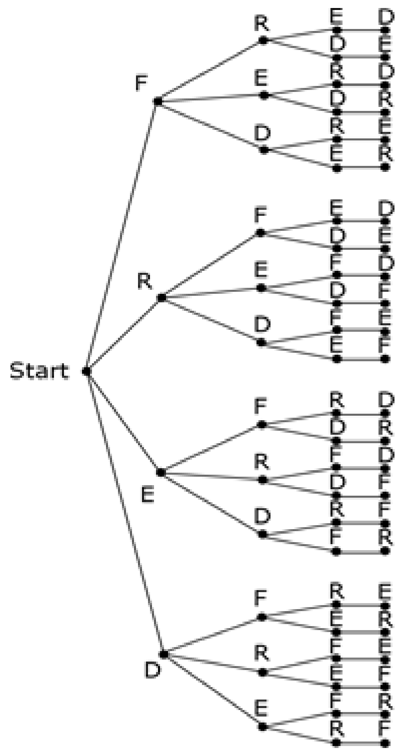
Students conduct a bag pull experiment. A bag contains 5 marbles. There is one red marble, two blue marbles and two purple marbles. Students will draw one marble without replacement and then draw another.

What is the sample space for this situation? Explain how you determined the sample space and how you will use it to find the probability of drawing one blue marble followed by another blue marble.

Show all possible arrangements of the letters in the word FRED using a tree diagram.

If each of the letters is on a tile and drawn at random, what is the probability that you will draw the letters F-R-E-D in that order?

What is the probability that your “word” will have an F as the first letter?



Instructional Strategies:

Students should begin to expand the knowledge and understanding of the probability of simple events, to find the probabilities of compound events by creating organized lists, tables and tree diagrams. This helps students create a visual representation of the data; i.e., a sample space of the compound event. From each sample space, students determine the probability or fraction of each possible outcome.

Students continue to build on the use of simulations for simple probabilities and now expand the simulation of compound probability. Providing opportunities for students to match situations and sample spaces assists students in visualizing the sample spaces for situations.

Students often struggle making organized lists or trees for a situation in order to determine the theoretical probability. Having students start with simpler situations that have fewer elements enables them to have successful experiences with organizing lists and trees diagrams. Ask guiding questions to help students create methods for creating organized lists and trees for situations with more elements.

Students often see skills of creating organized lists, tree diagrams, etc. as the end product. Provide students with experiences that require the use of these graphic organizers to determine the theoretical probabilities. Have them practice making the connections between the process of creating lists, tree diagrams, etc. and the interpretation of those models.

Additionally, students often struggle when converting forms of probability from fractions to percents and vice versa. To help students with the discussion of probability, don't allow the symbol manipulation/conversions to detract from the conversations. By having students use technology such as a graphing calculator or computer software to simulate a situation and graph the results, the focus is on the interpretation of the data. Students then make predictions about the general population based on these probabilities.

Resources/Tools:

Illustrative Mathematics Grade 7 tasks: **Scroll to the appropriate section to find named tasks.**

- 7.SP.C.8
    - Waiting Times
    - Rolling Twice
    - Red, Green, or Blue?
- 7.SP.C.8.a
    - Sitting across from Each Other
    - Tetrahedral Dice
- 7.SP.C.8.b
    - Sitting across from Each Other
    - Tetrahedral Dice

Common Misconceptions:

Students often expect the theoretical and experimental probabilities of the same data to match. By providing multiple opportunities for students to experience simulations of situations in order to find and compare the experimental probability to the theoretical probability, students discover that rarely are those probabilities the same.

Students often expect that simulations will result in all of the possibilities. All possibilities may occur in a simulation, but not necessarily. Theoretical probability does use all possibilities.

Note examples in simulations when some possibilities are not shown.

APPENDIX: TABLE 1. Common Addition and Subtraction Situations

Shading taken from OA progression

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Taken from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and variants (blue and green). Yellow indicates problems that are the difficult four problem subtypes or variants that students in Grade 1 work with but do not need to master until Grade 2.

