3/13/2019

2017 Kansas Mathematics Standards

Flip Book

2nd Grade

This project used work created by the Departments of Education in Ohio, North Carolina, Georgia and resources created by Achieve the Core, EngageNY, Illustrative Mathematics, and NCTM.

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About the Flip Books

This project attempts to organize some of the most valuable resources that help develop the intent, understanding and implementation of the 2017 Kansas Mathematics Standards. These documents provide a starting point for teachers and administrators to begin discussion and exploration into the standards. It is not the only resource to support implementation of the 2017 Kansas Mathematics Standards.

This project is built on the previous work started in the summer of 2012 from Melisa Hancock (Manhattan, KS), Debbie Thompson (Wichita, KS) and Patricia Hart (Wichita, KS) who provided the initial development of the “flip books.” The “flip books” are based on a model that Kansas had for earlier standards; however, this edition specifically targets the Kansas Mathematics Standards that were adopted in the summer of 2017. These flip books incorporate the resources from other state departments of education, the mathematics learning progressions, and other reliable sources including The National Council of Teachers of Mathematics and the National Supervisors of Mathematics. In addition, mathematics educators across the country have suggested changes/additions that could or should be made to further enhance its effectiveness. The document is posted on the KSDE Mathematics website at http://community.ksde.org/Default.aspx?tabid=5646 and will continue to undergo changes periodically. When significant changes/additions are implemented, the modifications will be posted and dated.

Planning Advice - Focus on the Clusters

The (mathematics standards) call for a greater focus. Rather than racing to cover topics in today's mile-wide, inch-deep curriculum, we need to use the power of the eraser and significantly narrow and deepen how time and energy is spent in the mathematics classroom. There is a necessity to focus deeply on the major work of each grade to enable students to gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems both in and out of the mathematics classroom. (www.achievethecore.org)

Not all standards should have the same instructional emphasis. Some groups of standards require a greater emphasis than others. In order to be intentional and systematic, priorities need to be set for planning, instruction, and assessment. "Not everything in the Standards should have equal priority" (Zimba, 2011). Therefore, there is a need to elevate the content of some standards over that of others throughout the K-12 curriculum.

When the Standards were developed the following were considerations in the identification of priorities: 1) the need to be qualitative and well-articulated; 2) the understanding that some content will become more important than other; 3) the creation of a focus means that some essential content will get a greater share of the time and resources "while the remaining content is limited in scope." 4) a "lower" priority does not imply exclusion of content, but is usually intended to be taught in conjunction with or in support of one of the major clusters.

"The Standards are built on the progressions, so priorities have to be chosen with an eye to the arc of big ideas in the Standards. A prioritization scheme that respects progressions in the Standards will strike a balance between the journey and the endpoint. If the endpoint is everything, few will have enough wisdom to walk the path, if the endpoint is nothing, few will understand where the journey is headed. Beginnings and the endings both need particular care. … It would also be a mistake to identify such standard as a locus of emphasis. (Zimba, 2011)

The important question in planning instruction is: "What is the mathematics you want the student to walk away with?" In order to accomplish this, educators need to think about "grain size" when planning instruction. Grain size corresponds to the knowledge you want the student to know. Mathematics is simplest at the right grain size. According to Phil Daro (*Teaching Chapters, Not Lessons—Grain Size of Mathematics*), strands are too vague and too large a grain size, while lessons are too small a grain size. Units or chapters produce about the right "grain size". In the planning process educators should attend to the clusters, and think of the standards as the ingredients of a cluster. Coherence of mathematical ideas and concepts exists at the cluster level across grades.

A caution--Grain size is important but can result in conversations that do not advance the intent of this structure. Extended discussions among teachers where it is argued for "2 days" instead of "3 days" on a topic because it is a lower priority can detract from the overall intent of suggested priorities. The reverse is also true. As Daro indicates, focusing on lessons can provide too narrow a view which compromises the coherence value of closely related standards.

The video clip Teaching Chapters, Not Lessons—Grain Size of Mathematics presents Phil Daro further explaining grain size and the importance of it in the planning process. (Click on photo to view video.)

Along with "grain size", clusters have been given **priorities** which have important implications for instruction. These priorities should help guide the focus for teachers as they determine allocation of time for both planning and instruction. The priorities provided help guide the focus for teachers as they determine distribution of time for both planning and instruction, helping to assure that students really understand mathematics before moving on. Each cluster has been given a priority level. As professional educators begin planning, developing and writing units, these priorities provide guidance in assigning time for instruction and formative assessment within the classroom.

Each cluster within the standards has been given a priority level influenced by the work of Jason Zimba. The three levels are referred to as — **Major, Supporting** and **Additional**. Zimba suggests that about 70% of instruction should relate to the **Major** clusters. The lower two priorities (**Supporting** and **Additional**) can work together by supporting the **Major** priorities. You can find the grade Level Focus Documents for the 2017 Kansas Math Standards at: http://community.ksde.org/Default.aspx?tabid=6340.

Recommendations for Cluster Level Priorities

Appropriate Use:

- Use the priorities as guidance to inform instructional decisions regarding time and resources spent on clusters by varying the degrees of emphasis.
- Focus should be on the major work of the grade in order to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through sense-making, reasoning, arguing and critiquing, modeling, etc.
- Evaluate instructional materials by taking the cluster level priorities into account. The major work of the grade must be presented with the highest possible quality; the additional work of the grade should support the major priorities and not detract from them.
- Set priorities for other implementation efforts such as staff development, new curriculum development, and revision of existing formative or summative testing at the state, district or school level.

Things to Avoid:

- Neglecting any of the material in the standards. Seeing Supporting and Additional clusters as optional.
- Sorting clusters (from Major to Supporting to Additional) and then teaching the clusters in order. This would remove the coherence of mathematical ideas and create missed opportunities to enhance the major work of the grade with the other clusters.
- Using the cluster headings as a replacement for the actual standards. All features of the standards matter—from the practices to surrounding text, including the particular wording of the individual content standards. Guidance for priorities is given at the cluster level as a way of thinking about the content with the necessary specificity yet without going so far into detail as to comprise the coherence of the standards (grain size).

Mathematics Teaching Practices

(High Leverage Teacher Actions)

[National Council of Teachers of Mathematics. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: National Council of Teachers of Mathematics.]

The eight Mathematics Teaching Practices should be the foundation for mathematics instruction and learning. This framework was informed by over twenty years of research and presented in *Principles to Actions* by the National Council of Teachers of Mathematics (NCTM). If teachers are guided by this framework, they can move “toward improved instructional practice” and support “one another in becoming skilled at teaching in ways that matter for ensuring successful mathematics learning for all students” (NCTM, 2014, p. 12).

1. **Establish mathematics goals to focus learning.**
   Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
2. **Implement tasks that promote reasoning and problem solving.**
   Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
3. **Use and connect mathematical representations.**
   Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
4. **Facilitate meaningful mathematical discourse.**
   Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
5. **Pose purposeful questions.**
   Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.
6. **Build procedural fluency from conceptual understanding.**
   Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
7. **Support productive struggle in learning mathematics.**
   Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
8. **Elicit and use evidence of student thinking.**
   Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Standards for Mathematical Practice in Second Grade

The Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students grades K-12. Below are a few examples of how these Practices may be integrated into tasks that Grade 2 students complete.

Practices	Explanations and Examples
1) Make Sense and Persevere in Solving Problems.	Mathematically proficient students in Second Grade examine problems and tasks, can make sense of the meaning of the task and find an entry point or a way to start the task. Second Grade students also develop a foundation for problem solving strategies and become independently proficient on using those strategies to solve new tasks. In Second Grade, students' work continues to use concrete manipulatives and pictorial representations as well as mental mathematics. Second Grade students also are expected to persevere while solving tasks; that is, if students reach a point in which they are stuck, they can reexamine the task in a different way and continue to solve the task. Lastly, mathematically proficient students complete a task by asking themselves the question, "Does my answer make sense?"
2) Reason abstractly and quantitatively.	Mathematically proficient students in Second Grade make sense of quantities and relationships while solving tasks. This involves two processes - decontexualizing and contextualizing. In Second Grade, students represent situations by decontextualizing tasks into numbers and symbols. For example, in the task, "There are 25 children in the cafeteria and they are joined by 17 more children. How many students are in the cafeteria? " Second Grade students translate that situation into an equation, such as: 25 + 17 = __ and then solve the problem. Students also contextualize situations during the problem solving process. The processes of reasoning are in other areas of mathematics, such as determining the length of quantities when measuring with standard units.
3) Construct viable arguments and critique the reasoning of others.	Mathematically proficient students in Second Grade accurately use definitions and previously established solutions to construct viable arguments about mathematics. During discussions about problem solving strategies, students constructively critique the strategies and reasoning of their classmates. For example, while solving 74 - 18, students may use a variety of strategies, and after working on the task, can discuss and critique each other's' reasoning and strategies, citing similarities and differences between strategies.
4) Model with mathematics.	Mathematically proficient students in Second Grade model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. Second Grade students use concrete manipulatives and pictorial representations to provide further explanation of the equation. Likewise, Second Grade students are able to create an appropriate problem situation from an equation. For example, students are expected to create a story problem for the equation 43 + 17 = ___ such as "There were 43 gumballs in the machine. Tom poured in 17 more gumballs. How many gumballs are now in the machine?"

Practice	Explanation and Example
5) Use appropriate tools strategically.	Mathematically proficient students in Second Grade have access to and use tools appropriately. These tools may include connecting cubes, place value (base ten) blocks, hundreds boards, number lines, rulers, and concrete geometric shapes (e.g., pattern blocks, 3-D solids). Students also have experiences with educational technologies, such as calculators and virtual manipulatives, which support conceptual understanding and higher-order thinking skills. During classroom instruction, students have access to various mathematical tools as well as paper, and determine which tools are the most appropriate to use. For example, while measuring the length of the hallway, students can explain why a yardstick is more appropriate to use than a foot ruler OR when solving $58 + 27$, students can explain why place value blocks are more appropriate than counters as a manipulative.
6) Attend to precision.	Mathematically proficient students in Second Grade are precise in their communication, calculations, and measurements. In all mathematical tasks, students in Second Grade communicate clearly, using grade-level appropriate vocabulary accurately as well as giving precise explanations and reasoning regarding their process of finding solutions. For example, while measuring an object, care is taken to line up the tool correctly in order to get an accurate measurement. During tasks involving number sense, students consider if their answer is reasonable and check their work to ensure the accuracy of solutions.
7) Look for and make use of structure.	Mathematically proficient students in Second Grade carefully look for patterns and structures in the number system and other areas of mathematics. For example, students notice number patterns within the tens place as they connect skip count by 10s off the decade to the corresponding numbers on a 100s chart. While working in the Numbers and Operations in Base Ten domain, students work with the idea that 10 ones equals a ten, and 10 tens equals 1 hundred. In addition, Second Grade students also make use of structure when they work with subtraction as missing addend problems, such as 50- 33 = __ can be written as 33+ __ = 50 and can be thought of as," How much more do I need to add to 33 to get to 50?"
8) Look for and express regularity in repeated reasoning.	Mathematically proficient students in Second Grade begin to look for regularity in problem structures when solving mathematical tasks. For example, after solving two digit addition problems by decomposing numbers (33+ 25 = 30 + 20 + 3 +5), students may begin to generalize and frequently apply that strategy independently on future tasks. Further, students begin to look for strategies to be more efficient in computations, including doubles strategies and making a ten. Lastly, while solving all tasks, Second Grade students accurately check for the reasonableness of their solutions during and after completing the task.

Adapted from the work of the State Department of Education of North Carolina.

Implementing Standards for Mathematical Practice

This guide was created to help educators implement these standards into their classroom instruction. These are the practices for the **students,** and the teacher can assist students in using them efficiently and effectively.

#1 – Make sense of problems and persevere in solving them.

Summary of this Practice:

- Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem.
- Plan a solution pathway instead of jumping to a solution.
- Monitor their progress and change the approach if necessary.
- See relationships between various representations.
- Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another.
- Continually ask themselves, "Does this make sense?"
- Understand various approaches to solutions.

Student Actions	Teacher Actions
• Actively engage in solving problems and thinking is visible (doing mathematics vs. following steps or procedures with no understanding). • Relate current "situation" to concepts or skills previously learned, and checking answers using different methods. • Monitor and evaluate their own progress and change course when necessary. • Always ask, "Does this make sense?" as they are solving problems.	• Allow students time to initiate a plan; using question prompts as needed to assist students in developing a pathway. • Constantly ask students if their plans and solutions make sense. • Question students to see connections to previous solution attempts and/or tasks to make sense of the current problem. • Consistently ask students to defend and justify their solution(s) by comparing solution paths.

What questions develop this Practice?

- How would you describe the problem in your own words? How would you describe what you are trying to find?
- What do you notice about...?
- What information is given in the problem? Describe the relationship between the quantities.
- Describe what you have already tried. What might you change? Talk me through the steps you've used to this point.
- What steps in the process are you most confident about? What are some other strategies you might try?
- What are some other problems that are similar to this one?
- How might you use one of your previous problems to help you begin? How else might you organize...represent...show...?

What are the characteristics of a good math task for this Practice?

- Requires students to engage with conceptual ideas that underlie the procedures to complete the task and develop understanding.
- Requires cognitive effort - while procedures may be followed, the approach or pathway is not explicitly suggested by the task, or task instructions and multiple entry points are available.
- Encourages multiple representations, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations to develop meaning.
- Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.

#2 – Reason abstractly and quantitatively.

Summary of this Practice:

- Make sense of quantities and their relationships.
- Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships.
- Understand the meaning of quantities and are flexible in the use of operations and their properties.
- Create a logical representation of the problem.
- Attend to the meaning of quantities, not just how to compute them.

Student Actions	Teacher Actions
• Use varied representations and approaches when solving problems. • Represent situations symbolically and manipulating those symbols easily. • Give meaning to quantities (not just computing them) and making sense of the relationships within problems.	• Ask students to explain the meaning of the symbols in the problem and in their solution. • Expect students to give meaning to all quantities in the task. • Question students so that understanding of the relationships between the quantities and/or the symbols in the problem and the solution are fully understood.

What questions develop this Practice?

- What do the numbers used in the problem represent? What is the relationship of the quantities?
- How is____related to_____?
- What is the relationship between ____ and ____?
- What does ______ mean to you? (e.g. symbol, quantity, diagram)
- What properties might you use to find a solution?
- How did you decide that you needed to use ____? Could we have used another operation or property to solve this task? Why or why not?

What are the characteristics of a good math task for this Practice?

- Includes questions that require students to attend to the meaning of quantities and their relationships, not just how to compute them.
- Consistently expects students to convert situations into symbols in order to solve the problem; and then requires students to explain the solution within a meaningful situation.
- Contains relevant, realistic content.

#3 – Construct viable arguments and critique the reasoning of others.

Summary of this Practice:

- Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments.
- Justify conclusions with mathematical ideas.
- Listen to the arguments of others and ask useful questions to determine if an argument makes sense.
- Ask clarifying questions or suggest ideas to improve/revise the argument.
- Compare two arguments and determine correct or flawed logic.

Student Actions	Teacher Actions
• Make conjectures and exploring the truth of those conjectures. • Recognize and use counter examples. • Justify and defend all conclusions and using data within those conclusions. • Recognize and explain flaws in arguments, which may need to be demonstrated using objects, pictures, diagrams, or actions.	• Encourage students to use proven mathematical understandings, (definitions, properties, conventions, theorems etc.), to support their reasoning. • Question students so they can tell the difference between assumptions and logical conjectures. • Ask questions that require students to justify their solution and their solution pathway. • Prompt students to respectfully evaluate peer arguments when solutions are shared. • Ask students to compare and contrast various solution methods • Create various instructional opportunities for students to engage in mathematical discussions (whole group, small group, partners, etc.)

What questions develop this Practice?

- What mathematical evidence would support your solution? How can we be sure that...? How could you prove that...?
- Will it still work if...?
- What were you considering when...? How did you decide to try that strategy?
- How did you test whether your approach worked?
- How did you decide what the problem was asking you to find? (What was unknown?)
- Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not?
- What is the same and what is different about...? How could you demonstrate a counter-example?

What are the characteristics of a good math task for this Practice?

- Structured to bring out multiple representations, approaches, or error analysis.
- Embeds discussion and communication of reasoning and justification with others.
- Requires students to provide evidence to explain their thinking beyond merely using computational skills to find a solution.
- Expects students to give feedback and ask questions of others' solutions.

#4 – Model with mathematics.

Summary of this Practice:

- Understand reasoning quantitatively and abstractly (able to decontextualize and contextualize).
- Apply the math they know to solve problems in everyday life.
- Simplify a complex problem and identify important quantities to look at relationships.
- Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation.
- Reflect on whether the results make sense, possibly improving/revising the model.
- Ask themselves, "How can I represent this mathematically?"

Student Actions	Teacher Actions
• Apply mathematics to everyday life. • Write equations to describe situations. • Illustrate mathematical relationships using diagrams, data displays, and/or formulas. • Identify important quantities and analyzing relationships to draw conclusions.	• Demonstrate and provide students experiences with the use of various mathematical models. • Question students to justify their choice of model and the thinking behind the model. • Ask students about the appropriateness of the model chosen. • Assist students in seeing and making connections among models.

What questions develop this Practice?

- What number model could you construct to represent the problem?
- How can you represent the quantities?
- What is an equation or expression that matches the diagram..., number line..., chart..., table...?
- Where did you see one of the quantities in the task in your equation or expression?
- What math do you know that you could use to represent this situation?
- What assumptions do you have to make to solve the problem?
- What formula might apply in this situation?

What are the characteristics of a good math task for this Practice?

- Structures represent the problem situation and their solution symbolically, graphically, and/or pictorially (may include technological tools) appropriate to the context of the problem.
- Invites students to create a context (real-world situation) that explains numerical/symbolic representations.
- Asks students to take complex mathematics and make it simpler by creating a model that will represent the relationship between the quantities.

