2017

Kansas Mathematics Standards

Grades K-12

Adopted August 2017

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Introduction

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of post-secondary success for all students.

Standards Review Process and Committee

The Kansas Mathematics Standards provide information on what students should know and be able to do at different grade levels in the area of Mathematics. These standards are guidelines school districts can use to develop their mathematics curriculum. **They are not the curriculum.** In Kansas, each school district develops its own mathematics curriculum and teachers decide on how they will provide instruction to ensure student's learning of mathematics. The Kansas State Department of Education reviews its curricular standards at least every seven years per Kansas Education Statute #72-6439.

Review Process

The Kansas Mathematics Standards Writing and Review Committee met regularly both in person and virtually between March 2016 and May 2017 to review, write, and edit the standards document. Documentation/minutes of these meetings were kept to explain and justify decisions made. Once the committee completed its task, the finished product was presented to the State Board of Education for adoption in July 2017. The committee then presented to the State Board of Education in August 2017 to ask for their adoption.

Committee Selection

Information about standards committee formation was shared with the education community via KSDE list serves, meetings, and the State Board of Education. A registration site was developed specifically with the purpose of obtaining nominations for the standards review committees. Individuals could either self-nominate or could recommend someone who was thought to be an excellent committee member. The registration site asked for name, address, email, board district, job title, gender, race, education level, committee group interest, years of work experience, highest level of education. KSDE staff were asked to ensure that committee members for the mathematics committees consisted of diversity of gender, race, ethnicity, and education level (K-12 and post-secondary). Furthermore, special care was taken to ensure that every state board district was represented.

In addition to the "official committee members" each standards committee had an "interested party" ad-hoc group which may be comprised of educational organizations, business community, legislators, KSDE staff, community members, and/or parents. These individuals were interested in the standards review process and their role was to participate in the discussions and provide feedback, however they did not have an official vote in the final product that was sent to the State Board for adoption.

Kansas Mathematics Standards Committee

Below is a list of the writing, review, and ad hoc team members and the district/capacity they served in at the time of its formation.

Writing Team	Review Team
***Debbie Thompson, USD 259**	*Christopher Carter, USD 233
Jerry Braun, USD 489	Bonnie Austin, USD 443
Cherryl Delacruz, USD 501	Jessica Barger, USD 512
Danira Fernandez-Flores, USD 497	Larry Diepenbrock, USD 423
Senator Forest Knox, Kansas Senate	Elisa Dorian, USD 469
Kim Lackey, USD 383	Maureen Engen, Archdiocese of Kansas City Kansas
Dr. Sherri Martinie, Kansas State University	Kelli Ireton, USD 475
Sheila Meggers, USD 308	Jamie Junker, USD 259
Maria Ortiz-Smith, USD 443	Sherry Kinderknecht, USD 315
Elizabeth Peyser, Math Consultant	Suzie Legg, USD 500
Senator Marc Rhoades, Kansas Senate	Dr. Connie Schrock, Emporia State University
John Scoggins, USD 320	Dr. Lucas Shivers, USD 383
Lynette Sharlow, USD 259	Jenny Wilcox, USD 437
Brian Shelton, Special Education Interlocal 607	Rhonda Willis, USD 250
Sarah Stevens, Math Consultant	
Rich Wilson, USD 233	
***Chair**	*Chair

Ad Hoc Team
Barbara Dayal, ECSETS Representative, Kansas State Department of Education
Mickey DeHook, Parent Representative
Shana Gorton, Parent Representative
Dr. Paula Hough, Multi-Tier System of Support Representative
Shelby Jensen, Wichita Area Technical College
Martin Kollman, CTE Representative, Kansas State Department of Education
Lisa Lajoie-Smith, Kansas Association of Teachers of Math Representative
Deb Matthews, ECSETS Representative, Kansas State Department of Education
Alicia Stoltenberg, Center for Education Testing and Evaluation
Dr. Vera Stroup, ECSETS Representative, Kansas State Department of Education
William Thompson, Business & Industry Representative
Ryan Willis, Coffeyville Community College
Patrick Woods, Business & Industry Representative
Paul Wallen, Business & Industry Representative
***ECSETS* – Early Childhood Special Education and Title Services**
***CTE* – Career and Technical Education**

Rose Capacities and Kansas Social, Emotional, and Character Development Model Standards

The Rose Capacities are a mandate from the Kansas Legislature. The Kansas Social, Emotional, and Character Development Model Standards (SECD) were adopted by the Kansas State Department of Education. Both sets of standards are an interplay of cognitive and non-cognitive or affective skills. Through careful teacher planning and implementation in the classroom, these non-cognitive or affective skills are supported and developed in the manner students interact with the Standards for Mathematical Practices and Modeling Standards.