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

TABLE 2. Common Multiplication and Division Situations

Grade level identification of introduction of problem situations taken from OA progression

	Unknown Product	**Group Size Unknown** **("How many in each group?" Division)**	**Number of Groups Unknown** **("How many groups?" Division)**
	3 x 6 = ?	3 x ? = 18; 18 ÷ 3 = ?	? x 6 = 18; 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[4], Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a × b = ?	a × ? = p, and p ÷ a = ?	? × b = p, and p ÷ b = ?

Multiplicative compare problems appear first in Grade 4 (green), with whole number values and with the "times as much" language from the table. In Grade 5, unit fractions language such as "one third as much" may be used. Multiplying and unit language change

TABLE 3. The Properties of Operations

the subject of the comparing sentence ("A red hat costs n times as much as the blue hat" results in the same comparison as "A blue hat is 1/n times as much as the red hat" but has a different subject.)

Name of Property	Representation of Property	Example of Property, Using Real Numbers
Properties of Addition		
Associative	$(a+b)+c=a+(b+c)$	$(78+25)+75=78+(25+75)$
Commutative	$a+b=b+a$	$2+98=98+2$
Additive Identity	$a+0=a$ *and* $0+a=a$	$9875+0=9875$
Additive Inverse	For every real number a, there is a real number $-a$ such that $a+-a=-a+a=0$	$-47+47=0$
Properties of Multiplication		
Associative	$(a\times b)\times c=a\times(b\times c)$	$(32\times 5)\times 2=32\times(5\times 2)$
Commutative	$a\times b=b\times a$	$10\times 38=38\times 10$
Multiplicative Identity	$a\times 1=a$ *and* $1\times a=a$	$387\times 1=387$
Multiplicative Inverse	*For every real number a,* $a\neq 0$, *there is a real number* $\frac{1}{a}$ *such that* $a\times\frac{1}{a}=\frac{1}{a}\times a=1$	$\frac{8}{3}\times\frac{3}{8}=1$
Distributive Property of Multiplication over Addition		
Distributive	$a\times(b+c)=a\times b+a\times c$	$7\times(50+2)=7\times 50+7\times 2$

(Variables a, b, and c represent real numbers.)
Excerpt from NCTM's ***Developing Essential Understanding of Algebraic Thinking****, grades 3-5 p. 16-17*

TABLE 4. The Properties of Equality

Name of Property	Representation of Property	Example of property
Reflexive Property of Equality	$a = a$	$3{,}245 = 3{,}245$
Symmetric Property of Equality	$If\ a = b, then\ b = a$	$2 + 98 = 90 + 10, then\ 90 + 10 = 2 + 98$
Transitive Property of Equality	$If\ a = b\ and\ b = c, then\ a = c$	$If\ 2 + 98 = 90 + 10\ and\ 90 + 10 = 52 + 48$ *then* $2 + 98 = 52 + 48$
Addition Property of Equality	$If\ a = b, then\ a + c = b + c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} + \frac{3}{5} = \frac{2}{4} + \frac{3}{5}$
Subtraction Property of Equality	$If\ a = b, then\ a - c = b - c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} - \frac{1}{5} = \frac{2}{4} - \frac{1}{5}$
Multiplication Property of Equality	$If\ a = b, then\ a \times c = b \times c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} \times \frac{1}{5} = \frac{2}{4} \times \frac{1}{5}$
Division Property of Equality	$If\ a = b\ and\ c \neq 0, then\ a \div c = b \div c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} \div \frac{1}{5} = \frac{2}{4} \div \frac{1}{5}$
Substitution Property of Equality	If $a = b$, then *b* may be substituted for *a* in any expression containing *a*.	$If\ 20 = 10 + 10$ *then* $90 + 20 = 90 + (10 + 10)$