#5 – Use appropriate tools strategically.

Summary of this Practice:

- Use available tools recognizing the strengths and limitations of each.
- Use estimation and other mathematical knowledge to detect possible errors.
- Identify relevant external mathematical resources to pose and solve problems.
- Use technological tools to deepen their understanding of mathematics.
- Use mathematical models for visualize and analyze information

Student Actions	Teacher Actions
• Choose tools that are appropriate for the task. • Know when to use estimates and exact answers. • Use tools to pose or solve problems to be most effective and efficient.	• Demonstrate and provide students experiences with the use of various math tools. A variety of tools are within the environment and readily available. • Question students as to why they chose the tools they used to solve the problem. • Consistently model how and when to estimate effectively, and requiring students to use estimation strategies in a variety of situations. • Ask student to explain their mathematical thinking with the chosen tool. • Ask students to explore other options when some tools are not available.

What questions develop this practice?

- What mathematical tools could we use to visualize and represent the situation?
- What information do you have?
- What do you know that is not stated in the problem? What approach are you considering trying first?
- What estimate did you make for the solution?
- In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? Why was it helpful to use...?
- What can using a______show us that ______may not?
- In what situations might it be more informative or helpful to use...?

What are the characteristics of a good math task for this Practice?

- Lends itself to multiple learning tools. (Tools may include; concrete models, measurement tools, graphs, diagrams, spreadsheets, statistical software, etc.)
- Requires students to determine and use appropriate tools to solve problems.
- Asks students to estimate in a variety of situations:
  - a task when there is no need to have an exact answer
  - a task when there is not enough information to get an exact answer
  - a task to check if the answer from a calculation is reasonable

#6 – Attend to precision.

Summary of this Practice:

- Communicate precisely with others and try to use clear mathematical language when discussing their reasoning.
- Understand meanings of symbols used in mathematics and can label quantities appropriately.
- Express numerical answers with a degree of precision appropriate for the problem context.
- Calculate efficiently and accurately.

Student Actions	Teacher Actions
• Use mathematical terms, both orally and in written form, appropriately. • Use and understanding the meanings of math symbols that are used in tasks. • Calculate accurately and efficiently. • Understand the importance of the unit in quantities.	• Consistently use and model correct content terminology. • Expect students to use precise mathematical vocabulary during mathematical conversations. • Question students to identify symbols, quantities and units in a clear manner.

What questions develop this Practice?

- What mathematical terms apply in this situation? How did you know your solution was reasonable?
- Explain how you might show that your solution answers the problem.
- Is there a more efficient strategy?
- How are you showing the meaning of the quantities?
- What symbols or mathematical notations are important in this problem?
- What mathematical language..., definitions..., properties can you use to explain...?
- How could you test your solution to see if it answers the problem?

What are the characteristics of a good math task for this Practice?

- Requires students to use precise vocabulary (in written and verbal responses) when communicating mathematical ideas.
- Expects students to use symbols appropriately.
- Embeds expectations of how precise the solution needs to be (some may more appropriately be estimates).

#7 – Look for and make use of structure.

Summary of this Practice:

- Apply general mathematical rules to specific situations.
- Look for the overall structure and patterns in mathematics.
- See complicated things as single objects or as being composed of several objects.

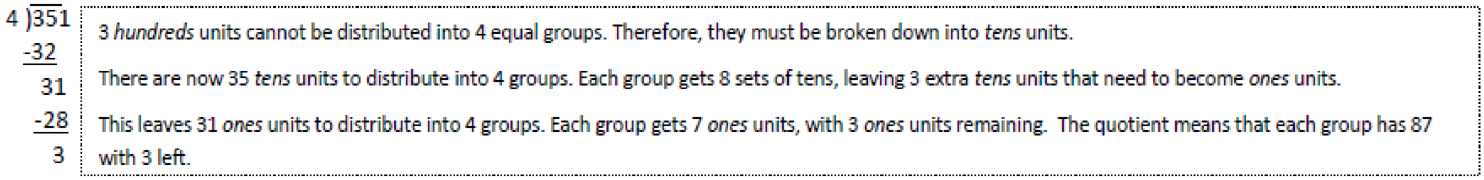
Student Actions	Teacher Actions
• Look closely at patterns in numbers and their relationships to solve problems. • Associate patterns with the properties of operations and their relationships. • Compose and decompose numbers and number sentences/expressions.	• Encourage students to look for something they recognize and having students apply the information in identifying solution paths (i.e. compose/decompose numbers and geometric figures, identify properties, operations, etc.) • Expect students to explain the overall structure of the problem and the big math idea used to solve the problem.

What questions develop this Practice?

- What observations do you make about...? What do you notice when...?
- What parts of the problem might you eliminate..., simplify...?
- What patterns do you find in...?
- How do you know if something is a pattern?
- What ideas that we have learned before were useful in solving this problem?
- What are some other problems that are similar to this one? How does this relate to...?
- In what ways does this problem connect to other mathematical concepts?

What are the characteristics of a good math task for this Practice?

- Requires students to look for the structure within mathematics in order to solve the problem. (i.e. – decomposing numbers by place value; working with properties; etc.)
- Asks students to take a complex idea and then identify and use the component parts to solve problems. i.e. Building on the structure of equal sharing, students connect the understanding to the traditional division algorithm. When "unit size" cannot be equally distributed, it is necessary to break down into a smaller "unit size". (example below)



$$\begin{array}{r} 4\,\overline{)351} \\ -32 \\ 31 \\ -28 \\ 3 \end{array}$$

3 *hundreds* units cannot be distributed into 4 equal groups. Therefore, they must be broken down into *tens* units.

There are now 35 *tens* units to distribute into 4 groups. Each group gets 8 sets of tens, leaving 3 extra *tens* units that need to become *ones* units.

This leaves 31 *ones* units to distribute into 4 groups. Each group gets 7 *ones* units, with 3 *ones* units remaining. The quotient means that each group has 87 with 3 left.

- Expects students to recognize and identify structures from previous experience(s) and apply this understanding in a new situation. i.e. $7 \times 8 = (7 \times 5) + (7 \times 3)$ OR $7 \times 8 = (7 \times 4) + (7 \times 4)$ new situations could be, distributive property, area of composite figures, multiplication fact strategies.

#8 – Look for and express regularity in repeated reasoning.

Summary of this Practice:

- See repeated calculations and look for generalizations and shortcuts.
- See the overall process of the problem and still attend to the details.
- Understand the broader application of patterns and see the structure in similar situations.
- Continually evaluate the reasonableness of their intermediate results.

Student Actions	Teacher Actions
• Notice if processes are repeated and look for both general methods and shortcuts. • Evaluate the reasonableness of intermediate results while solving. • Make generalizations based on discoveries and constructing formulas when appropriate.	• Ask what math relationships or patterns can be used to assist in making sense of the problem. • Ask for predictions about solutions at midpoints throughout the solution process. • Question students to assist them in creating generalizations based on repetition in thinking and procedures.

What questions develop this Practice?

- Will the same strategy work in other situations?
- Is this always true, sometimes true or never true? How would we prove that...?
- What do you notice about...?
- What is happening in this situation? What would happen if…?
- Is there a mathematical rule for...?
- What predictions or generalizations can this pattern support? What mathematical consistencies do you notice?

What are the characteristics of a good math task for this Practice?

- Present several opportunities to reveal patterns or repetition in thinking, so students can make a generalization or rule.
- Requires students to see patterns or relationships in order to develop a mathematical rule.
- Expects students to discover the underlying structure of the problem and come to a generalization.
- Connects to a previous task to extend learning of a mathematical concept.

Critical Areas for Mathematics in 2nd Grade

In Grade 2, instructional time should focus on **four** critical areas:

1. **Extending understanding of base-ten notation.**
Students extend their understanding of the base-ten system. This includes ideas of counting in twos, fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones). Students extend this understanding to include decomposition of numbers to assist with later work in operations (e.g., 853 can also be decomposed into 85 tens and 3 ones OR 7 hundreds, 15 tens, and 3 ones OR 8 hundreds, 4 tens, and 13 ones, etc.)

2. **Building fluency with addition and subtraction.**
Students use their understanding of addition to develop <u>fluency</u> (efficiency, accuracy, and flexibility) with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods (students are expected to use more than the traditional algorithm) to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations (e.g., Commutative Property and Associative Property). They select and accurately apply methods that are appropriate for the context and the numbers involved to <u>mentally</u> calculate sums and differences for numbers with only tens or only hundreds. Students understand that a word problem can be represented with an equation based on the situation, but the solution may use a related equation that is easier to manipulate (e.g., a word problem may be represented with a situation equation such as $25+?=62$; and students understand that even though the word problem is a joining situation, it is easier to solve using a subtraction equation $\{62-25=?\}$).

3. **Using standard units of measure.**
Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

4. **Describing and analyzing shapes.**
Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Dynamic Learning Maps (DLM) and Essential Elements

The Dynamic Learning Maps and Essential Elements are knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities.

For more information please visit the Dynamic Learning Maps and Essential Elements website.

Growth Mindset

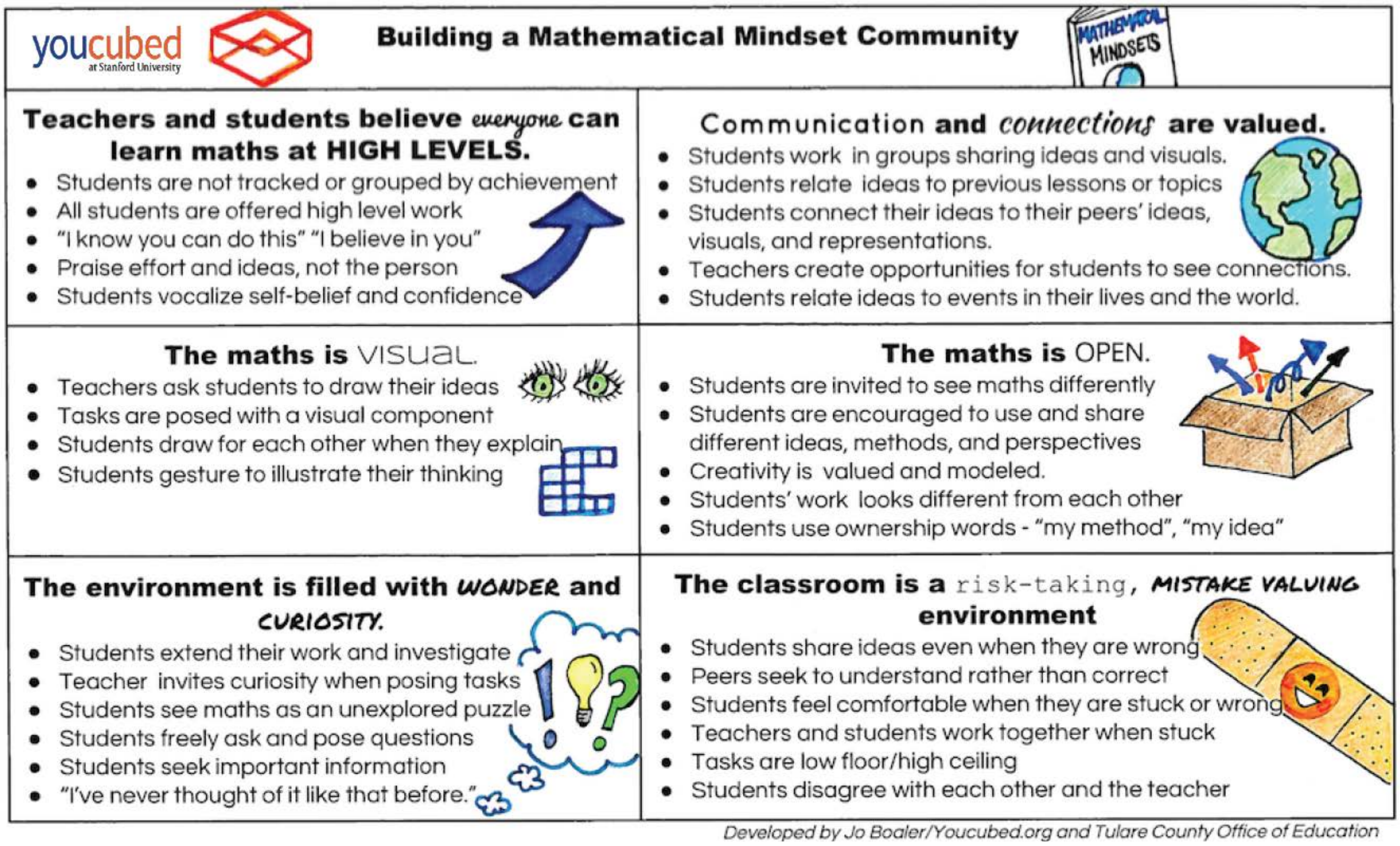
The term "growth mindset" comes from the groundbreaking work of Carol Dweck. She identified that everyone holds ideas about their own potential. Some people believe that their intelligence is more or less fixed in math – that you can do math or you can't, while others believe they can learn anything and that their intelligence can grow.

In a fixed mindset, people believe their basic qualities, like their intelligence or talent, are simply fixed traits. They spend their time documenting their intelligence or talent instead of developing it. They also believe that talent alone creates success—without effort. Students with a fixed mindset are those who are more likely to give up easily.

In a **growth mindset**, people believe that their most basic abilities can be developed through dedication and hard work—brains and talent are just the starting point. This view creates a love of learning and a resilience that is essential for great accomplishment. Students with a growth mindset are those who keep going even when work is hard, and who are persistent.

It is possible to change mindsets and to shift students' mindsets from fixed to growth and cause higher mathematics achievement and success in life. Watch this short video to get a better understanding of what Growth Mindset is and the benefits it can bring our students.

You can find a variety of resources related to **Growth Mindset** at: http://community.ksde.org/Default.aspx?tabid=6383.



youcubed at Stanford University

Building a Mathematical Mindset Community

Teachers and students believe *everyone* can learn maths at HIGH LEVELS.

- Students are not tracked or grouped by achievement
- All students are offered high level work
- "I know you can do this" "I believe in you"
- Praise effort and ideas, not the person
- Students vocalize self-belief and confidence

Communication and *connections* are valued.

- Students work in groups sharing ideas and visuals.
- Students relate ideas to previous lessons or topics
- Students connect their ideas to their peers' ideas, visuals, and representations.
- Teachers create opportunities for students to see connections.
- Students relate ideas to events in their lives and the world.

The maths is VISUAL.

- Teachers ask students to draw their ideas
- Tasks are posed with a visual component
- Students draw for each other when they explain
- Students gesture to illustrate their thinking

The maths is OPEN.

- Students are invited to see maths differently
- Students are encouraged to use and share different ideas, methods, and perspectives
- Creativity is valued and modeled.
- Students' work looks different from each other
- Students use ownership words - "my method", "my idea"

The environment is filled with *WONDER* and *CURIOSITY*.

- Students extend their work and investigate
- Teacher invites curiosity when posing tasks
- Students see maths as an unexplored puzzle
- Students freely ask and pose questions
- Students seek important information
- "I've never thought of it like that before."

The classroom is a risk-taking, *MISTAKE VALUING* environment

- Students share ideas even when they are wrong
- Peers seek to understand rather than correct
- Students feel comfortable when they are stuck or wrong
- Teachers and students work together when stuck
- Tasks are low floor/high ceiling
- Students disagree with each other and the teacher

Developed by Jo Boaler/Youcubed.org and Tulare County Office of Education

Grade 2 Content Standards Overview

Operations and Algebraic Thinking (2.OA)

A. Represents and solves problems involving addition and subtraction
   OA.1
B. Add and subtract within 20
   OA.2
C. Work with equal groups of objects to gain foundations for multiplication
   OA.3 OA.4

Number and Operations in Base Ten (2.NBT)

A. Understand place value.
   NBT.1 NBT.2 NBT.3 NBT.4
B. Use place value understanding and properties of operations to add and subtract.
   NBT.5 NBT.6 NBT.7 NBT.8
   NBT.9

Measurement and Data (2.MD)

A. Measure and estimate lengths in standard units
   MD.1 MD.2 MD.3 MD.4
B. Relate addition and subtraction to length
   MD.5 MD.6
C. Work with time and money
   MD.7 MD.8 MD.9
D. Represent and interpret data
   MD.10 MD.11

Geometry (2.G)

A. Reason with shapes and their attributes
   G.1 G.2 G.3

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

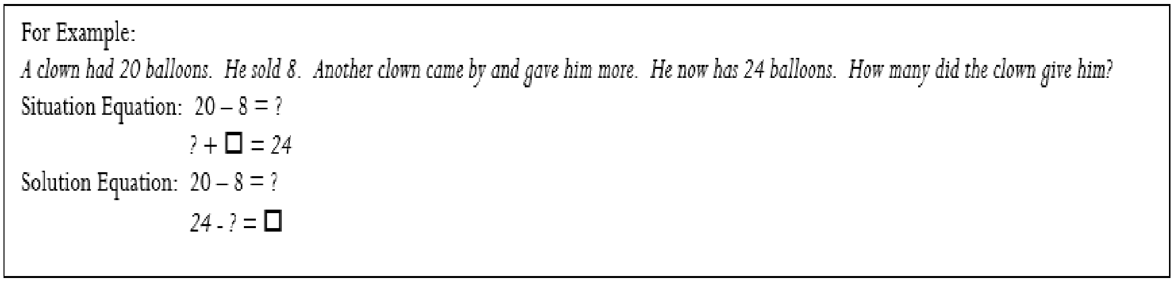
▶ Major Clusters ◆ Supporting Clusters ● Additional Clusters

Domain: Operations and Algebraic Thinking (OA)

▶ ***Cluster A:*** *Represent and solve problems involving addition and subtraction.*

Standard: 2.OA.1

Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, *with unknowns in all positions, (e.g. by using drawings and situation equations and/or solution equations with a symbol for the unknown number to represent the problem.) Refer to shaded section of* [*Table 1*](#) *for specific situation types.* **(2.OA.1)**



For Example:

A clown had 20 balloons. He sold 8. Another clown came by and gave him more. He now has 24 balloons. How many did the clown give him?