Rose Capacities

1. Sufficient oral and written communication skills to enable students to function in a complex and rapidly changing civilization
2. Sufficient knowledge of economic, social, and political systems to enable the students to make informed choices
3. Sufficient understanding of governmental processes to enable the students to understand the issues that affect his or her community, state, and nation
4. Sufficient self-knowledge and knowledge of his or her mental and physical wellness
5. Sufficient grounding in the arts to enable each student to appreciate his or her cultural and historical heritage
6. Sufficient training or preparation for advanced training in either academic or vocational fields so as to enable each child to choose and pursue life work intelligently
7. Sufficient levels of academic or vocational skills to enable public school students to compete favorably with their counterparts in surrounding states, in academics or in the job market

The Kansas Social, Emotional, Character Development (SECD) Standards, adopted at the April 2012 meeting of the Kansas State Department of Education are designed to help keep children safe and successful while developing their academic and life skills. Students who meet the **Social-Emotional and Character Development Standards** reflect the descriptions below.

- They demonstrate character in their actions by treating others as they wish to be treated and giving their best effort.
- They assume responsibility for their thoughts and actions.
- They demonstrate a growth mindset and continually develop cognitively, emotionally and socially.
- They exhibit the skills to work independently and collaboratively with efficiency and effectiveness.
- They strive for excellence by committing to hard work, persistence and internal motivation.
- They exhibit creativity and innovation, critical thinking and effective problem solving.
- They use resources, including technology and digital media, effectively, strategically capably and appropriately.
- They demonstrate an understanding of other perspectives and cultures.
- They model the responsibility of citizenship and exhibit respect for human dignity.

Learning Progressions for Mathematics

The Kansas State Standards in Mathematics use learning progressions that describe the progression of a topic across grade levels. These narrative documents which are informed by research on children's cognitive development and by the logical structure of mathematics. The progressions justify the sequence of the standards, point out cognitive difficulties and solutions, and provide clarity to particularly 'knotty' areas of mathematics. They are useful in teacher preparation, professional development, curriculum organization, and textbook support. Progression documents also provide a transmission mechanism among mathematics education research, standards, and instruction.

The learning progressions are linked throughout the standards document to provide insight into the content standards. The progressions are structured so the right column of each page identifies the specific standard(s) that are explained to the left. The narrative gives up-to-date information about the content and how students learn that content.

There are a total of 15 learning progressions for grades K – 12. The list of the documents are provided below as a reference.

- Counting & Cardinality and Operations & Algebraic Thinking: Grades K-5
- Number & Operations in Base Ten: Grades K-5
- Measurement & Data (data part): Grades K-5
- Measurement & Data (measurement part): Grades K-5
- Geometry: Grades K-6
- Number & Operations-Fractions: Grades 3-5
- Ratios & Proportional Relationships: Grades 6-7
- Expressions & Equations: Grades 6-8
- Statistics & Probability: Grades 6-8
- The Number System: Grades 6-8 & Number and Quantity Standards (number part): High School
- Progression on Functions: Grades 8-High School
- Algebra: High School
- Statistics & Probability: High School
- Modeling: High School
- Geometry: Grades 7 to High School

The learning progressions are periodically updated. Please refer to the website listed below for the most current edition.

All learning progressions are located at http://community.ksde.org/Default.aspx?tabid=6174.

Mathematics Teaching Practices
(High Leverage Teacher Actions)

[National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: National Council of Teachers of Mathematics.]

The eight Mathematics Teaching Practices should be the foundation for mathematics instruction and learning. This framework was informed by over twenty years of research and presented in *Principles to Actions* by the National Council of Teachers of Mathematics (NCTM). If teachers are guided by this framework, they can move "toward improved instructional practice" and support "one another in becoming skilled at teaching in ways that matter for ensuring successful mathematics learning for all students" (NCTM, 2014, p. 12).