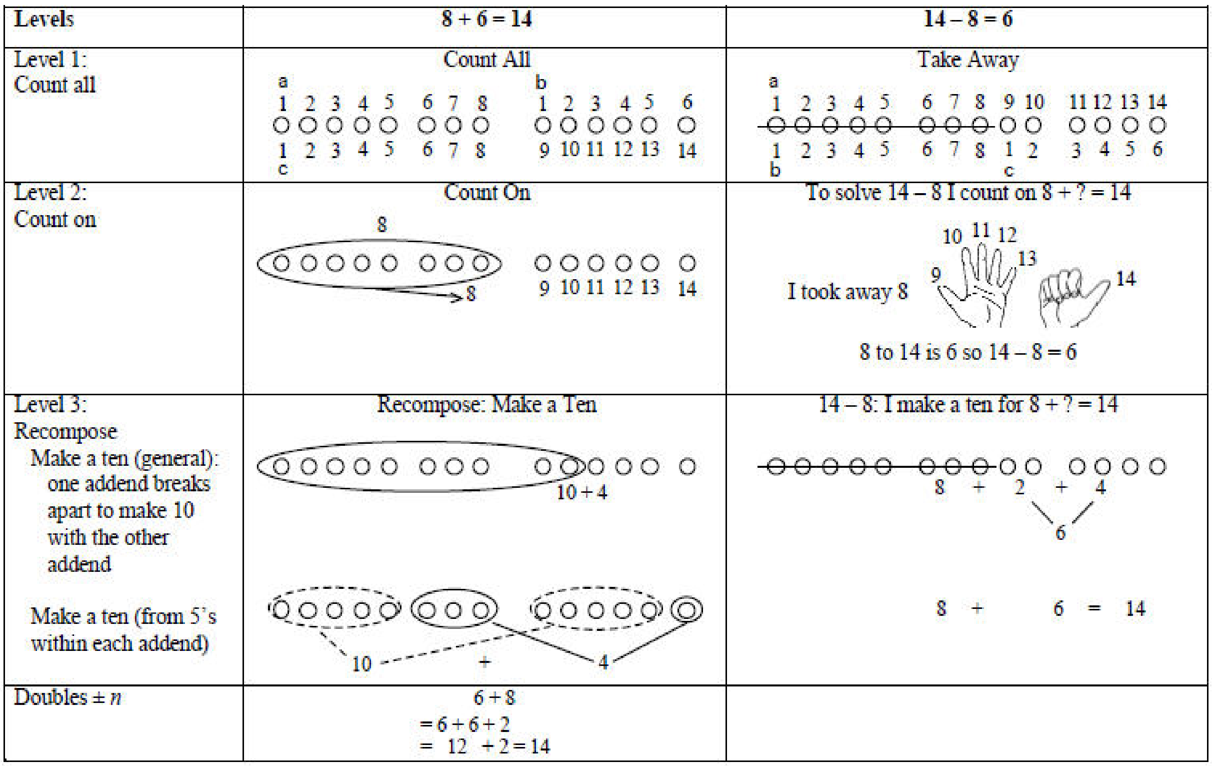
(Variables a, b, and c can represent any number in the rational, real, or complex number systems.)

TABLE 5. The Properties of Inequality

Exactly one of the following is true: $a < b, a = b, a > b$.
$If\ a > b\ and\ b > c\ then\ a > c$.
$If\ a > b, then\ b < a$.
$If\ a > b, then - a < -b$.
$If\ a > b, then\ a \pm c > b \pm c$.
$If\ a > b\ and\ c > 0, then\ a \times c > b \times c$.
$If\ a > b\ and\ c < 0, then\ a \times c < b \times c$.
$If\ a > b\ and\ c > 0, then\ a \div c > b \div c$.
$If\ a > b\ and\ c < 0, then\ a \div c < b \div c$.

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

TABLE 6. Development of Counting in K-2 Children



Levels	8 + 6 = 14	14 – 8 = 6
Level 1: Count all	Count All a 1 2 3 4 5 6 7 8 / b 1 2 3 4 5 6 1 2 3 4 5 6 7 8 9 10 11 12 13 14 c	Take Away a 1 2 3 4 5 6 7 8 9 10 11 12 13 14 b 1 2 3 4 5 6 7 8 1 2 c 3 4 5 6
Level 2: Count on	Count On 8 8 9 10 11 12 13 14	To solve 14 – 8 I count on 8 + ? = 14 I took away 8 9 10 11 12 13 14 8 to 14 is 6 so 14 – 8 = 6
Level 3: Recompose Make a ten (general): one addend breaks apart to make 10 with the other addend Make a ten (from 5's within each addend)	Recompose: Make a Ten 10 + 4 10 + 4	14 – 8: I make a ten for 8 + ? = 14 8 + 2 + 4 6 8 + 6 = 14
Doubles ± n	6 + 8 = 6 + 6 + 2 = 12 + 2 = 14	

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

Beginning--A child can count very small collections (1-4) collection of items and understands that the last word tells "how many" even. Beyond on small numbers the number words may be said without the lack of numerical understanding. This is often referred to as rote counting.