Situation Equation: $20 - 8 = ?$

$? + \square = 24$

Solution Equation: $20 - 8 = ?$

$24 - ? = \square$

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

D. *Represent and solve problems involving addition and subtraction* and *Work with addition and subtraction equations* in Grade 1, to *relate addition and subtraction to length* and *Work with time and money* in Grade 2, and to *Solve problems involving the four operations, and identify and explain patterns in arithmetic* in Grade 3.

Explanation and Examples:

This standard calls for calls for students to add and subtract numbers within 100 in the context of one- and two-step word problems. Students should have ample experiences working on **all the subtypes** of problems illustrated within [Table 1](#) and that have unknowns in all positions, including:

Results Unknown:	**Change Unknown:**	**Start Unknown:**
There are 29 students on the playground. Then 18 more students showed up. How many students are there now? $29 + 18 = ?$	There are 29 students on the playground. Some more students show up. There are now 47 students. How many students came? $29 + ? = 47$	There are some students on the playground. Then 18 more students came. There are now 47 students. How many students were on the playground at the beginning? $? + 18 = 47$

This standard also calls for students to solve one- and two-step problems using drawings, objects and equations. Students should be able to use place value blocks, hundreds charts, number lines, or create drawings of any of these tools to support their work. See **Table** 1 in the Appendix to see examples of the different situation types that second grade students are expected to master for one-step problems.

Two step-problems include situations where students have to add and subtract within the same problem.
Example:
First there are 25 students in the cafeteria. Eighteen more students come in. After a few minutes, some students have to leave. If there are 14 students still in the cafeteria, how many students left the cafeteria? *Write an equation for your problem.*

EXPECT students to use *place value blocks (base 10)*, *number line, hundreds chart*, etc. to show, solve and **explain their reasoning**. Just explaining by telling the steps of the procedure will not be enough. Students need to understand the operations and the process. Instead of asking for the "answer", say: "Use the model and explain," "what are the relationships," "what is the structure," or "justify your answer."

Word problems that are connected to students' lives can be used to develop fluency with addition and subtraction. Table 1 in the Appendix describes the different addition and subtraction situations and their relationship to the position of the unknown.

Example:

- *Take From (Result unknown)*: David had 63 stickers. He gave 37 to Susan. How many stickers does David have now? $63 - 37 = ?$
- *Add To*: David had $37. His grandpa gave him some money for his birthday. Now he has $63. How much money did David's grandpa give him? $\$37 + ? = \$63$
- *Compare*: David has 63 stickers. Susan has 37 stickers. How many more stickers does David have than Susan? $63 - 37 = ?$
  - Even though the modeling of the two problems above is different, the equation, $63 - 37 = ?$ can represent both situations (How many more do I need to make 63?)
- *Take From (Start Unknown):* David had some stickers. He gave 37 to Susan. Now he has 26 stickers. How many stickers did David have before? $? - 37 = 26$

It is important to attend to the difficulty level of the problem situations in relation to the position of the unknown.

- **Result Unknown, Total Unknown**, and **Both Addends Unknown** problems are the least complex for students.
- The next level of difficulty includes **Change Unknown**, **Addend Unknown**, and **Difference Unknown.**
- The most difficult are **Start Unknown** and versions of **Bigger and Smaller Unknown** (**Compare** problems).

This standard focuses on developing an algebraic representation of a word problem through addition and subtraction. The intent is **NOT** to introduce traditional algorithms or rules, but to "**make meaning**" of operations.
Second graders should work on **ALL** problem types regardless of the level of difficulty. Mastery is expected in second grade.

Instructional Strategies:

Solving algebraic problems requires emphasizing the most crucial problem-solving strategy—understanding the situation.

Students now build on their work with one-step problems to solve two-step problems and model and represent their solutions with equations for all the situations shown in **Table 1**, see Appendix. The problems should involve sums and differences less than or equal to 100 using the numbers 0 to 100. It is important that students develop the habit of checking their answer to a problem to determine if it makes sense for the situation and the questions being asked.

Ask students to write word problems for their classmates to solve. Start by giving students the answer to a problem, then tell them whether it is an addition or subtraction problem situation. For example, ask students to write an addition word problem for their classmates to solve which requires adding four two-digit numbers with 100 as the answer. Students then share, discuss, and compare their solution strategies after they solve the problems.

Tools/Resources

See: "Pies for Sale," NCSM, Great Tasks for Mathematics K-5, (2013).
See: "Show What You Know," NCSM, Great Tasks for Mathematics K-5, (2013).

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.OA.A.1
    - Pencil and a Sticker
    - NBT Saving Money 2

See: Progressions for Common Core State Standards in Mathematics: K-5, Number and Operations in Base Ten for detailed information.

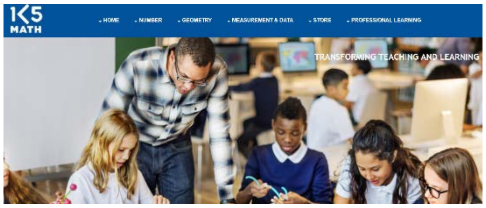
Dr. Douglas Clements and Dr. Julia Sarama are well respected early numeracy educators and researchers. Their book, Learning and Teaching Early Math: The Learning Trajectories Approach, provides a fabulous learning trajectory for *Addition and Subtraction* from the ages of 1 to 7. These researchers explain more about the different situations that students are expected to know and solve. In addition to this trajectory, they have others that are useful when diagnosing students' gaps in mathematical understanding. Clements and Sarama have provided educators access to their online trajectories. Make sure you register so you can use their developmental progressions and the activities and lessons to go with each stage of learning.

LT²

Learning and Teaching with *Learning Trajectories* (LT²)

Visit **K-5 Math Teaching Resources** section showing the developmental stages of addition and subtraction with corresponding games and activities.

These links from the **K-5 Math Teaching Resources** site offer word problems with result unknown for both addition and subtraction problems. There are card available for the other situation types but they are only available with the purchase of their Math Centers.



Greg Tang's website has some great resources for teachers but one hidden gem is the Word Problem Generator. Click here to access an online tool that will provide you with one or a list of problems targeted to specific situations as shown on **Table** 1 (see Appendix). *NOTE: This will only generate one-step word problems. Second grade students need to quickly move to two-step word problems*.

Thinking Blocks on Math Playground allows students several ways to model problems.

Common Misconceptions:

Some students end their solution of a two-step problem after they complete just the first step. They may have misunderstood the question or only focused on finding the first part of the problem. Frequently key words will only provide a clue to one of the operations needed to solve a problem and so kids are misled in thinking that they have solved the problem. Instead of focusing on key words, have students think about what is happening in the situation and then determine what equations are needed to solve the problem. This will help alleviate the tendency to stop after solving just one of the pieces in a two-step word problem.

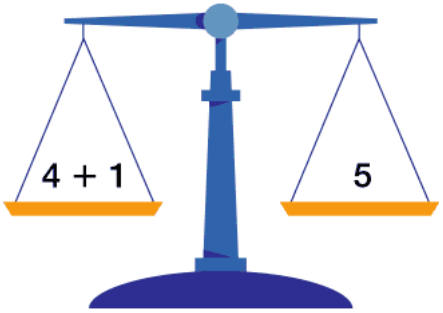
Students need to solve problems where key words are contrary to such thinking.

- For example, the use of the word *left* does not indicate subtraction as a solution method: Debbie took the 8 stickers she no longer wanted and gave them to Anna. Now Debbie has 11 stickers *left*. How many stickers did Debbie have to begin with?

It is important that students not rely on using key words to solve problems. The goal is for students to make sense of the problem and understand what it is asking them to do, rather than search for "tricks" and/or guess at the operation needed to solve the problem.

Students need many opportunities to solve a variety of two-step problems and develop the habit of reviewing their solution after they think they have finished.

Many children have misconceptions about the equal sign. Students can misunderstand the use of the equal sign even if they have proficient computational skills. The equal sign means "is the same as" or "has the same value as", however, many primary students think that the equal sign tells you that the "answer is coming up." Using an equations balance can help students with this misconception.



Students need to see examples of number sentences with an operation to the right of the equal sign and the answer on the left and with number expressions of both sides, so they do not over-generalize from limited examples.

A few examples:

- 9 = 12 – 3
- 15 = 5 + 5 + 5
- 7 + 4 = 8 + 3
- 15 – 8 = 7 + 8

Domain: Operations and Algebraic Thinking (OA)

▶ ***Cluster B:*** *Add and subtract within 20.*

Standard: 2.OA.2

Fluently (efficiently, accurately, and flexibly) add and subtract within 20 using mental strategies (counting on, making a ten, decomposing a number, creating an equivalent but easier and known sum, and using the relationship between addition and subtraction) Work with equal groups of objects to gain foundations for multiplication. **(2.OA.2)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

- *Represent and solve problems involving addition and subtraction* and *Add and subtract within 20* in Grade 1, and to *Use place value understanding and properties of operations to add and subtract* in Grade 2

Explanation and Examples:

This standard mentions the word fluently when students are adding and subtracting numbers within 20. Fluency means accurately (correct answer), efficiently (within 4-5 seconds), and flexibly (using strategies, such as "making 10" or "breaking apart numbers") finding solutions.

Mental Strategies

- Counting on (+1 and +2)
- Making ten (8 + 6 can be solved by decomposing the 6 into 2 + 4 and then combining the 8 and 2 to make a ten. Then 10 plus the leftover 4 is 14.)
- Using the relationship between addition and subtraction (e.g., knowing that $8+4=12$, one knows $12-8=4$)
- Creating equivalent but easier or known sums (e.g., adding 6+ 7 by creating the known

Second Graders internalize facts and develop fluency by repeatedly using strategies that make sense to them. This leads to long-term retention and better outcomes than using timed tests. Timing students before they know the solutions to the problems makes them retain wrong answers.

Research indicates that teachers' can best support students' memorization of sums and differences through varied experiences such as, making 10, breaking numbers apart and working on mental strategies. These strategies replace the use of repetitive timed tests in which students try to memorize operations as if there were not any relationships among the various facts. When teachers teach facts for automaticity, rather than memorization, they encourage students to THINK about the relationships among the facts. (Fostnot & Dolk, 2001)

When students are able to demonstrate fluency they are accurate, efficient, and flexible. Students must have efficient strategies in order to know sums from memory.

It is no accident that the standard says “know from memory” rather than “memorize”. The first describes an outcome, whereas the second is describing a method of achieving that outcome. So the standards are not dictating timed tests. (McCallum, 2011)

Example: 9 + 5 = ____
?

Student 1	Student 2
Counting On I started at 9 and then counted 5 more. I landed on 14.	*Decomposing a Number Leading to a Ten* I know that 9 and 1 is 10, so I broke 5 into 1 and 4. 9 plus 1 is 10. Then I have to add 4 more, which gets me to 14.

Which one is more efficient? Have these discussions with students so they will be flexible in their thinking.

Example: 13 – 9 = _____

=?

Student 1	Student 2
Using the Relationship between Addition and Subtraction I know that 9 plus 4 equals 13. So, 13 minus 9 equals 4.	*Creating an easier problem using compatible numbers* Instead of 13 minus 9. I added 1 to each of the numbers to make the problem 14 minus 10. I know the answer is 4. So 13 minus 9 is also 4.

This standard is strongly connected to all the rest of the standards in this domain. It focuses on students being able to fluently add and subtract numbers to 20. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Instructional Strategies:

An efficient strategy is one that can be done mentally and quickly. Provide many activities that will help students develop a strong understanding of number relationships, addition and subtraction so they can develop, share and use efficient strategies for mental computation.

Students gain computational fluency, using efficient and accurate methods for computing, as they come to understand the role and meaning of arithmetic operations in number systems. Efficient mental processes become automatic with use.

- Have students study how numbers are related to the anchor numbers 5 and 10, so they can apply these relationships to their strategies for knowing 5 + 4 or 8 + 3.
- Students might picture 5 + 4 on a ten-frame to mentally see 9 as the answer, or 1 less than 10.
- For remembering 8 + 7, students might think, since 8 is 2 away from 10, take 2 away from 7 to make 10+5=15 or know that 7+7= 14 and one more makes 15.
- Another example: After multiple experiences with ten-frames, when students add to 9, they mentally SEE 9, but THINK 10 and generalize that 9 + 8 is the same thing as 10 + 7. Then, apply this same thinking to 19 + 8 is the same thing as 20 + 7, SEE 19, THINK 20, and so on.

Provide activities in which students apply the commutative and associative properties to their mental strategies for sums less or equal to 20 using the numbers 0 to 20.

Provide simple word problems designed for students to invent and try a particular strategy as they solve it. Have students explain their strategies so their classmates can understand it.

Guide the discussion so the focus is on the methods that are most useful. Encourage students to try the strategies that were shared so they can eventually adopt efficient strategies that work for them.

Make posters for student-developed, mental strategies for addition and subtraction within 20. Use names for the strategies that make sense to the students and include examples of the strategies.

Present a particular strategy along with the specific addition and subtraction facts relevant to the strategy. Have students use objects and drawings to explore how these facts are alike.

Tools/Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.OA.B.2
    - Building toward fluency
    - Hitting The Target Number
    - MD Delayed Gratification

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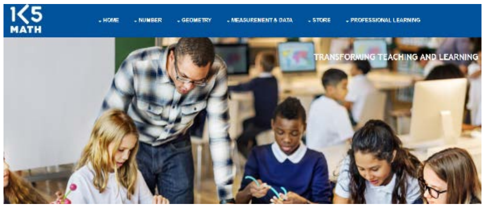


Visit **K-5 Math Teaching Resources** section showing the developmental stages of addition and subtraction with corresponding games and activities.

Common Misconceptions:

Students may over-generalize and begin to think that answers to addition problems must be greater. (*Example*: Adding 0 to any number results in a sum that is equal to that number and not greater.) Provide word problems involving 0 and have students model using drawings with an empty space for 0.

Students are usually proficient when they focus on a strategy relevant to particular facts. When these facts are mixed with others, students may revert to counting as a strategy and ignore the efficient strategies they learned. Provide a list of facts from two or more strategies and ask students to name a strategy that would work for that fact. Students should be expected to explain why they chose that strategy then show how to use it. This relates to efficiency.

Domain: Operations and Algebraic Thinking (OA)

◆ ***Cluster C:*** *Work with equal groups of objects to gain foundations for multiplication.*

Standard: 2.OA.3

Determine whether a group of objects (up to 20) has an odd or even number of members, *(e.g. by pairing objects or counting them by 2s)*; write an equation to express an even number as a sum of two equal addends. **(2.OA.3)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

- *Work with addition and subtraction equations* and *Use place value understanding and properties of operations to add and subtract* in Grade 1, and to *Use place value understanding and properties of operations to add and subtract* in Grade 2.

Explanation and Examples:

This standard calls for students to apply their work with the doubles addition facts to the concept of odd and even numbers. Students should have ample experiences exploring that if a number can be decomposed (broken apart) into two equal (whole number) addends (e.g. 10 = 5 + 5), then that number (*10 in this case*) is an even number. If it cannot be decomposed into two equal addends, then it is an odd number.

Students should explore this concept with concrete objects (e.g., counters, connecting cubes, etc.) before moving towards pictorial representations of groups or arrays.

Example:

Is 8 an even number? Prove your answer.

Student 1	**Student 2**
I grabbed 8 counters. I paired the counters up into groups of 2. Since I didn't have any counters left over, I know that 8 is an even number	I know that 4 plus 4 equals 8. So 8 is an even number.
Student 3	**Student 4**
I drew 8 boxes in a rectangle that had two columns. Since every box on the left matches a box on the right, I know that 8 is even.	I drew 8 circles. I matched one on the left with one on the right. Since they all match up I know that 8 is an even number.

Students explore odd and even numbers in a variety of ways including the following:

1. Students may investigate if a number is odd or even using concrete models.
2. Determining if the number of objects can be divided into two equal sets, arranged into pairs or counted by twos.
3. After the above experiences, students may discover that they only need to look at the digit in the ones place to determine if a number is odd or even since any number of tens will always split into two even groups.

Example:
Students need opportunities writing equations representing sums of two equal addends, such as: 2 + 2 = 4, 3 + 3 = 6, 5 + 5 = 10, 6 + 6 = 12, or 8 + 8 =16. This understanding will lay the foundation for multiplication and is closely connected to 2.OA.4.

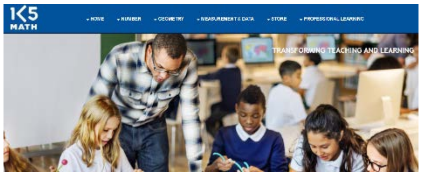
Instructional Strategies: 2.OA.3 & 2.OA.4
Students need to understand that a collection of objects can be one thing (or one group) and that a group contains a given number of objects.

- Investigate separating no more than 20 objects into two equal groups.
- Find the numbers that will have some objects remaining and no objects remaining after separating the collections into two equal groups.
- Odd numbers will have some objects remaining while even numbers will not. For an even number of objects in a collection, show the total as the sum of equal addends (repeated addition).

Tools/Resources:
Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.0A.C.3
    - Red and Blue Tiles
    - Buttons odd and even
- 2.OA.C.4
    - Counting Dots in Arrays

Visit K-5 Math Teaching Resources click on **Number**, then on **2nd Grade**. Scroll down to 2.OA.3 to access resources specifically for this standard.



Common Misconceptions:
Students will look at the number of digits to determine if the number is odd or even instead of the quantity itself. Example: 53 is an even number because it has 2 digits. This is a misconception.

Students will determine whether a number is odd or even by the first digit in the number instead of the digit in the ones place.