1. **Establish mathematics goals to focus learning.**
   Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
2. **Implement tasks that promote reasoning and problem solving.**
   Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
3. **Use and connect mathematical representations.**
   Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
4. **Facilitate meaningful mathematical discourse.**
   Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
5. **Pose purposeful questions.**
   Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
6. **Build procedural fluency from conceptual understanding.**
   Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
7. **Support productive struggle in learning mathematics.**
   Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
8. **Elicit and use evidence of student thinking.**
   Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Standards for Mathematical Practice
(High Leverage Student Actions)

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $\frac{(y-2)}{(x-1)} = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Standards for Mathematical Practice with Guiding Questions

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
1. Make sense of problems and persevere in solving them. ● Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem. ● Plan a solution pathway instead of jumping to a solution. ● Can monitor their progress and change the approach if necessary. ● See relationships between various representations. ● Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. ● Continually ask themselves, "Does this make sense?" Can understand various approaches to solutions.	How would you describe the problem in your own words? How would you describe what you are trying to find? What do you notice about...? What information is given in the problem? Describe the relationship between the quantities. Describe what you have already tried. What might you change? Talk me through the steps you've used to this point. What steps in the process are you most confident about? What are some other strategies you might try? What are some other problems that are similar to this one? How might you use one of your previous problems to help you begin? How else might you organize...represent... show...?
2. Reason abstractly and quantitatively. ● Make sense of quantities and their relationships. ● Are able to decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. ● Understand the meaning of quantities and are flexible in the use of operations and their properties. ● Create a logical representation of the problem. ● Attends to the meaning of quantities, not just how to compute them.	What do the numbers used in the problem represent? What is the relationship of the quantities? How is__________related to_________? What is the relationship between__________and___? What does_________mean to you? (e.g. symbol, quantity, diagram) What properties might we use to find a solution? How did you decide in this task that you needed to use...? Could we have used another operation or property to solve this task? Why or why not?
3. Construct viable arguments and critique the reasoning of others. ● Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. ● Justify conclusions with mathematical ideas. ● Listen to the arguments of others and ask useful questions to determine if an argument makes sense. ● Ask clarifying questions or suggest ideas to improve/revise the argument. ● Compare two arguments and determine correct or flawed logic.	What mathematical evidence would support your solution? How can we be sure that...? / How could you prove that...? Will it still work if...? What were you considering when...? How did you decide to try that strategy? How did you test whether your approach worked? How did you decide what the problem was asking you to find? (What was unknown?) Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? What is the same and what is different about...? How could you demonstrate a counter-example?
4. Model with mathematics. ● Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). ● Apply the math they know to solve problems in everyday life. ● Are able to simplify a complex problem and identify important quantities to look at relationships. ● Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. ● Reflect on whether the results make sense, possibly improving/revising the model. ● Ask themselves, "How can I represent this mathematically?"	What number model could you construct to represent the problem? What are some ways to represent the quantities? What's an equation or expression that matches the diagram..., number line..., chart..., table...? Where did you see one of the quantities in the task in your equation or expression? What math do you know that you could use to represent this situation? What assumptions do you have to make to solve the problem? What formula might apply in this situation?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
5. Use appropriate tools strategically. ● Use available tools recognizing the strengths and limitations of each. ● Use estimation and other mathematical knowledge to detect possible errors. ● Identify relevant external mathematical resources to pose and solve problems. ● Use technological tools to deepen their understanding of mathematics.	What mathematical tools could we use to visualize and represent the situation? What information do you have? What do you know that is not stated in the problem? What approach are you considering trying first? What estimate did you make for the solution? In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? Why was it helpful to use...? What can using a__________show us that______may not? In what situations might it be more informative or helpful to use...?
6. Attend to precision. ● Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. ● Understand meanings of symbols used in mathematics and can label quantities appropriately. ● Express numerical answers with a degree of precision appropriate for the problem context. ● Calculate efficiently and accurately.	What mathematical terms apply in this situation? How did you know your solution was reasonable? Explain how you might show that your solution answers the problem. Is there a more efficient strategy? How are you showing the meaning of the quantities? What symbols or mathematical notations are important in this problem? What mathematical language..., definitions..., properties can you use to explain...? How could you test your solution to see if it answers the problem?
7. Look for and make use of structure. ● Apply general mathematical rules to specific situations. ● Look for the overall structure and patterns in mathematics. ● See complicated things as single objects or as being composed of several objects.	What observations do you make about...? What do you notice when...? What parts of the problem might you eliminate..., simplify...? What patterns do you find in...? How do you know if something is a pattern? What ideas that we have learned before were useful in solving this problem? What are some other problems that are similar to this one? How does this relate to...? In what ways does this problem connect to other mathematical concepts?
8. Look for and express regularity in repeated reasoning. ● See repeated calculations and look for generalizations and shortcuts. ● See the overall process of the problem and still attend to the details. ● Understand the broader application of patterns and see the structure in similar situations. ● Continually evaluate the reasonableness of their intermediate results	Will the same strategy work in other situations? Is this always true, sometimes true or never true? How would we prove that...? What do you notice about...? What is happening in this situation? What would happen if...? Is there a mathematical rule for...? What predictions or generalizations can this pattern support? What mathematical consistencies do you notice?