Level 1—The child uses one to one correspondence to find the number of objects in two sets. Even if the quantity is known for the first set, the child will start with the first set and continue counting on into the second set. The child begins the count with one. This also connects to Piaget's Hierarchical Inclusion – that in order to have 7 – you have to have 6, 5, 4, etc.

Level 2 – At this level the student begins the counting, starting with the known quantity of the first set and "counts on" from that point in the sequence to establish how many. This method is used to find the total in two sets. This counting is not rote. This level of counting requires several connections between cardinality and counting meanings of the number words

Table 7. Cognitive Rigor Matrix/Depth of Knowledge (DOK)

Kansas Math Standards require high-level cognitive demand asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target the depth(s) of knowledge (DOK) that the student needs to bring to the item/task will be identified, using the Cognitive Rigor Matrix shown below.

Depth of Thinking (Webb)+ Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	• Recall conversions, terms, facts			
Understand	• Evaluate an expression • Locate points on a grid or number on number line • Solve a one-step problem • Represent math relationships in words, pictures, or symbols	• Specify, explain relationships • Make basic inferences or logical predictions from data/observations • Use models/diagrams to explain concepts • Make and explain estimates	• Use concepts to solve non-routine problems • Use supporting evidence to justify conjectures, generalize, or connect ideas • Explain reasoning when more than one response is possible • Explain phenomena in terms of concepts	• Relate mathematical concepts to other content areas, other domains • Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	• Follow simple procedures • Calculate, measure, apply a rule (e.g., rounding) • Apply algorithm or formula • Solve linear equations • Make conversions	• Select a procedure and perform it • Solve routine problem applying multiple concepts or decision points • Retrieve information to solve a problem • Translate between representations	• Design investigation for a specific purpose or research question • Use reasoning, planning, and supporting evidence • Translate between problem & symbolic notation when not a direct translation	• Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	• Retrieve information from a table or graph to answer a question • Identify a pattern/trend	• Categorize data, figures • Organize, order data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern	• Compare information within or across data sets or texts • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph	• Analyze multiple sources of evidence or data sets
Evaluate			• Cite evidence and develop a logical argument • Compare/contrast solution methods • Verify reasonableness	• Apply understanding in a novel way, provide argument or justification for the new application
Create	• Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	• Generate conjectures or hypotheses based on observations or prior knowledge and experience	• Develop an alternative solution • Synthesize information within one data set	• Synthesize information across multiple sources or data sets • Design a model to inform and solve a practical or abstract situation