Domain: Operations and Algebraic Thinking (OA)

◆ ***Cluster C:*** *Work with equal groups to gain foundations for multiplication.*

Standard: 2.OA.4

Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **(2.OA.4)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 2.OA.3

Explanation and Examples:

This standard expects students to use rectangular arrays to work with repeated addition. This is a building block for multiplication in 3rd Grade. Students should explore this concept with concrete objects (e.g., counters, square tiles, connecting cubes, etc.) as well as pictorial representations on grid paper or other drawings.

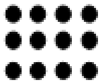
Based on the commutative property of addition, students can add either the rows or the columns and still arrive at the same solution.

Example

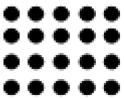
Find the total number of objects below. (Students use concrete objects to show, solve and explain)

Student 1	Student 2
I see 3 counters in each column and there are 4 columns. So I added 3 + 3 + 3 + 3. That equals 12.	I see 4 counters in each row and there are 3 rows. So I added 4+4+4. That equals 12.

Students arrange any set of objects into a rectangular array. Objects can be cubes, buttons, counters, etc. Objects do not have to be square to make an array. *Geoboards* can also be used to demonstrate rectangular arrays. Students then write equations that represent the total as the sum of equal addends as shown below



4 + 4 + 4 + 4 = 16

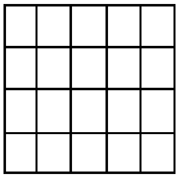


5 + 5 + 5 + 5 = 20

Instructional Strategies:

A rectangular array is an arrangement of objects in horizontal rows and vertical columns. Arrays can be made out of any number of objects that can be put into rows and columns.

- All rows contain the same number of items and all columns contain an equal number of items.
- Have students use objects to build all the arrays possible with no more than 25 objects. Their arrays can have up to 5 rows and 5 columns.
- Ask students to draw the arrays on grid paper and write two different equations under the arrays. One showing the total as a sum by rows and the other showing the total as a sum by columns.
- Both equations will show the total as a sum of equal addends.



The equation by rows: 20 = 5 + 5 + 5 +5
The equation by columns: 20 = 4 + 4 + 4 + 4 + 4

Build on knowledge of composing and decomposing numbers to investigate arrays with up to 5 rows and up to 5 columns in different orientations. For example, form an array with 3 rows and 4 objects in each row. Represent the total number of objects with equations showing a sum of equal addends two different ways: by rows, 12 = 4 + 4 + 4; by columns, 12 = 3 + 3 + 3 + 3.

Show that by rotating the array 90° to form 4 rows with 3 objects in each row. Write two different equations to represent 12 as a sum of equal addends: by rows, 12 = 3 + 3 + 3 + 3; by columns, 12 = 4 + 4 + 4. Have students discuss this statement and explain their reasoning. *The two arrays are different and yet the same*.

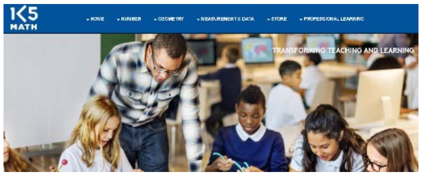
Ask students to think of a full ten-frame showing 10 circles as an array. One view of the ten-frame is 5 rows with 2 circles in each row. Students count by rows to 10 and write the equation 10 = 2 + 2 + 2 + 2 + 2. Then students put two full ten-frames together end-to-end so they form 10 rows of 2 circles or (10 columns of 2 circles). They use this larger array to count by 2s up to 20 and write an equation that shows 20 equal to the sum of ten 2s.

Tools/Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.OA.C.4
    - Counting Dots in Arrays

See Learning Progressions for Operations & Algebraic Thinking for detailed information



Visit K-5 Math Teaching Resources click on **Number**, then on **2nd Grade**. Scroll down to 2.OA.4 to access resources specifically for this standard.

Common Misconceptions:

Students may not be consistent in how they describe arrays. They may not remember the convention that was discussed in class. They cannot justify that the commutative property applies.

Domain: Number and Operations in Base Ten (NBT)

▶ *Cluster A:* Understand place value.

Standard: **2.NBT.1**

Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; *(e.g. 706 equals 7 hundreds, 0 tens, and 6 ones.)* Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens—called a "hundred." **(2.NBT.1a)**

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds **(2.NBT.1b)**

c. Show flexibility in composing and decomposing hundreds, tens and ones *(e.g. 207 can be composed from 2 hundreds 7 ones OR 20 tens 7 ones OR 207 ones OR 1 hundred 10 tens 7 ones OR 1 hundred 9 tens 17 ones, etc.)* **(2017)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

- *Extend the counting sequence* and *Understand place value* in Grade 1
- *Work with equal groups of objects to gain foundations for multiplication* in Grade 2
- *Use place value understanding and properties of operations to perform multi-digit arithmetic* in Grade 3.

Explanation and Examples:

This standard expects students to work on decomposing numbers by place value. Students should have ample experiences with concrete materials and pictorial representations examining that all numbers between 100 and 999 can be decomposed into hundreds, tens, and ones and then into several different combinations within those place values.

Example:

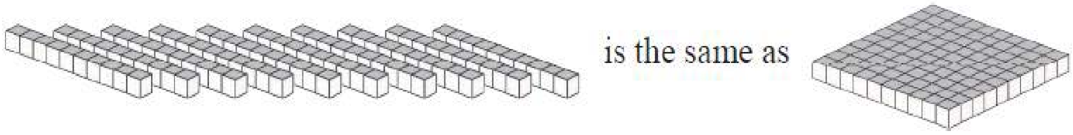
285 can be shown as **2 hundreds, 8 tens, and 5 ones** but it is also correct to show this number as **28 tens and 5 ones** OR **1 hundred, 18 tens, and 5 ones** OR **2 hundreds, 7 tens, and 15 ones** and so on.

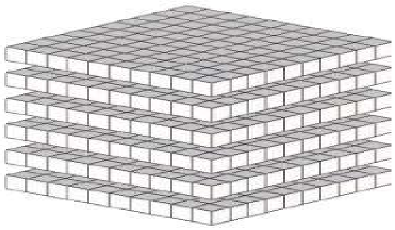
Interpret the value of a digit (1-9 and 0) in a multi-digit numeral by its position within the number with models, words and numerals.

Use 10 as a benchmark number to compose and decompose when adding and subtracting whole numbers.

2.NBT.1a calls for students to extend their work from 1st Grade by exploring a hundred as a unit (or bundle) of ten tens.

2.NBT.1b builds on the work of **2.NBT.2a.** Students should explore the idea that numbers such as 100, 200, 300, etc., are groups of hundreds that have no tens or ones. Students can represent this with place value (base 10) blocks.
2.NBT.1c extends the thinking to various decompositions by place value. This is needed for regrouping later on.





6 hundreds are the same as 600

Understanding that 10 ones make one ten and that 10 tens make one hundred is fundamental to students' mathematical development. It is also important that students can reverse this thinking and know that 1 ten can be decomposed into 10 ones and 1 hundred can be decomposed into 10 tens.

Students need multiple opportunities counting and "bundling" groups of tens in first grade. In second grade, students build on their understanding by making bundles of 100s with or without leftovers using base ten blocks, cubes in towers of 10, ten frames, etc. This emphasis on bundling hundreds will support students' discovery of place value patterns. Equally important is the idea of being able to "unbundle" to decompose numbers.

As students are representing the various amounts, it is important that emphasis is placed on the language associated with the quantity. For example, 243 **can be expressed in multiple ways** such as 2 groups of hundred, 4 groups of ten and 3 ones, as well as 24 tens and 3 ones.

When students read numbers, they should read in standard form as well as using place value concepts. For example, 243 should be read as "two hundred forty-three" as well as two hundreds, 4 tens, 3 ones.

Instructional Strategies: 2.NBT.1 through 2.NBT.4
The understanding that 100 is 10 tens or 100 ones is critical to the understanding of place value. Using proportional models like base-ten blocks and bundles of tens along with numerals on place-value mats provides connections between physical and symbolic representations of a number. These models can be used to compare two numbers and identify the value of their digits. It is essential to find out if students truly understand how base-ten blocks represent numbers before doing extensive work. If students have not had enough time with groupable models (connecting cubes, building ten frames, etc.) before using tradeable models for place value (base-ten blocks), then they may be just "going through the motions" and not really understanding what they are doing to represent the numbers. View this video by Marilyn Burns as she works with Jonathan and discovers that this student has gaps in place value understanding.

Model three-digit numbers using base-ten blocks in multiple ways. For example, 236 can be **236 ones**, OR **23 tens and 6 ones**, OR **2 hundreds, 3 tens and 6 ones**, OR **20 tens and 36 ones**. Use activities and games that have students match different representations of the same number.

Provide games and other situations that allow students to practice skip-counting. Students can use nickels, dimes and dollar bills to skip count by 5, 10 and 100. Pictures of the coins and bills can be attached to models familiar to students: a nickel on a five-frame with 5 dots or pennies and a dime on a ten-frame with 10 dots or pennies.

On a number line, have students use a clothespin or marker to identify the number that is ten more than a given number or five more than a given number.

Have students create and compare all the three-digit numbers that can be made using numbers from 0 to 9. For instance, using the numbers 1, 3, and 9, students will write the numbers 139, 193, 319, 391, 913 and 931.
When students compare the numerals in the hundreds place, they should conclude that the two numbers with 9 hundreds would be greater than the numbers showing 1 hundred or 3 hundreds. When two three-digit numbers have the same digit in the hundreds place, students need to compare their digits in the tens place to determine which number is larger.

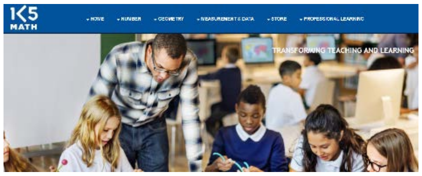
Tools/Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.NBT.A.1
    - Regrouping
    - Three composing/decomposing problems
    - Bundling and Unbundling
    - Boxes and Cartons of Pencils
    - Largest Number Game
    - Ten $10s make $100
    - Counting Stamps
    - One, Ten, and One Hundred More and Less
    - Making 124
    - Looking at Numbers Every Which Way
- 2.NBT.A.1.a
    - Party Favors

See: Progression for Common Core State Standards in Mathematics: K-5, Number and Operations in Base Ten for detailed information.

Visit K-5 Math Teaching Resources click on **Number**, then on **2nd Grade**. Scroll down to 2.NBT.1 to access resources specifically for this standard.



Common Misconceptions: 2.NBT.1 through 2.NBT.4

Some students may not move beyond thinking of the number 358 as 300 ones plus 50 ones plus 8 ones to the concept of 8 singles, 5 bundles of tens, and 3 bundles of hundreds. You may have to go back to groupable models (connecting cubes or ten frames) to establish understanding. Then when students use base-ten blocks, they need to model the collecting of 10 ones (singles) to make a ten or 10 tens to make a hundred. It is important that students connect a group of 10 ones with the word *ten* and a group of 10 tens with the word *hundred.*

Domain: Number and Operations in Base Ten (NBT)

▶ ***Cluster A:*** *Understand place value.*

Standard: 2.NBT.2

Count within 1000; skip-count by 2s, 5s, 10s, and 100s; explain and generalize the patterns. **(2.NBT.2)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 2.NBT.1

Explanation and Examples:

The standard expects students to count within 1,000. This means that students are expected to "count on" from any number and say the next few numbers that come afterwards. They also need to be able to tell the numbers that come before numbers.

Understand that counting by 2s, 5s, 10s, and 100s is counting groups of items by that amount.

Example:

What are the next 3 numbers after 498? *499, 500, 501*.

When you count back from 201, what are the first 3 numbers that you say? *200, 199, 198*.

Students should explore the patterns of numbers when they skip count. When students skip count by 5s, the ones digit alternates between 5 and 0, if they start at 0. When students skip count by 100s, the hundreds digit is the only digit that changes, and it increases by one number. Students should skip count starting at a number other than 0. *Example: start at 5 and skip count by 2s;* 5, 7, 9, 11, 13, ... *Start at 7 and skip count by 5s*; 7, 12, 17, 22, 27, ... *Discuss the patterns created. What do they notice?*

Examples:

- The use of the 100s chart is great tool for students to identify the counting patterns.
- The use of money (nickels, dimes, dollars) or base ten blocks may be helpful visual cues.

The ultimate goal for second graders is to be able to count in multiple ways with no visual support.

Instructional Strategies: See 2.NBT.1

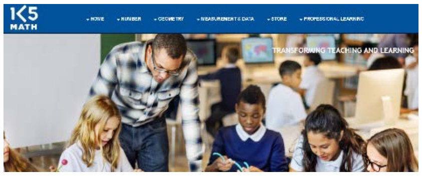
Use the "Counting Around the Class" routine to support skip counting and then discuss the patterns of the numbers.

Tools/Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.NBT.A.2
    - Saving Money 2

"Counting Around the Class" questions.



Visit K-5 Math Teaching Resources click on **Number**, then on **2nd Grade**. Scroll down to 2.NBT.2 to access resources specifically for this standard.

Common Misconceptions: See 2.NBT.1

Domain: Number and Operations in Base Ten (NBT)

▶ ***Cluster A:*** *Understand place value.*

Standard: 2.NBT.3

Read and write numbers within 1000 using base-ten numerals, number names, expanded form, and unit form. **(2.NBT.3)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 2.NBT.1

Explanation and Examples:

This standard calls for students to read, write and represent a number of objects with various forms. These representations can include place value (base 10) blocks, pictorial representations or other concrete materials.

Base-ten numerals – 726
Number names – seven hundred twenty-six
Expanded form – 700 + 20 + 6
Unit form – 7 hundreds, 2 tens, 6 ones

When writing the number in words (number names), remember that the word "*and*" should **not** be used between any of the whole-number words – "*and*" represents the decimal point.

Examples:
When students say the expanded form, it may sound like this: "6 hundreds plus 3 tens plus 7 ones" OR 600 plus 30 plus 7." That is acceptable.

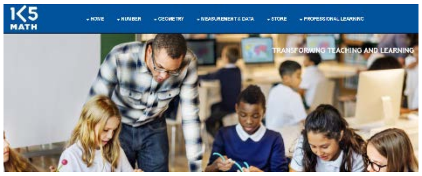
Instructional Strategies: See 2.NBT.1

Tools/Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.NBT.A.3
    - Looking at Numbers Every Which Way

Visit K-5 Math Teaching Resources click on **Number**, then on **2nd Grade**. Scroll down to 2.NBT.3 to access resources specifically for this standard.



Common Misconceptions: See 2.NBT.1

Domain: Number and Operations in Base Ten (NBT)

▶ ***Cluster A:*** *Understand place value*.

Standard: 2.NBT.4

Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, <, =, and ≠ relational symbols to record the results of comparisons. **(2.NBT.4)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 2.NBT.1

Explanation and Examples:

This standard builds on the work of **2.NBT.1** and **2.NBT.3** by having students compare two numbers by examining the amount of hundreds, tens and ones in each number. Do not use TRICKS such as an alligator or Pac Man. Use math.

Students were introduced to the symbols greater than (>), less than (<), equal to (=), and not equal (≠) in First Grade, and are expected to use them in Second Grade with numbers within 1,000.

Students should have ample experiences communicating their comparisons **in words** before using only symbols.

Example: 452 _____ 455

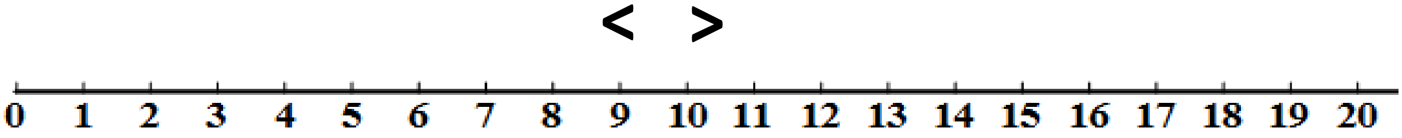
Student 1	**Student 2**
452 has 4 hundreds 5 tens and 2 ones. 455 has 4 hundreds 5 tens and 5 ones. They have the same number of hundreds and the same number of tens, but 455 has 5 ones and 452 only has 2 ones,452 is less than 455. 452 < 455	452 is less than 455. I know this because when I count up I say 452 before I say 455.

To compare, students apply their understanding of place value. They first attend to the numeral in the hundreds place, then the numeral in tens place, then, if necessary, to the numeral in the ones place. Comparative language includes but is not limited to: more than, less than, greater than, most, greatest, least, same as, equal to and not equal to. Students use the appropriate symbols to record the comparisons.

Instructional Strategies: See 2.NBT.1

These symbols are relational (=, ≠, <,>) but many students view them the same as the operational symbols they are more familiar with (+, -). To alleviate this misconception, make sure you use the symbols in conjunction with the number line.

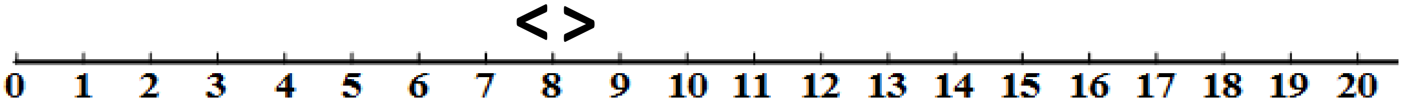
Display a number line with the relational symbols (<,>) above it. Make sure these symbols are easily moved.



Now put up the two numbers you want to compare:

15 8

Explain that you want to compare where the number **15** is in relation to the number **8**. So you move the two relational symbols above the **8**.

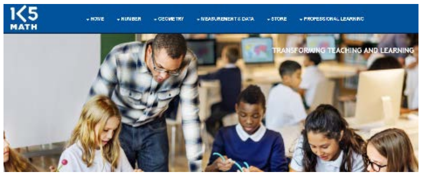


Ask the students where the **15** is in relation to the **8** – is to the left, which would show it to be a lesser number, or is it to the right, which would show it to be a greater number? This is using the relationship between the numbers and not on tricks. When using an alligator or Pac Man, students are getting the relational understanding of the symbols. These symbols are asking where the first number is on the number line in relation to the second number. So use the number line when establishing understanding.