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

How to Read the Standards Document

Interactive Nature of Document

The standards committee intended for this document to be a resource to educators. Therefore, the team decided to add links to resources and reference materials. Tables and graphics have been updated and linked to resources to provide more guidance. Due to the importance of the *Learning Progressions*, links are referenced throughout the document.

Identifying Links and Resources within Document

- Hyperlinks are in blue font and underlined.
- **Red** words are roll-over words. Hovering over the word provides a definition of the term identified.

Grade Level Overview Pages

Each grade level has an overview page which provides a listing of the domains within that grade level as well as critical areas of emphasis within each domain. On these pages, you will find hyperlinks to each standard which allows the user to immediately jump to the desired standard. Additionally, you will find a link to a critical area document which provides grade specific details within each of the critical areas. Lastly, you will find a box that includes the *Standards for Mathematical Practice* which links to a document that provides grade level details related to each practice.

Grade Seven Content Standards Overview
Critical Areas for Grade Seven

Ratios and Proportional Relationships (7.RP)
A. Analyze proportional relationships and use them to solve real-world and mathematical problems.
7.RP.1 7.RP.2 7.RP.3

The Number System (7.NS)
A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
7.NS.1 7.NS.2 7.NS.3

Expressions and Equations (7.EE)
A. Use properties of operations to generate equivalent expressions.
7.EE.1 7.EE.2
B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
7.EE.3 7.EE.4

Geometry (7.G)
A. Draw, construct, and describe geometrical figures and describe the relationships between them.
7.G.1 7.G.2 7.G.3
B. Solve real-life and mathematical problems involving area, surface area, and volume.
7.G.4 7.G.5 7.G.6

Statistics and Probability (7.SP)
A. Use random sampling to draw inferences about a population.
7.SP.1 7.SP.2
B. Draw informal comparative inferences about two populations.
7.SP.3 7.SP.4
C. Investigate chance processes and develop, use, and evaluate probability models.
7.SP.5 7.SP.6 7.SP.7 7.SP.8

Standards for Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Click on the box to open specific details related to Grade Seven!

Grade Level Standards

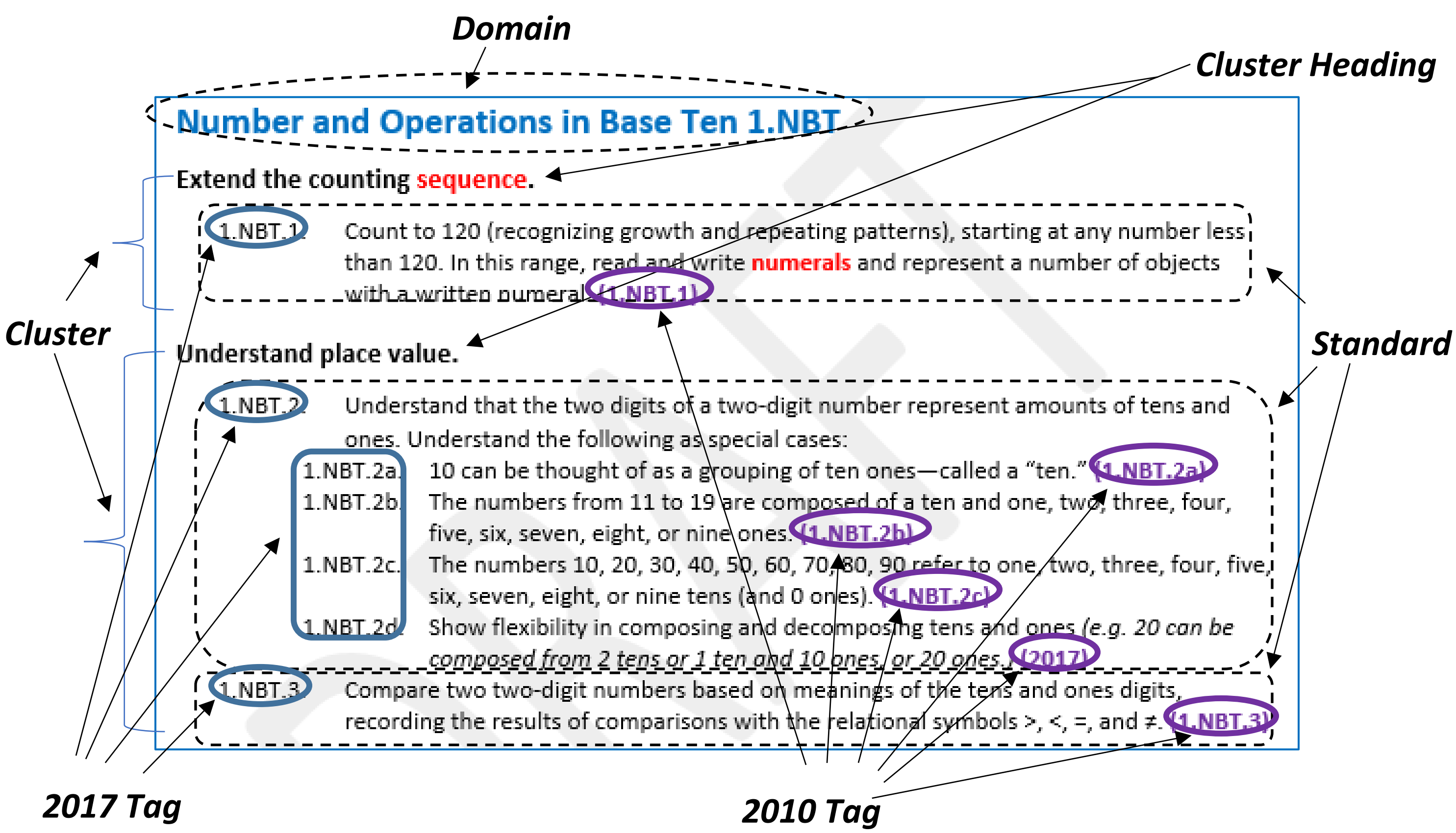
Standards define what *students* should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject. At the beginning of each cluster, the bold label is referred to as the ***Cluster Heading***.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Tagging System