References, Resources, and Links

1. Achieve the Core. http://www.achievethecore.org/.
2. Bolam, R., McMahon, A., Stoll, L., Thomas, S., & Wallace, M. (2005). Creating and sustaining professional learning communities. *Research Report Number 637*. General Teaching Council for England. London, England: Department for Education and Skills.
3. Clements, D. & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach*. New York, NY: Routledge
4. Croft, A., Coggshall, J. G., Dolan, M., Powers, E., with Killion, J. (2010). *Job-embedded professional development: What it is, who is responsible, and how to get it done well.* National Comprehensive Center for Teacher Quality. Retrieved March 11, 2013, from http://www.tqsource.org/
5. Darling-Hammond, L., Wei, R.C., Andree, A., Richardson, N., & Orphanos, S. (2009). *Professional learning in the learning profession: a status report on teacher development in the United States and abroad.* Oxford, OH: National Staff Development Council and the School Redesign Network at Stanford University.
6. engageNY Modules: http://www.engageny.org/mathematicshttp://www.engageny.org/mathematics
7. Focus by Grade Level, Content Emphases by Jason Zimba: http://achievethecore.org/page/774/focus-by-grade-level
8. Garet, M. S., Porter, A. C., Desimone, L., Birman, B., & Yoon, K. (2001). What makes professional development effective? *American Educational Research Journal,* 38(4), 915–945. Retrieved March 11, 2013, from www.aztla.asu.edu/ProfDev1.pdf
9. Georgie Frameworks: https://www.georgiastandards.org/Standards/Pages/BrowseStandards/MathStandards9-12.aspx
10. Guskey, T. (2000). *Evaluating professional development.* Thousand Oaks, CA: Corwin Press.
11. Illustrative Mathematics. http://www.illustrativemathematics.org/.
12. Inside Mathematics. http://www.insidemathematics.org/.
13. Kanold, T. & Larson, M. (2012). *Common Core Mathematics in a PLC at Work, Leaders Guide.*
14. Kansas Mathematics Standards (CCSS). (2012). http://www.ksde.org/Default.aspx?tabid=4754
15. Killion, J. (2002). *Assessing impact: Evaluating staff development.* Oxford, OH: National Staff Development Council.
16. Learning Forward. (2011). *Standards for Professional Learning.*
17. Learn NC. http://www.learnnc.org/lp/editions/ccss2010-mathematics.
18. Learn Zillion. http://learnzillion.com/.
19. Mathematics Assessment Project. http://map.mathshell.org/materials/index.php.
20. McCallum, W., Daro, P., & Zimba, J. *Progressions Documents for the Common Core Math Standards.* Retrieved March 11, 2013, from *http://ime.math.arizona.edu/progressions/#about* .
21. Mid-continent Research for Education and Learning. (2000). *Principles in action: Stories of award-winning professional development* [video]. Aurora, CO: Author.
22. National Council of Teachers of Mathematics. (1991). *Professional Standards for Teaching Mathematics.*
23. National Council of Teachers of Mathematics. (2000). *Principles & Standards for School Mathematics.*
24. National Council of Teachers of Mathematics. (2006). *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence.*
25. National Council of Teachers of Mathematics. (2009) *Focus in high school mathematics: Reasoning and sense making.* Reston, VA: NCTM.
26. National Council of Teachers of Mathematics. (2010). *Developing essential understanding of algebraic thinking*. Reston, VA: NCTM.
27. National Council of Teachers of Mathematics. (2010). *Developing essential understanding of number and*

numeration. Reston, VA: NCTM.

28. National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: National Council of Teachers of Mathematics.
29. National Council of Teachers of Mathematics. *Core Math Tools. http://www.nctm.org/resources/content.aspx?id=32702.*
30. National Council of Teachers of Mathematics. *Illuminations. http://illuminations.nctm.org/.*
31. National Mathematics Advisory Panel. (2008). *The final report of the national mathematics advisory panel.* U.S. Department of Education.
32. National Staff Development Council. (2001). *Standards for staff development.* Oxford, OH: Author. Retrieved March 11, 2013, from http://www.nsdc.org/standards/index.cfm
33. Porter, A., Garet, M., Desimone, L., Yoon, K., & Birman, B. (2000). *Does professional development change teaching practice? Results from a three-year study.* Washington, DC: U.S. Department of Education. Retrieved March 11, 2013, from http://www.ed.gov/rschstat/eval/teaching/epdp/report.pdf
34. Publishers Criteria: www.corestandards.org
35. South Dakota Department of Education. *Curation Review Rubric for Mathematics Learning Resources.* Retrieved March 11, 2013, from http://www.livebinders.com/play/play?id=367846
36. Steiner, L. (2004). *Designing effective professional development experiences: What do we know?* Naperville, IL: Learning Point Associates.
37. Tools for the Common Core Standards. http://commoncoretools.me/.
38. Van de Walle, J., Lovin, L., Karp, K., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: developmentally appropriate instruction for grades pre-K – 2*. Boston, MA:Pearson.
39. Wisconsin Department of Public Instruction. (2010) *Characteristics of high-quality and evidence-based professional learning*. Retrieved March 11, 2013 from, http://www.betterhighschools.org/MidwestSIG/documents/Rasmussen_Handout1.pdf
40. Wright, R., Stranger, G., Stafford, A., & Martland, J. (2006). *Teaching number in the classroom with 4-8 year olds*. London: Sage.
41. Yoon, K. S., Duncan T., Lee, S. W.-Y., Scarloss, B., & Shapley, K. L. (2007). *Reviewing the evidence on how teacher professional development affects student achievement*. (Issues & Answers Report, REL 2007–No. 033). Washington, DC: National Center for Education Evaluation and Regional Assistance, Regional Education Laboratory Southwest. Retrieved March 11, 2013, from http://ies.ed.gov/