Tools/Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.NBT.A.4
    - Ordering 3-digit numbers
    - Comparisons 2
    - Comparisons 1
    - Digits 2-5-7
    - Number Line Comparisons
    - Using Pictures to Explain Number Comparisons



Visit K-5 Math Teaching Resources click on **Number**, then on **2nd Grade**. Scroll down to 2.NBT.4 to access resources specifically for this standard.

Common Misconceptions: See 2.NBT.1

Often students think of these relational symbols as operational symbols instead. In order to alleviate this misconception, allow students to use the number line and the hundred chart to see the relationship between one number and the other. See *Instructional Strategies* above.

Domain: Number and Operations in Base Ten (NBT)

▶ ***Cluster B:*** *Use place value understanding and properties of operations to add and subtract.*

Standard: 2.NBT.5

Fluently (efficiently, accurately, and flexibly) add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction *(e.g. composing/decomposing by like base-10 units, using friendly or benchmark numbers, using related equations, compensation, number line, etc.).* **(2.NBT.5)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: 2.NBT.5 through 2.NBT.9

This cluster is connected to:

- *Understand and apply properties of operations and the relationship between addition and subtraction* and *Add and subtract within 20 and Use place value understanding and properties of operations to add and subtract* in Grade 1
- *Add and subtract within 20* in Grade 2
- *Use place value understanding and properties of operations to perform multi-digit arithmetic* in Grade 3.

Explanation and Examples:

This standard mentions the word fluently when students are adding and subtracting numbers within 100. Fluency means accuracy (correct answer), efficiency (reasonable steps and time in computing), and flexibility (using strategies; such as making 10 or breaking numbers apart).

Example: 67 + 25 =

Place Value Strategy	**Counting On and Decomposing a Number Leading to a Ten**	**Associative Property:**
I broke both 67 and 25 into tens and ones. 6 tens plus 2 tens equals 8 tens. Then I added the ones. 7 ones plus 5 ones equals 12 ones. I then combined my tens and ones. 8 tens plus 12 ones equals 92.	I wanted to start with 67 and then break 25 apart. I started with 67 and counted on to my next ten. 67 plus 3 gets me to 70. I then added 2 more to get to 72. I then added my 20 and got to 92.	I broke 67 and 25 into tens and ones so I had to add 60+7+20+5. I added 60 and 20 first to get 80. Then I added 7 to get 87. Then I added 5 more. My answer is 92.

Example: 63 - 32 =

Relationship between Addition and Subtraction
I broke apart both 63 and 32 into tens and ones. I know that 2 plus 1 equals 3, so I have 1 left in the ones place. I know that 3 plus 3 equals 6, so I have a 3 in my tens place. My answer has a 1 in the ones place and 3 in the tens place, so my answer is 31.

Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Students should have experiences solving problems written both horizontally and vertically. They need to communicate their thinking and be able to justify their strategies both verbally and with paper and pencil.

Addition strategies based on place value for 48 + 37 may include:

- Adding by place value: 40 + 30 = 70 and 8 + 7 = 15 and 70 + 15 = 85.
- Incremental adding (breaking one number into tens and ones); 48 + 10 = 58, 58 + 10 = 68, 68 + 10 = 78, 78 + 7 = 85
- Compensation (making a friendly number): 48 + 2 = 50, 37 – 2 = 35, 50 + 35 = 85

Subtraction strategies based on place value for 81 - 37 may include:

- Adding up (from smaller number to larger number): 37 + 3 = 40, 40 + 40 = 80, 80 + 1 = 81, and 3 + 40 + 1 = 44.
- Incremental subtracting: 81 -10 = 71, 71 – 10 = 61, 61 – 10 = 51, 51 – 7 = 44
- Subtracting by place value: 81 – 30 = 51, 51 – 7 = 44

Properties that students should know and use are:

- Commutative property of addition (Example: 3 + 5 = 5 + 3)
- Associative property of addition (Example: (2 + 7) + 3 = 2 + (7+3))
- Identity property of 0 (Example: 8 + 0 = 8)

Students in second grade need to communicate their understanding of why some properties work for some operations and not for others.

- **Commutative Property:** In first grade, students investigated whether the commutative property works with subtraction. The intent was for students to recognize that taking 5 from 8 is not the same as taking 8 from 5. Students should also understand that they will be working with numbers in later grades that will allow them to subtract greater numbers from smaller numbers. This exploration of the commutative property continues in second grade.

Instructional Strategies:

Provide many activities that will help student develop a strong understanding of number relationships, as well as, develop strategies for addition and subtraction that will lead to efficient strategies for mental computation. An efficient strategy is one that can be done mentally and fairly quickly. Students gain computational fluency, using efficient and accurate methods for computing as they come to understand the role and the meaning of arithmetic operations in the number system.

Students need to build on their flexible strategies for adding within 100 from Grade 1 to using numbers 0 to 1000.

Initially students apply base-ten concepts and use direct modeling with physical objects or drawings to find different ways to solve problems. They move to inventing strategies that do not involve physical material or counting by ones to solve problems. *Student invented strategies likely will be based on place value concepts, the commutative and associative properties and the relationship between addition and subtraction. These strategies should be done mentally or with written record for support.*

It is vital that student-invented strategies be shared, explored, recorded and tried by others. Recording expressions and equations in the strategies horizontally encourages student to think about the numbers and the quantities they represent instead of the digits.

Not every student will invent strategies, but all student can and will try strategies they have seen that makes sense to them. Different strategies will be chosen by different students.

Students will decompose and compose tens and hundreds when they develop their own strategies for solving problems where regrouping is necessary. They might use the make-ten strategy (37 + 8 = 40 + 5) add 3 to 37 then 5, or (62 + 9 = 60 + 7) take off 2 and get 60 then add 7 more) because no ones are exchanged for a ten or a ten for ones.

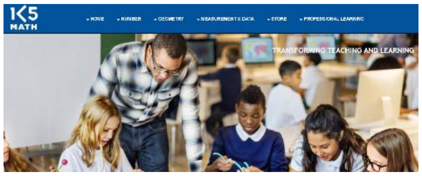
Have students analyze problems before they solve them. Present a variety of subtraction problems within 1000. Ask students to identify the problems requiring them to decompose the tens or hundreds to find a solution and explain their reasoning.

Tools/Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.NBT.B.5
    - Jamir's Penny Jar
    - Saving Money 2
    - Saving Money 1

See: "Show What You Know," NCSM, Great Tasks for Mathematics K-5, (2013).



Visit K-5 Math Teaching Resources click on **Number**, then on **2nd Grade**. Scroll down to 2.NBT.5 to access resources specifically for this standard.

Thinking Blocks on Math Playground allows students several ways to model problems.

Common Misconceptions: 2.NBT.5 through 2.NBT.9

Students may think that the 4 in 46 represents 4, not 40 or 4 tens. Students need many experiences representing two- and three-digit numbers with manipulatives that group (base ten blocks) and those that do NOT group, such as counters, etc.

When adding two-digit numbers, some students might start with the digits in the ones place and record the entire sum. Then they add the digits in the tens place and record this sum. Assess students' understanding of *a ten* and provide more experiences modeling addition with grouped and pre-grouped base-ten materials as mentioned above.

When subtracting two-digit numbers, students might start with the digits in the ones place and subtract the smaller digit from the greater digit. Then they move to the tens and the hundreds places and subtract the smaller digits from the greater digits. Assess students' understanding of *a ten* and provide more experiences modeling subtraction with grouped and pre-grouped base-ten materials.

Domain: Number and Operations in Base Ten (NBT)

▶ ***Cluster B:*** *Use place value understanding and properties of operations to add and subtract.*

Standard: 2.NBT.6

Add up to four two-digit numbers using strategies based on place value and properties of operations. **(2.NBT.6)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 2.NBT.5

Explanation and Examples:

This standard calls for students to add a string of two-digit numbers (up to four numbers) by applying place value strategies and properties of operations.

Example: 43 + 34 + 57 + 24 = __________

Student 1 - Associative Property I saw the 43 and 57 and added them first, since I know 3 plus 7 equals 10. When I added them 100 was my answer. Then I added 34 and had 134. Then I added 24 and had 158.	**Student 2 - Place Value Strategies** I broke up all of the numbers into tens and ones. First I added the tens. 40 + 30 + 50 + 20 = 140. Then I added the ones. 3 + 4 + 7 + 4 = 18. Then I combined the tens and ones and had 158 as my answer.
Student 3 - Place Value Strategies and Associative Property I broke up all the numbers into tens and ones. First I added up the tens. 40 + 30 + 50 + 20. I changed the order of the numbers to make adding easier.	**Student 4** I added up the ones. 3 + 4 + 7 + 4. I changed the order of the numbers to make adding easier. I know that 3 plus 7 equals 10 and 4 plus 4 equals 8. 10 plus 8 equals 18. I then combined my tens and my ones. 140 plus 18 equals 158.

Students demonstrate addition strategies with up to four two-digit numbers either with or without regrouping. Problems should often be provided in a story problem format to help develop a stronger understanding of larger numbers and their values.

Instructional Strategies: See 2.NBT.5

Tools/Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.NBT.B.6
    - Toll Bridge Puzzle

Thinking Blocks on Math Playground allows students several ways to model problems.

Common Misconceptions: See 2.NBT.5

Domain: Number and Operations in Base Ten (NBT)

▶ ***Cluster B:*** *Use place value understanding and properties of operations to add and subtract.*

Standard: **2.NBT.7**

Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, like base-ten units such as hundreds and hundreds, tens and tens, ones and ones are used; and sometimes it is necessary to compose or decompose tens or hundreds. **(2.NBT.7)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 2.NBT.5

Explanation and Examples:

This standard builds on the work from 2.NBT.5 by increasing the size of numbers (two 3-digit numbers). Students should have ample experiences in using concrete materials (place value blocks, ten frames) and pictorial representations to support their work.

This standard also references composing and decomposing a ten when adding and subtracting. This work should include strategies such as making a 10, making a 100, breaking apart a 10, or creating an easier problem. While the standard (traditional) algorithm could be used here, students' experiences should extend beyond only working with that algorithm. The partial sums algorithm naturally emphasizes place value and so should be used along with the compensation strategy and the counting up strategy. View these video clips sharing the compensation strategy. View this video clip for the counting up or counting on strategy.

There is a strong connection between this standard and place value understanding with addition and subtraction of smaller numbers. Students may use concrete models or drawings to support their addition or subtraction of larger numbers.

Strategies are similar to those stated in 2.NBT.5, as students extend their learning to include greater place values moving from tens to hundreds to thousands. Interactive whiteboards and document cameras may also be used to model and justify student thinking.

Students use number lines, base ten blocks, etc. to show, solve and explain reasoning. Explanation of thinking is a critical component of this standard.

Example: 354 + 287 =

Student 1	Student 2	Student 3
Uses a number line. "I started at 354 and jumped 200. I landed on 554. I then made 8 jumps of 10 and landed on 634. I then jumped 7 and landed on 641."	Uses base ten blocks & mat. "I broke all of the numbers up by place using a place value chart. I first added the ones (4+7), then the tens (50+80) and then the hundreds 200=500) I then combined my answers: 500+130=630. 630+11=641".	Uses place value blocks. "I made a pile of 354. I then added 287. That gave me 5 hundreds, 13 tens and 11 ones. I noticed that I could trade some pieces. I had 11 ones, and traded 10 ones for a ten. I then had 14 tens, so I traded 10 tens for a hundred and ended up with 6 hundreds, 4 tens and 1 one

Instructional Strategies: See 2.NBT.5

Tools/Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

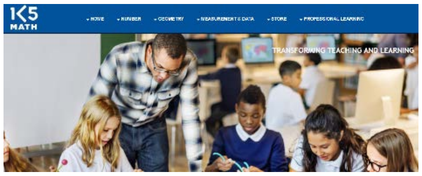
- 2.NBT.B.7
  - How Many Days Until Summer Vacation?
  - Many Ways to do Addition 2

See: Progression for Common Core State Standards in Mathematics: K-5, Number and Operations in Base Ten detailed information.

Thinking Blocks on Math Playground allows students several ways to model problems.

See: "Creating Story Problems ", Georgia Department of Education. In this lesson, students will add and subtract numbers less than 100 and understand the relationship between addition and subtraction. The activity applies reading/listening to story problems. Students write and solve problems involving a variety of situations, choosing strategies including- part-part- whole, comparing, grouping, doubling, counting on and counting back situations. Students will use drawings, equations, and written responses to solve single story problems

Visit K-5 Math Teaching Resources click on **Number**, then on **2nd Grade**. Scroll down to 2.NBT.7 to access resources specifically for this standard.



Common Misconceptions: See 2.NBT.5

Domain: Number and operations in Base Ten (NBT)

▶ ***Cluster B:*** *Use place value understanding and properties of operations to add and subtract.*

Standard: 2.NBT.8

Mentally add 10 or 100 to a given number 100 – 900, and mentally subtract 10 or 100 from a given number 100 – 900. **(2.NBT.8)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 2.NBT.5

Explanation and Examples:

This standards expects students to mentally add and subtract multiples of 10 or 100 to any number between 100 and 900. When computing sums of 3-digit numbers, students might use strategies based on Level 3 *composition and decomposition* and Level 2 *counting-on* strategies when finding the value of an expression such as 148 + 473. (See **Table** 1 in Appendix) Example: they might say "100 and 400 is 500. And 70 and 30 is another hundred, so 600. Then, 8, 9, 10, 11…..and the other 10 is 21. So, 621". (In order to discuss the strategies being used, the teacher should write down the student's thinking as it is being shared.) There are two kinds of decompositions in this strategy. Both addends are decomposed into 100s, 10s, and ones; and the first addend decomposed successively into the part already added and the part still to add.

Students should have ample experiences developing proficiency with mental computation. Mentally adding and subtracting 10 or 100 to a given number understanding that they are only changing the digit in the tens place (multiples of ten) or the digit in the hundreds place (multiples of 100). Working with place value blocks before moving to mental computation will be beneficial for most students.

Mental math strategies may include:

- Counting on; 300, 400, 500, etc.
- Counting back; 550, 450, 350, etc.

Examples:

- 100 more than 653 is _______(753)
- 10 less than 87 is _______(77)
- "Start at 248. Count up by 10s until I tell you to stop."

Instructional Strategies: See 2.NBT.5

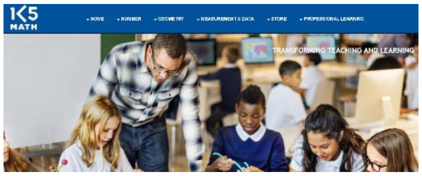
See: "Piggy Banks", NCSM, Great Tasks for Mathematics K-5, (2013).

Tools and Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.NBT.B.8
  - Choral Counting

Visit K-5 Math Teaching Resources click on **Number**, then on **2nd Grade**. Scroll down to 2.NBT.8 to access resources specifically for this standard.



Common Misconceptions: See 2.NBT.5

Domain: Number and Operations in Base Ten (NBT)

▶ ***Cluster B***: *Use place value understanding and properties of operations to add and subtract.*

Standard: 2.NBT.9

Explain why addition and subtraction strategies work using place value and the properties of operations. The explanations given may be supported by drawings or objects. **(2.NBT.9)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 2.NBT.5

Explanation and Examples:

This standard calls for the student to explain using concrete objects, pictures and words (oral or written) **why** addition or subtraction strategies work. The expectation is that students apply their knowledge of place value and the properties of operations in their explanation. This is where it is critical that they see the connection between standards.

Understanding place value and then being able to describe how it works when computing, along with the properties of operations, is expected. Students don't need to name the properties but they need to know that they can use them and how to use them when computing.

Students should have the opportunity to solve problems and then explain why their strategies work – using place value language and/or the use of the properties of operations.

Example:

There are 36 birds in the park. 25 more birds arrive. How many birds are there? Solve the problem and show your work.

Student 1	Student 2
I broke 36 and 25 into tens and ones and then added them. 30 + 6 + 20 + 5. I can change the order of my numbers, so I added 30+20 and got 50. Then I added on 6 to get 56. Then I added 5 to get 61. This strategy works because I broke all the numbers up by their place value.	I used place value blocks and made a pile of 36. Then I added 25. I had 5 tens and 11 ones. I had to trade 10 ones for a 10. Then I had 6 tens and 1 one. That makes 61. This strategy works because I added up the tens and then added up the ones and traded if I had more than 10 ones.

Students could also have experiences examining strategies and explaining why they work. Also include incorrect examples for students to examine. Operations embedded within meaningful context promote development of reasoning and justification.

Example:
One of your classmates solved the problem 56 - 34 = __ by writing —*I know that I need to add 2 to the number 4 to get 6. I also know that I need to add 20 to 30 to get 20 to get to 50. So, the answer is 22.*‖ Is their strategy correct? Explain why or why not?

Example:
One of your classmates solved the problem 25 + 35 by adding 20 + 30 + 5 + 5. Is their strategy correct? Explain why or why not?

Example:
Mason read 473 pages in June. He read 227 pages in July. How many pages did Mason read altogether?

Karla's explanation:	**Debbie's explanation:**	**Becky's explanation:**
473 + 227 = _____. I added the ones together (3 + 7) and got 10. Then I added the tens together (70 + 20) and got 90. I knew that 400 + 200 was 600. So I added 10 + 90 for 100 and added 100 + 600 and found out that Mason had read 700 pages altogether.	473 + 227 = ______. I started by adding 200 to 473 and got 673. Then I added 20 to 673 and I got 693 and finally I added 7 to 693 and I knew that Mason had read 700 pages altogether.	I used base ten blocks on a base ten mat to help me solve this problem. I added 3 ones (units) plus 7 ones and got 10 ones which made one ten. I moved the 1 ten to the tens place. I then added 7 tens rods plus 2 tens rods plus 1 tens rod and got 10 tens or 100. I moved the 1 hundred to the hundreds place. Then I added 4 hundreds plus 2 hundreds plus 1 hundred and got 7 hundreds or 700. So Mason read 700 books.

Students should be able to connect different representations and explain the connections. Representations can include numbers, words (including mathematical language), pictures, number lines, and/or physical objects. Students should be able to use any/all of these representations as needed.

Instructional Strategies: See 2.NBT.5

Visit K-5 Math Teaching Resources click on **Number**, then on **2nd Grade**. Scroll down to 2.NBT.9 to access resources specifically for this standard.

Tools and Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.NBT.B.9
    - Peyton and Presley Discuss Addition

Thinking Blocks on Math Playground allows students several ways to model problems.

Common Misconceptions: See 2.NBT.5

Domain: Measurement and Data (MD)

▶ ***Cluster A:*** *Measure and estimate lengths in standard units.*

Standard: 2.MD.1

Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **(2.MD.1)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections:

This cluster is connected to:

- *Measure lengths indirectly and by iterating length units* in Grade 1, and to *Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures* in Grade 3.

Explanation and Examples:

This standard calls for students to measure the length of objects in both customary (inches and feet) and metric (centimeters and meters). Students should have ample experiences choosing objects, identifying the appropriate tool and unit, and then measuring the object. The teacher should allow students to determine which tools and units to use.

Foundational understandings to help with measure concepts:

- Understand that larger units can be subdivided into equivalent smaller units (partition).
- Understand that the same unit can be repeated to determine the measure (iteration).
- Understand the relationship between the size of a unit and the number of units needed (compensatory principal).
- Understand measuring two-dimensional space (area) using non-standard units.

Students in second grade will build upon what they learned in first grade from measuring length with non-standard units to the new skill of measuring length in metric and U.S. Customary with standard units of measure.

They should have many experiences measuring the length of objects with rulers, yardsticks, meter sticks, and tape measures.

Students will need to be taught how to use a ruler appropriately to measure the length of an object, especially where to begin the measuring. It is important to help students locate the starting point on the measuring instrument. Some have a protected edge (this is the process of justification of object and the instrument). The use of how to use rulers needs to be targeted after students understand the **attributes** that are to be measured. Dr. John Van de Walle has noted that if we don't make sure students understand the attributes sufficiently, then our lessons focus on using the instrument and the understanding of "what" is being measured is missed and leads to misconceptions.

Ask students questions such as: “Do you start at the end of the ruler or at the zero?” helps them focus on where to start on the instrument. Then ask them: “Why do we have to start at the zero?” and “Are we looking at the spaces or the tic marks on the rulers?” (The spaces indicate the length of the object – not the tic marks. The tic marks delineate the end of the space.)

Instructional Strategies: 2.MD.1 through 2.MD.4

Second graders are transitioning from measuring lengths with informal or nonstandard units to measuring with standard units: inches, feet, centimeters, and meters. The measure of length is a count of how many units are needed to match the length of the object or distance being measured. Emphasize that the space is what is being measured, not the tic marks on the ruler.

Students have to understand what a length unit is and how it is used to find a measurement. They need many experiences measuring lengths with appropriate tools so they can become very familiar with the standard units and estimate lengths.

Use language that reflects the approximate nature of measurement, such as the length of the room is about 26 feet.

Insist that students always estimate lengths before they measure. Estimation helps them focus on the attribute to be measured, the length units, and the process. After they find measurements, have students discuss the estimates, their procedures for finding the measurements and the differences between their estimates and the measurements.

If asking student to estimate and measure more than one object, the sequence is –estimate, measure, estimate, measure…. Using this sequence helps the student refine their ability to estimate.

Rulers that have only one system (either customary or metric) work most effectively with student beginning this stage of learning to measure. And using rulers with just the units marked and not the subunits (such as halves and fourths) assists students in focusing on the correct units of measure.

Resources/Tools:

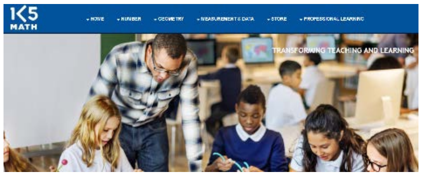
Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.MD.A
    - How Big is a Foot?

For detailed information--See Learning Progressions on Measurement

Centimeter rulers and tapes
Inch rulers and tapes
Yardsticks and Meter sticks

Visit K-5 Math Teaching Resources click on **Measurement and Data**, then on **2nd Grade**. Scroll down to 2.MD.1 to access resources specifically for this standard.



Common Misconceptions:

When some students see standard rulers with numbers on the markings, they believe that the numbers are counting the marks instead of the units or spaces between the marks.

Have students use informal or standard length units to make their own rulers by marking each whole unit with a number in the middle. They will see that the ruler is a representation of a row of units and focus on the spaces.

Some students might think that they can only measure lengths with a ruler starting at the left edge. Provide situations where the ruler does not start at zero. For example, a ruler is broken and the first inch number that can be seen is 2. If a pencil is measured and it is 9 inches on this ruler, the students must subtract 2 inches from the 9 inches to adjust for where the measurement started. Some student become confused when the ruler they are using have both customary and metric measures on it. By covering on scale with masking tape the student becomes less confused.

Domain: Measurement and Data (MD)

▶ ***Cluster A**: Measure and estimate lengths in standard units.*

Standard: 2.MD.2

Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **(2.MD.2)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See 2.MD.1

Explanation and Examples:

This standard expects students to measure an object using two units of different lengths. Concentrate on the "spaces" for the units and not the marks on the rulers.

Example:
A student measures the length of their desk and finds that it is 3 feet and 36 inches.

Students should explore the idea that the number of units for length of the desk is greater in inches than in feet. This concept is referred to as the compensatory principle.

Students need multiple opportunities to measure using different units of measure. They should not be limited to measuring to one standard unit. Students should have access to various tools, both U.S. Customary and metric. The more students work with a specific unit of measure, the better they become at choosing the appropriate tool when measuring.

Students measure the length of the same object using different tools (ruler with inches, ruler with centimeters, a yardstick, or meter stick). This will help students learn which tool is more appropriate for measuring a given object.

They describe the relationship between the size of the measurement unit and the number of units needed to measure something. For instance, a student might say, "The longer the unit, the fewer I need."

Instructional Strategies: See 2.MD.1

Common Misconceptions: See 2.MD.1

Domain: Measurement and Data (MD)

▶ *Cluster A: Measure and estimate lengths in standard units.*

Standard: 2.MD.3

Estimate lengths using whole units of inches, feet, centimeters, and meters. **(2.MD.3)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.

Connections: See 2.MD.1

Explanation and Examples:

This standard expects students to estimate the lengths of objects using inches, feet, centimeters, and meters. Students should make estimates after seeing a benchmark unit, such as the length of one inch, before making their estimate.

It can be helpful for students to find personal benchmarks. Often the width of one of their fingers is a cm. For some students the space between their elbow and the end of their hand is a foot. You need to explain that their personal benchmarks may change as they get older.

Example:

Estimation helps develop familiarity with the specific unit of measure being used. To measure the length of a shoe, knowledge of an inch or a centimeter is important so that one can approximate the length in inches or centimeters. Setting up personal benchmarks can be beneficial.

Students should begin practicing estimation with items which are familiar to them (length of desk, pencil, favorite book, etc.).

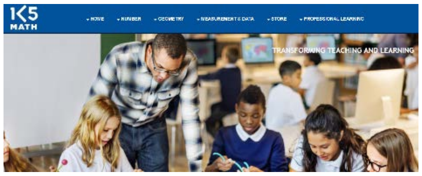
Instructional Strategies: See 2.MD.1

Resources/Tools:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.MD.A.1
    - Determining Length

Visit K-5 Math Teaching Resources click on **Measurement and Data**, then on **2nd Grade**. Scroll down to 2.MD.3 to access resources specifically for this standard.



Domain: Measurement and Data (MD)

▶ *Cluster A:* *Measure and estimate lengths in standard units.*

Standard: 2.MD.4

Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit (inches, feet, centimeters, and meters). **(2.MD.4)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.

Connections: See 2.MD.1

Explanation and Examples:

This standard expects students to determine the difference in length between two objects. Students should choose objects, identify appropriate tools and units, measure both objects, and then determine the differences in lengths using the same unit of measure

Second graders should be familiar enough with inches, feet, yards, centimeters, and meters to be able to compare the differences in lengths of two objects. They can make direct comparisons by measuring the difference in length between two objects by laying them side by side and selecting an appropriate standard length unit of measure.

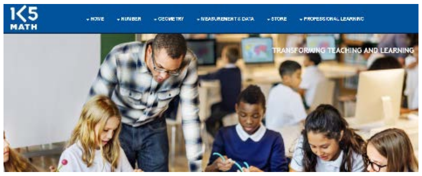
Students should use comparative phrases such as "It is longer by 2 inches" or "It is shorter by 5 centimeters" to describe the difference between two objects.

It is important that students have multiple opportunities to work with actual objects in the process of measuring.

Instructional Strategies: See 2.MD.1

Resources/Tools:

Visit K-5 Math Teaching Resources click on **Measurement and Data**, then on **2nd Grade**. Scroll down to 2.MD.4 to access resources specifically for this standard.



Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.MD.A.3
    - Determining Length

Common Misconceptions: See 2.MD.1

Domain: Measurement and Data (MD)

▶ ***Cluster B**: Relate addition and subtraction to length.*

Standard: 2.MD.5

Use addition and subtraction within 100 to solve one- and two-step word problems involving lengths that are given in the same units, *e.g. by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.* **(2.MD.5)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: 2.MD.5 through 2.MD.6

This cluster is connected to:

- *Use place value understanding and properties of operations to add and subtract* in Grade 1,
- *Represent and solve problems involving addition and subtraction* in Grade 2,
- *Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures* in Grade 3.

Explanation and Examples:

This standard applies the concept of length to solve addition and subtraction word problems with numbers within 100. Students must use the same unit in these problems and discuss why it doesn't work to use different units of measure.

Example:

In P.E. class Kate jumped 14 inches. Mary jumped 23 inches. How much farther did Mary jump than Kate? Write an equation and then solve the problem.

Student 1	Student 2
My equation is 14 + __ = 23 since I am trying to find out the difference between Kate and Mary's jump. I used **place value blocks** and counted out 14. I then added blocks until I got to 23. I needed to add 9 blocks. Mary jumped 9 more inches than Kate.	My equation is 23 - 14 = __. I drew a **number line**. I started at 23. I moved back to 14 and counted how far I moved (the units). I moved back 9 spots. Mary jumped 9 more inches than Kate.

Students need experience working with addition and subtraction to solve word problems (make connections to all the subtypes within Table 1 (See Appendix) which include measures of length. It is important that word problems stay within the same unit of measure.

Some representations students can use include drawings, number lines, rulers, pictures, and/or physical objects.

As students begin to work measurement problems, remember to consider the different types of equations that can be used to create problems.

Equations include:

- $10 + 15 = c$
- $c - 20 = 5$
- $c - 10 = 25$
- $20 + b = 35$
- $15 + a = 35$
- $35 = a + 15$
- $35 = 20 + b$

Example:

- A word problem for $5 - n = 2$ could be: Mary is making a dress. She has 5 yards of fabric. She uses some of the fabric and has 2 yards left. How many yards did Mary use?

There is a strong connection between this standard and demonstrating fluency of addition and subtraction facts. Addition facts through 10 + 10 and the related subtraction facts should be included.

Instructional Strategies: **(2.MD.5-6)**

Connect the whole-number units on rulers, yardsticks, meter sticks and measuring tapes to number lines showing whole-number units starting at 0. Use these measuring tools to model different representations for whole-number sums and differences less than or equal to 100 using the numbers 0 to 100.

Use the meter stick to view units of ten (10 cm) and hundred (100 cm), and to skip count by 5s and 10s.

Provide one- and two-step word problems that include different lengths measurement made with the same unit (inches, feet, centimeters, or meters). Students add and subtract within 100 to solve problems for these situations:

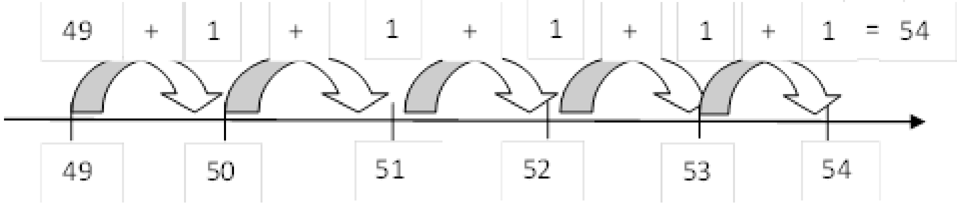
- adding to, taking from,
- putting together,
- taking apart,
- comparing,

with **unknowns in all positions**.

Students use drawings and write equations with a symbol for the unknown to solve the problems.

- Have students represent their addition and subtraction within 100 on a number line. They can use notebook or grid paper to make their own number lines.
- First have them mark and label a line on paper with whole-number units that are equally spaced and relevant to the addition or subtraction problem.

- Then have them show the addition or subtraction using curved lines segments above the number line and between the numbers marked on the number line. For 49 + 5, start at 49 on the line and draw a curve to 50 Continue drawing curves to 54.
- Drawing the curves or making the "hops" between the numbers will help students focus on a space as the length of a unit and the sum or difference as a length.



Resources/Tools

- Rulers
- Yardsticks
- Meter sticks
- Measuring tapes
- Cash register tapes or paper strips

See: Progression for Common Core State Standards in Mathematics: K-5, Geometric Measurement for detailed information about this standard.

NCTM Illuminations – NCTM has many great resources available to educators, some of these resources (i.e. interactives) are open to any educator while others (i.e. lessons) require an individual or institutional membership. If you find that a resource referenced in the flip books requires membership access, check with your school/district to see if they have an institutional membership which would grant you access all NCTM documents. If they do not have a membership, this would be a valuable resource to request.

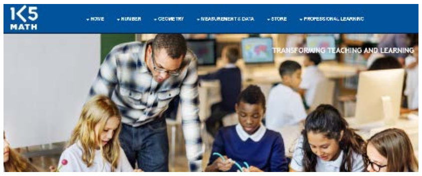
- "Hopping Backward to Solve Problems". In this lesson students determine differences using the number line to compare lengths.
- " Where Will I Land?". In this lesson students find differences using the number line, a continuous model for subtraction.

Thinking Blocks on Math Playground allows students several ways to model problems.

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.MD.B.5
    - High Jump Competition

Visit K-5 Math Teaching Resources click on **Measurement and Data**, then on **2nd Grade**. Scroll down to 2.MD.5 to access resources specifically for this standard.



Common Misconceptions:

Students may think that they always have to start at zero. Adjustments can be made if measured from a different starting location than zero.

Help students develop and understanding of what the problem is asking. Often "key words" can be misleading and usually will only help with one step of the problem. This is a limitation when working with multi-step word problems. The teaching of a "key word approach" limits the development of understanding what the problems is actually asking.

Domain: Measurement and Data (MD)

▶ *Cluster B:* *Relate addition and subtraction to length.*

Standard: 2.MD.6

Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram. **(2.MD.6)**

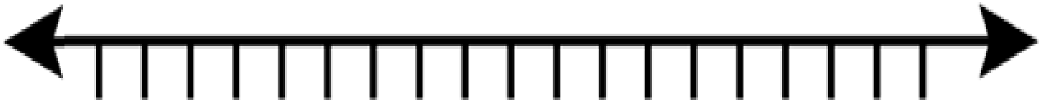
Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.

Connections: See 2.MD.5

Explanation and Examples:

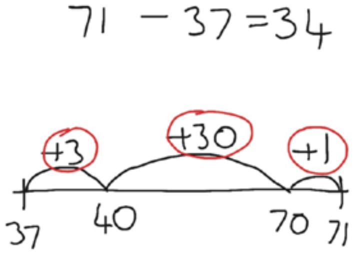
This standard expects the student to create number lines within 100 to solve addition and subtraction problems. Students should create the number line with evenly spaced points corresponding to the numbers.



Students represent their thinking when adding and subtracting within 100 by using a number line. An interactive whiteboard or document camera can be used to help students demonstrate their thinking. Their thinking should connect to strategies that expand beyond one by one counting.

Example:

Using an open number line allows students to think beyond one-to-one counting. This counting up method is very effective and can frequently be done mentally.



Instructional Strategies: See 2.MD.5

Resources/Tools:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.MD.B.6
    - Frog and Toad on the Number Line

Common Misconceptions: See 2.MD.5

Domain: Measurement and Data (MD)

◆ ***Cluster C:*** *Work with time and money.*

Standard: 2.MD.7

Tell and write time from analog and digital clocks to the nearest five minutes. **(2.MD.7)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.

Connections: 2.MD.7 through 2.MD.8

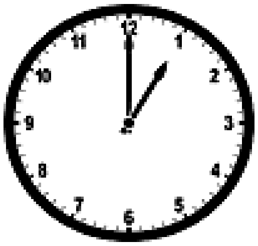
This cluster is connected to:

- *Tell and write time in* Grade 1
- *Represent and solve problems involving addition and subtraction* in Grade 2
- *Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses* of objects in Grade 3.

Explanation and Examples:

This standard calls for students to tell and write time after reading analog and digital clocks.

Time should be to 5 minute intervals. Teachers should help students make the connection between skip counting by 5s (2.NBT.2) and telling time on an analog clock.



In first grade, students learned to tell time to the nearest hour and half-hour. Students build on this understanding in second grade by skip-counting by 5 to recognize 5-minute intervals on the clock. They need exposure to both digital and analog clocks.

It is important that they can recognize time in both formats and communicate their understanding of time using both numbers and language. Common time phrases include the following: quarter till ___, quarter after ___, ten till ___, ten after ___, and half past ___.

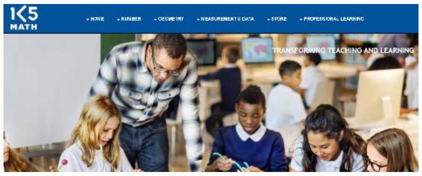
Instructional Strategies: 2.MD.7 through 2.MD.8

Second graders expand their work with telling time from analog and digital clocks to the nearest hour or half-hour in Grade 1 to telling time to the nearest five minutes.

Tools/Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.MD.C.7
    - Ordering Time



Visit K-5 Math Teaching Resources click on **Measurement and Data**, then on **2nd Grade**. Scroll down to 2.MD.7 to access resources specifically for this standard.

Common Misconceptions: 2.MD.7 through 2.MD.8

Some students might confuse the hour and minutes hands. For the time of 3:45, they say the time is 9:15. Also, some students name the numeral closest to the hands, regardless of whether this is appropriate. For instance, for the time of 3:45 they say the time is 3:09 or 9:03.

One way to avoid this confusion is to use Dr. John Van de Walle's strategy of using a one-handed clock to begin telling time. This method is explained in his book (Teaching Student-Centered Mathematics: PreK-2) and is called the **One-handed Clock**. The hour hand gives the most information about the time. To give students a better understanding of this you will need to buy two inexpensive clocks. Place both clocks in an area so all students can see them but are easy for you to access. Make sure both clocks are set to the same correct time and then remove the minute hand from one of the clocks. The clock with both hands should then be covered so that students will see just the one-handed clock. At various times during the day, draw your students' attention to the one-handed clock and ask them to tell you the time. Then remove the cover from the two-handed clock to verify the time. Students will begin to see that the hour hand gives them an idea of how many minutes past the hour it is based on how far it is between two numbers.

Provide opportunities for students to experience and measure times to the nearest five minutes and the nearest hour. Have them focus on the movement and features of the hands on real or geared manipulative clocks.

Domain: Measurement and Data (MD)

◆ ***Cluster C:*** *Work with time and money.*

Standard: 2.MD.8

Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately (Do not use decimal point, if showing 25 cents, use the word cents or ¢). *For example: If you have 2 dimes and 3 pennies, how many cents do you have?* **(2.MD.8)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 2.MD.7

Explanation and Examples:

This standard expects students to solve word problems involving **either** dollars or cents – not a combination.
Since students have not been introduced to decimals, problems should either have only dollars or only cents.

Example:
What are some possible combinations of coins (pennies, nickels, dimes, and quarters) that equal 37 cents?

Example:
What are some possible combinations of dollar bills ($1, $5 and $10) that equal 12 dollars?

Since money is not specifically addressed in kindergarten or first grade, students should have multiple opportunities to identify, count, recognize, and use coins and bills. They should also experience making equivalent amounts using both coins and bills, but not mixed together. "Dollar bills" should include denominations up to one hundred ($1, $5, $10, $20, $100).

Students should solve story problems connecting the different representations. These representations may include objects, pictures, charts, tables, words, and/or numbers. Teachers should make sure that students are using all subtypes of problems from **Table** 1 from the standards document.
Example:
Sandra went to the store and received 76¢ in change. What are three different sets of coins she could have received?

Instructional Strategies: See 2.MD.7

The topic of money begins at Grade 2 and builds on the work in other clusters in this and previous grades. Help students learn money concepts and solidify their understanding of other topics by providing activities where students make connections between them.
Students use the context of money to find sums and differences less than or equal to 100 using the numbers 0 to 100. They add and subtract to solve one- and two-step word problems involving money situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.

Students use drawings and equations with a symbol for the unknown number to represent the problem. The dollar sign, $, is used for labeling whole-dollar amounts without decimals, such as $29.

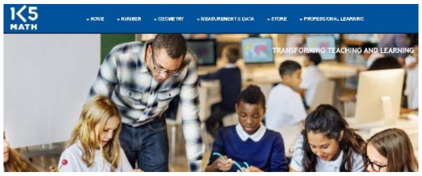
Students need to learn the relationships between the values of a penny, nickel, dime, quarter and dollar bill.

Tools/Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.MD.C.8
    - Jamir's Penny Jar
    - Susan's Choice
    - Visiting the Arcade
    - Saving Money 1
    - Alexander, Who Used to be Rich Last Sunday
    - Pet Shop
    - Choices, Choices, Choices

Visit K-5 Math Teaching Resources click on **Measurement and Data**, then on **2nd Grade**. Scroll down to 2.MD.8 to access resources specifically for this standard.



Thinking Blocks on Math Playground allows students several ways to model problems.

Common Misconceptions:

Sometimes students will record twenty-nine dollars as 29$. Remind them that the dollar sign goes in front. The cent sign goes after the number and there is no decimal point used with the cent sign nor can both signs be used in the same amount.

Students might over-generalize the value of coins when they count them. They might count them as individual objects. Also, some students think that the value of a coin is directly related to its size, so the bigger the coin, the more it is worth.

Domain: Measurement and Data (MD)

◆ ***Cluster C:*** *Work with time and money.*

Standard: 2.MD.9

Identify coins and bills and their values. **(2017)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.

Connections: See 2.MD.7

Explanation and Examples:

This standard expects students to know all coins and bills and their values.

Since money is not specifically addressed in kindergarten or first grade, students should have multiple opportunities to identify, count, recognize, and use coins and bills. They should also experience making equivalent amounts using both coins and bills, but not mixed together. "Dollar bills" should include denominations up to one hundred ($1, $5, $10, $20, $100).

Instructional Strategies: See 2.MD.7

The topic of money begins at Grade 2 and builds on the work in other clusters in this and previous grades. Help students learn money concepts and solidify their understanding of other topics by providing activities where students make connections between them.

Students use drawings and equations with a symbol for the unknown number to represent the problem. The dollar sign, $, is used for labeling whole-dollar amounts without decimals, such as $29.

Students need to learn the relationships between the values of a penny, nickel, dime, quarter and dollar bill.

Common Misconceptions:

Students might over-generalize the value of coins. Some students think that the value of a coin is directly related to its size, so the bigger the coin, the more it is worth.

Place pictures of a nickel on the top of five-frames that are filled with pictures of pennies. In a like manner, attach pictures of dimes and pennies to ten-frames and pictures of quarters to 5 x 5 grids filled with pennies. Have students use these materials to determine the value of a set of coins in cents.

Domain: Measurement and Data (MD)

◆ ***Cluster D:*** *Represent and interpret data.*

Standard: 2.MD.10

Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object using different units. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. **(2.MD.9)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: 2.MD.10 through 2.MD.11

This cluster connects to:

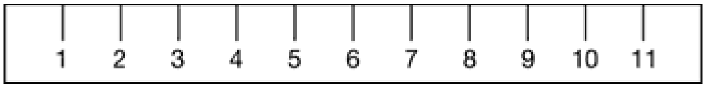
- *Measure lengths indirectly and by iterating length units* and *Represent and interpret data* in Grade 1
- *Represent and interpret data* in Grade 3.

Explanation and Examples:

This standard calls for students to represent the length of several objects by making a line plot. Students should round their lengths to the nearest whole unit.

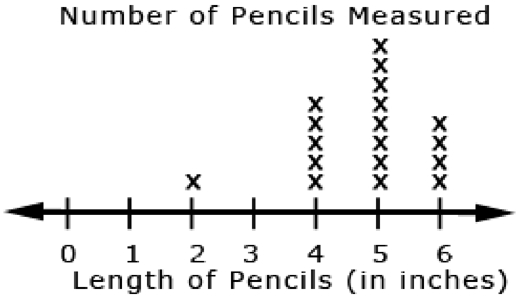
Example:

Measure objects in your desk to the nearest inch, display data collected on a line plot. How many objects measured 2 inches? 3 inches? Which length had the most number of objects? How do you know?



This standard emphasizes representing data using a line plot. Students will use the measurement skills learned in earlier standards to measure objects. Line plots are first introduced in this grade level.

A line plot can be thought of as plotting data on a number line. An interactive whiteboard may be used to create and/or model line plots as well as "Class" line-plots on chart paper.



Instructional Strategies: 2.MD.10 through 2.MD.11

Line plots are useful tools for collecting data because they show the number of things along a numeric scale.

The line plot is made by simply drawing a number line then placing an X above the corresponding value on the line that represents each piece of data. Make sure students understand that the Xs need to be of consistent size and are lined up similar to a bar graph.

Line plots are essentially bar graphs with a potential bar for each value on the number line but generally are quicker to make which also allows for more depth of instruction when less time is used for the “drawing/shading” of a bar graph. It also reinforces the ideas presented on a number line. Students need to make sure their Xs are all the same size so their line plot is not distorted.

Pose a question related to the lengths of several objects. Measure the objects to the nearest whole inch, foot, centimeter or meter. Create a line plot with whole-number units (0, 1, 2, ...) on the number line to represent the measurements.

At first students should create real object or picture graphs (pictograph) (where the object is drawn rather than a number). On picture graphs record the number of countable parts.

These graphs show items in a category and do not have a numerical scale. For example, a real object graph could show the students’ shoes (one shoe per student) lined end to end in horizontal or vertical rows by their color.
Students would simply count to find how many shoes are in each row or bar. The graphs should be limited to 2 to 4 rows or bars.

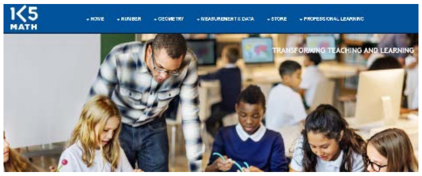
Students then move to making horizontal or vertical bar graphs with two to four categories and a single-unit scale. Use the information in the graphs to pose and solve simple put together, take-apart, and compare problems illustrated in **Table 1** (Appendix).

Tools/Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.MD.D.9
    - The Longest Walk
    - Growing Bean Plants
    - Hand Span Measures

Visit K-5 Math Teaching Resources click on **Measurement and Data**, then on **2nd Grade**. Scroll down to 2.MD.9 (to use this resource you must look at the purple tag at the end of the standard) to access resources specifically for this standard.



Common Misconceptions: 2.MD.10 through 2.MD.11

The attributes for the same kind of object can vary. This will cause equal values in an object graph to appear unequal. For example, when making an object graph using shoes for boys and girls, five adjacent boy shoes would likely appear longer than five adjacent girl shoes. To standardize the objects, place the objects on the same-sized construction paper or sticky-note, then make the object graph.

Domain: Measurement and Data (MD)

◆ ***Cluster D:** Represent and interpret data.*

Standard: 2.MD.11

Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph (See **Table** 1). **(2.MD.10)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 2.MD.10

Explanation and Examples:

This standard calls for students to work with categorical data by organizing, representing and interpreting data. Students should have experiences posing a question with four possible responses and then work with the data that they collect.

Example:

Students pose a question and up to four possible responses. Which is your favorite flavor of ice cream? Chocolate, vanilla, strawberry, or cherry?

Students collect their data by using tallies or another way of keeping track.

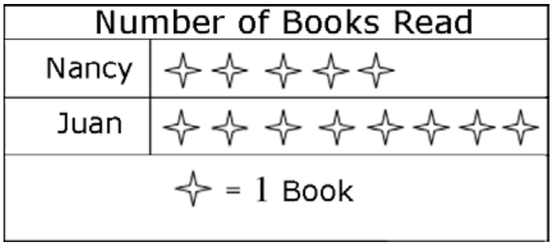
Students organize their data by totaling each category in a chart or table. Picture and bar graphs are introduced in Second Grade.

Flavor	Number of People
Chocolate	12
Vanilla	5
Strawberry	6
Cherry	9

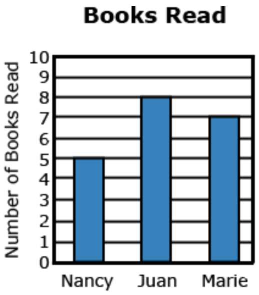
Students display their data using a picture graph or bar graph using a single unit scale. Students answer simple problems related to addition and subtraction that ask them to put together, take apart, and compare numbers. (See **Table** 1 in the Appendix) for examples of these.

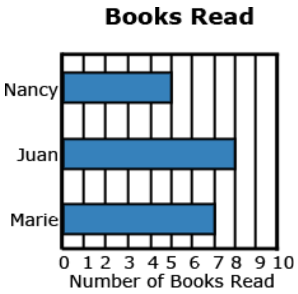
Students should draw both picture and bar graphs representing data that can be sorted up to four categories using single unit scales (e.g., scales should count by ones). The data should be used to solve put together, take-apart, and compare problems as listed in **Table 1**, page 49.

In second grade, picture graphs (pictographs) include symbols that represent single units. Pictographs should include a title, categories, category label, key, and data.



Second Graders should draw both horizontal and vertical bar graphs. Bar Graphs include a title, scale, scale label, category label and data.





Instructional Strategies: See 2.MD.10

Tools/Resources:

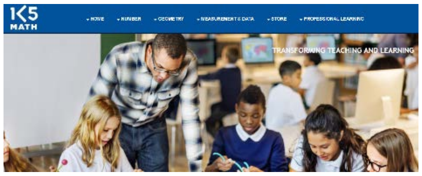
Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.MD.D.10
    - Favorite Ice Cream Flavor

See: "Pies for Sale," NCSM, Great Tasks for Mathematics K-5, (2013).

Georgia Department of Education:

- "Barn Yard Legs" This activity uses children's literature connections and has students create graph and tables and interpret their meaning.



Visit K-5 Math Teaching Resources click on **Measurement and Data**, then on **2nd Grade**. Scroll down to 2.MD.10 (to use this resource you must look at the purple tag at the end of the standard) to access resources specifically for this standard.

Common Misconceptions: See 2.MD.10

Domain: Geometry (G)

● ***Cluster A:*** *<u>Reason with shapes and their attributes</u>.*

Standard: 2.G.1

Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. **(2.G.1)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.

Connections:

This cluster is connected to:

- *Reason with shapes and their attributes* in Grade 1
- *Develop understanding of fractions as numbers* and *Reason with shapes and their attributes* in Grade 3.

Explanation and Examples:

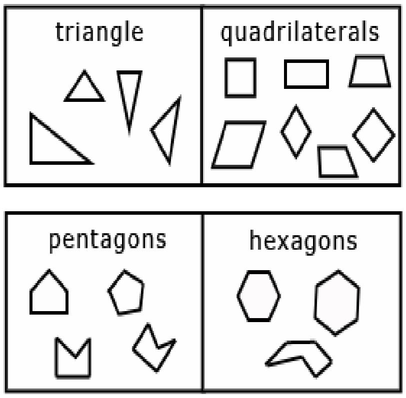
This standard calls for students to identify (recognize) and draw shapes based on a given set of attributes. These include triangles, quadrilaterals (squares, rectangles, and trapezoids), pentagons, hexagons and cubes.

Example:

Draw a closed shape that has five sides. What is the name of the shape? *Student - I drew a shape with 5 sides. It is called a pentagon*.

Students should be able to identify, describe, and draw triangles, quadrilaterals, pentagons, and hexagons. Pentagons, triangles, and hexagons should appear as both **regular** (equal sides and equal angles) and **irregular**.

Students recognize all four sided shapes as quadrilaterals. Students use the vocabulary word "angle" in place of "corner" but they do not need to name angle types.



Instructional Strategies: 2.G.1 through 2.G.3

Geosticks, geoboards, interactive whiteboards and document cameras may be used to help identify shapes and their attributes. Shapes should be presented in a variety of orientations and configurations.

Tools/Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

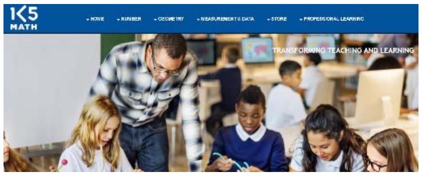
- 2.G.A.1
    - Polygons

See: "Creative Cards", NCSM, Great Tasks for Mathematics K-5, (2013).

Georgia Department of Education:

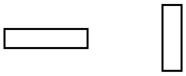
- "3-D Detectives". In this activity students describe and classify plane figures (triangles, square, rectangle, trapezoid, quadrilateral, pentagon, hexagon, and irregular polygonal shapes).
- "What's in a Name". In this activity students will describe and classify plane figures (triangles, square, rectangle, trapezoid, quadrilateral, pentagon, hexagon, and irregular polygonal shapes) according to the number of edges and vertices.

Visit K-5 Math Teaching Resources click on **Geometry**, then on **2nd Grade**. Scroll down to 2.G.1 to access resources specifically for this standard.



Common Misconceptions: (2.G.1 through 2.G.3)

Some students may think that a shape is changed by its orientation. They may see a rectangle with the longer side as the base, but claim that the same rectangle with the shorter side as the base is a different shape. This is why it is so important to have young students handle shapes and physically feel that the shape does not change regardless of the orientation, as illustrated below.



If students are only shown equilateral triangles, then when they see scalene or isosceles triangles, they do not recognize them as triangles even though they have three sides. You must make sure you are always showing students various types of shapes and not just the regular shapes that they see in pattern blocks and on posters.

Domain: Geometry (G)

● ***Cluster A:*** *Reason with shapes and their attributes.*

Standard: 2.G.2

Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. **(2.G.2)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

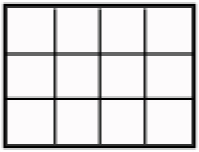
Connections: See 2.G.1

Explanation and Examples:

This standard expects students to partition a rectangle into squares (or square-like regions) and then determine the total number of squares. This relates to the standard 2.OA.4 where students are arranging objects in an array of rows and columns.

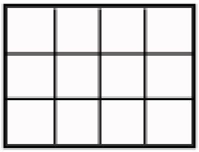
Example:

Split the rectangle into 3 rows and 4 columns. How many small squares did you make?



This standard is a precursor to learning about the area of a rectangle and using arrays for multiplication. An interactive whiteboard or manipulatives such as square tiles or other square shaped objects can be used to help students' partition rectangles.

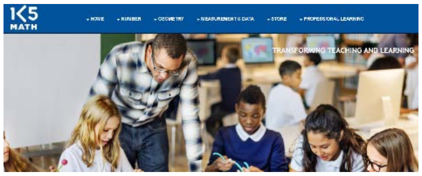
Rows are horizontal and columns are vertical.



Instructional Strategies: See 2.G.1

Modeling multiplication with partitioned rectangles promotes students' understanding of multiplication. Tell students that they will be drawing a square on grid paper. The length of each side is equal to 2 units.

- Ask them to guess how many 1 unit by 1 unit squares will be inside this 2 unit by 2 unit square.
- Students now draw this square and count the 1 by 1 unit squares inside it.
- They compare this number to their guess.
- Next, students draw a 2 unit by 3 unit rectangle and count how many 1 unit by 1 unit squares are inside.
- Now they choose the two dimensions for a rectangle, predict the number of 1 unit by 1 unit squares inside, draw the rectangle, count the number of 1 unit by 1 unit squares inside and compare this number to their guess.
- Students repeat this process for different-size rectangles. Finally, ask them to what they observed as they worked on the task.



Resources/Tools:

Visit K-5 Math Teaching Resources click on **Geometry**, then on **2nd Grade**. Scroll down to 2.G.2 to access resources specifically for this standard.

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.G.A.2
  - Partitioning a Rectangle into Unit Squares

Common Misconceptions: See 2.G.1

Domain: Geometry (G)

● ***Cluster A:*** *Reason with shapes and their attributes.*

Standard: 2.G.3

Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. *Note: fraction notation* $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ *is not expected at this grade level.* Recognize that equal shares of identical wholes need not have the same shape. **(2.G.3)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.6 Attend to precision.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 2.G.1

Explanation and Examples:

This standard calls for students to partition (divide) circles and rectangles into 2, 3 or 4 equal shares (regions).

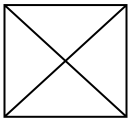
Students should be given ample experiences to explore this concept with paper strips and pictorial representations.

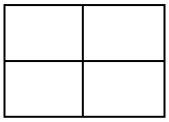
Students should also work with the vocabulary terms halves, thirds, half of, third of, and fourth (or quarter) of. While students are working on this standard, teachers should help them to make the connection that a "whole" is composed of two halves, three thirds, or four fourths.

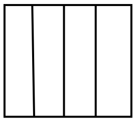
This standard also addresses the idea that equal shares of identical wholes may not have the same shape.

Example:

Divide each square into fourths a different way.

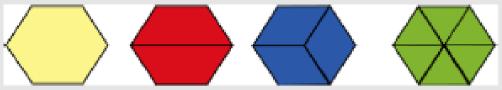






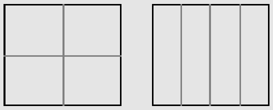
This standard introduces fractions in an **area model**. Students need experiences with different sizes, circles, and rectangles. These different partitions of a square afford the opportunity for students to identify correspondences between the differently-shaped fourths (MP.1), and to explain how one of the fourths shown in these squares can be transformed into one of the "other" fourths shown. (MP.7).

Different pattern blocks compose a regular hexagon



2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Squares partitioned into fourths

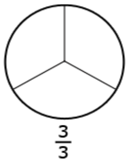


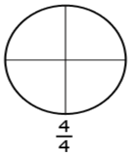
These different partitions of a square afford the opportunity for students to identify correspondences between the differently-shaped fourths (MP.1), and to explain how one of the fourths on the left can be transformed into one of the fourths on the right (MP.7).

2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

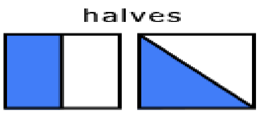
See [Learning Progressions on Geometry]() for dialed information.

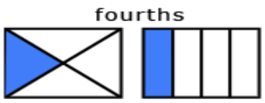
For example, students should recognize that when they cut a circle into three equal pieces, each piece will equal one third of its original whole. In this case, students should describe the whole as three thirds. If a circle is cut into four equal pieces, each piece will equal one fourth of its original whole and the whole is described as four fourths. Circles are difficult to show equal parts so you may want to use rectangular shapes such as strips of construction paper before using circles.





It is important for students to see circles and rectangles partitioned in multiple ways so they learn to recognize that equal shares can be different shapes within the same whole. An interactive whiteboard may be used to show partitions of shapes.





Instructional Strategies: See 2.G.1

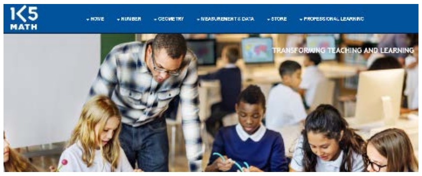
Tools/Resources:

Illustrative Mathematics Grade 2 tasks: **Scroll to the appropriate section to find named tasks.**

- 2.G.A.3
  - Representing Half of a Rectangle
  - Which Pictures Represent One Half?

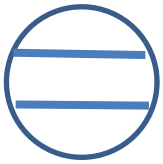
Resources/Tools:

Visit K-5 Math Teaching Resources click on **Geometry**, then on **2nd Grade**. Scroll down to 2.G.3 to access resources specifically for this standard.



Common Misconceptions: See 2.G.1

Students also may believe that a region model represents one out of two, three or four fractional parts without regard to the fact that the parts have to be equal shares, e.g., a circle divided by two equally spaced horizontal lines represents three thirds.



It is vital that students understand different representations of fair shares. Provide a collection of different-size circles and rectangles cut from paper. Ask students to fold some shapes into halves, some into thirds, and some into fourths. They compare the locations of the folds in their shapes as a class and discuss the different representations for the fractional parts.

To fold rectangles into thirds, ask students if they have ever seen how letters are folded to be placed in envelopes. Have them fold the paper very carefully to make sure the three parts are the same size. Ask them to discuss why the same process does not work to fold a circle into thirds.

APPENDIX: TABLE 1. Common Addition and Subtraction Situations

Shading taken from OA progression

	Result Unknown	**Change Unknown**	**Start Unknown**
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Taken from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and variants (blue and green). Yellow indicates problems that are the difficult four problem subtypes or variants that students in Grade 1 work with but do not need to master until Grade 2.

[1] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

TABLE 2. Common Multiplication and Division Situations

Grade level identification of introduction of problem situations taken from OA progression

	Unknown Product	**Group Size Unknown** **("How many in each group?" Division)**	**Number of Groups Unknown** **("How many groups?" Division)**
	3 x 6 = ?	3 x ? = 18; 18 ÷ 3 = ?	? x 6 = 18; 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[4], Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a × b = ?	a × ? = p, and p ÷ a = ?	? × b = p, and p ÷ b = ?

Multiplicative compare problems appear first in Grade 4 (green), with whole number values and with the "times as much" language from the table. In Grade 5, unit fractions language such as "one third as much" may be used. Multiplying and unit language change the subject of the comparing sentence ("A red hat costs n times as much as the blue hat" results in the same comparison as "A blue hat is 1/n times as much as the red hat" but has a different subject.)

TABLE 3. The Properties of Operations

Name of Property	Representation of Property	Example of Property, Using Real Numbers
Properties of Addition		
Associative	$(a+b)+c=a+(b+c)$	$(78+25)+75=78+(25+75)$
Commutative	$a+b=b+a$	$2+98=98+2$
Additive Identity	$a+0=a$ *and* $0+a=a$	$9875+0=9875$
Additive Inverse	For every real number a, there is a real number $-a$ such that $a+-a=-a+a=0$	$-47+47=0$
Properties of Multiplication		
Associative	$(a\times b)\times c=a\times(b\times c)$	$(32\times 5)\times 2=32\times(5\times 2)$
Commutative	$a\times b=b\times a$	$10\times 38=38\times 10$
Multiplicative Identity	$a\times 1=a$ *and* $1\times a=a$	$387\times 1=387$
Multiplicative Inverse	*For every real number* a, $a\neq 0$, *there is a real number* $\frac{1}{a}$ *such that* $a\times\frac{1}{a}=\frac{1}{a}\times a=1$	$\frac{8}{3}\times\frac{3}{8}=1$
Distributive Property of Multiplication over Addition		
Distributive	$a\times(b+c)=a\times b+a\times c$	$7\times(50+2)=7\times 50+7\times 2$

(Variables a, b, and c represent real numbers.)

Excerpt from NCTM's ***Developing Essential Understanding of Algebraic Thinking****, grades 3-5 p. 16-17*

TABLE 4. The Properties of Equality

Name of Property	Representation of Property	Example of property
Reflexive Property of Equality	$a = a$	$3{,}245 = 3{,}245$
Symmetric Property of Equality	$If\ a = b, then\ b = a$	$2 + 98 = 90 + 10, then\ 90 + 10 = 2 + 98$
Transitive Property of Equality	$If\ a = b\ and\ b = c, then\ a = c$	$If\ 2 + 98 = 90 + 10\ and\ 90 + 10 = 52 + 48$ *then* $2 + 98 = 52 + 48$
Addition Property of Equality	$If\ a = b, then\ a + c = b + c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} + \frac{3}{5} = \frac{2}{4} + \frac{3}{5}$
Subtraction Property of Equality	$If\ a = b, then\ a - c = b - c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} - \frac{1}{5} = \frac{2}{4} - \frac{1}{5}$
Multiplication Property of Equality	$If\ a = b, then\ a \times c = b \times c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} \times \frac{1}{5} = \frac{2}{4} \times \frac{1}{5}$
Division Property of Equality	$If\ a = b\ and\ c \neq 0, then\ a \div c = b \div c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} \div \frac{1}{5} = \frac{2}{4} \div \frac{1}{5}$
Substitution Property of Equality	If $a = b$, then b may be substituted for a in any expression containing a.	$If\ 20 = 10 + 10$ *then* $90 + 20 = 90 + (10 + 10)$

(Variables a, b, and c can represent any number in the rational, real, or complex number systems.)

TABLE 5. The Properties of Inequality

Exactly one of the following is true: $a < b, a = b, a > b$.

$If\ a > b\ and\ b > c\ then\ a > c.$

$If\ a > b, then\ b < a.$

$If\ a > b, then\ -a < -b.$

$If\ a > b, then\ a \pm c > b \pm c.$

$If\ a > b\ and\ c > 0, then\ a \times c > b \times c.$

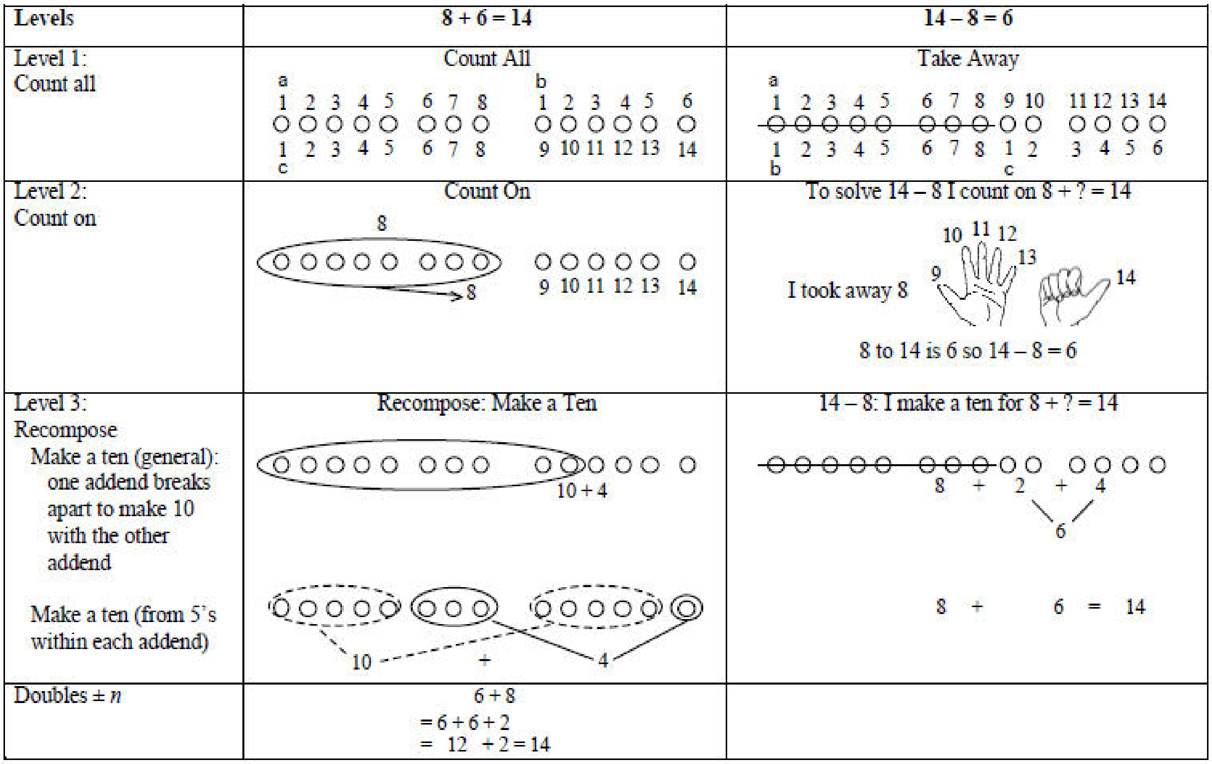
$If\ a > b\ and\ c < 0, then\ a \times c < b \times c.$

$If\ a > b\ and\ c > 0, then\ a \div c > b \div c.$

$If\ a > b\ and\ c < 0, then\ a \div c < b \div c.$

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

TABLE 6. Development of Counting in K-2 Children



Levels	8 + 6 = 14	14 – 8 = 6
Level 1: Count all	Count All a b 1 2 3 4 5 6 7 8 1 2 3 4 5 6 ○○○○○ ○○○ ○○○○○ ○ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 c	Take Away a 1 2 3 4 5 6 7 8 9 10 11 12 13 14 ~~○○○○○ ○○○~~○○ ○○○○ 1 2 3 4 5 6 7 8 1 2 3 4 5 6 b c
Level 2: Count on	Count On 8 ○○○○○ ○○○ ○○○○○ ○ 8 9 10 11 12 13 14	To solve 14 – 8 I count on 8 + ? = 14 10 11 12 9 13 14 I took away 8 8 to 14 is 6 so 14 – 8 = 6
Level 3: Recompose Make a ten (general): one addend breaks apart to make 10 with the other addend Make a ten (from 5's within each addend)	Recompose: Make a Ten ○○○○○ ○○○ ○○○○○ ○ 10 + 4 ○○○○○ ○○○ ○○○○○ ○ 10 + 4	14 – 8: I make a ten for 8 + ? = 14 ~~○○○○○ ○○○~~○○ ○○○○ 8 + 2 + 4 6 8 + 6 = 14
Doubles ± *n*	6 + 8 = 6 + 6 + 2 = 12 + 2 = 14	

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone.
Children are much more successful with counting on; it makes subtraction as easy as addition.

Beginning--A child can count very small collections (1-4) collection of items and understands that the last word tells "how many" even. Beyond on small numbers the number words may be said without the lack of numerical understanding. This is often referred to as rote counting.

Level 1—The child uses one to one correspondence to find the number of objects in two sets. Even if the quantity is known for the first set, the child will start with the first set and continue counting on into the second set. The child begins the count with one. This also connects to Piaget's Hierarchical Inclusion – that in order to have 7 – you have to have 6, 5, 4, etc.

Level 2 – At this level the student begins the counting, starting with the known quantity of the first set and "counts on" from that point in the sequence to establish how many. This method is used to find the total in two sets. This counting is not rote. This level of counting requires several connections between cardinality and counting meanings of the number words.

Level 3 - At this level the student begins using known facts to solve for unknown facts. For example, the student uses "make a ten" where one addend breaks apart to make 10 with another addend OR a doubles plus/minus one strategy. Students begin to implicitly use the properties of operations.

Table 7. Cognitive Rigor Matrix/Depth of Knowledge (DOK)

Kansas Math Standards require high-level cognitive demand asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target the depth(s) of knowledge (DOK) that the student needs to bring to the item/task will be identified, using the Cognitive Rigor Matrix shown below.

Depth of Thinking (Webb)+ Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	• Recall conversions, terms, facts			
Understand	• Evaluate an expression • Locate points on a grid or number on number line • Solve a one-step problem • Represent math relationships in words, pictures, or symbols	• Specify, explain relationships • Make basic inferences or logical predictions from data/observations • Use models/diagrams to explain concepts • Make and explain estimates	• Use concepts to solve non-routine problems • Use supporting evidence to justify conjectures, generalize, or connect ideas • Explain reasoning when more than one response is possible • Explain phenomena in terms of concepts	• Relate mathematical concepts to other content areas, other domains • Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	• Follow simple procedures • Calculate, measure, apply a rule (e.g., rounding) • Apply algorithm or formula • Solve linear equations • Make conversions	• Select a procedure and perform it • Solve routine problem applying multiple concepts or decision points • Retrieve information to solve a problem • Translate between representations	• Design investigation for a specific purpose or research question • Use reasoning, planning, and supporting evidence • Translate between problem & symbolic notation when not a direct translation	• Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	• Retrieve information from a table or graph to answer a question • Identify a pattern/trend	• Categorize data, figures • Organize, order data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern	• Compare information within or across data sets or texts • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph	• Analyze multiple sources of evidence or data sets
Evaluate			• Cite evidence and develop a logical argument • Compare/contrast solution methods • Verify reasonableness	• Apply understanding in a novel way, provide argument or justification for the new application
Create	• Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	• Generate conjectures or hypotheses based on observations or prior knowledge and experience	• Develop an alternative solution • Synthesize information within one data set	• Synthesize information across multiple sources or data sets • Design a model to inform and solve a practical or abstract situation

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