Each standard has both a 2017 and a 2010 tag identified within it. Tags are used to help identify resources associated with the standard. Since there were changes made from the 2010 Standards to the 2017 Standards some of the tags associated with a standard also changed. In order to better help educators locate appropriate resources related to a particular standard the standards review committee inserted both tags. For example, if an educator wanted to find resources associated with a particular standard they would want to search under the 2010 tag. In some cases the 2017 and 2010 tag will be the same while in others they will be different. Furthermore, some standards were created in 2017 and therefore will not have a 2010 tag, these standards will have **(2017)** list as the 2010 tag.



These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

Kindergarten Content Standards Overview

Critical Areas for COHERENCE in Kindergarten

Counting and Cardinality (K.CC)

A. Know number names and the count **sequence**.
   CC.1 CC.2 CC.3
B. Count to tell the number of objects.
   CC.4 CC.5
C. Compare numbers.
   CC.6 CC.7

Operations and Algebraic Thinking (K.OA)

A. Understand addition as putting together and adding to and understand subtraction as taking apart and taking from.
   OA.1 OA.2 OA.3
   OA.4 OA.5

Number and Operations in Base Ten (K.NBT)

A. Work with numbers 11-19 to gain foundations for place value.
   NBT.1

Measurement and Data (K.MD)

A. Describe and compare measurable attributes.
   MD.1 MD.2
B. Classify objects and count the number of objects in each category.
   MD.3

Geometry (K.G)

A. Identify and describe shapes.
   G.1 G.2 G.3
B. Analyze, compare, create, and compose shapes.
   G.4 G.5 G.6

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Click on the box to open specific details related to Kindergarten!

Counting and Cardinality K.CC

(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 1-5)

Know number names and the count sequence.

(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 4-5)

K.CC.1 Count to 100 by ones and by tens and identify as a growth pattern. **(K.CC.1)**
K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1). **(K.CC.2)**
K.CC.3 Read and write **numerals** from 0 to 20. **(K.CC.3)**

Count to tell the number of objects.

(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 4-5)

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality. **(K.CC.4)**
- K.CC.4a. When counting objects, say each number's name in sequential order, pairing each object with one and only one number name and each number name with one and only one object (Click here for a video showing this concept). **(K.CC.4a)**
- K.CC.4b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. **(K.CC.4b)**
- K.CC.4c. Understand that each successive number name refers to a quantity that is one larger. **(K.CC.4c)**
- K.CC.4d. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). **(K.CC.4d)**

K.CC.5 Count to answer "how many?" up to20 concrete or pictorial objects arranged in a line, a rectangular array, or a circle, or as many as 10 objects in a scattered configuration **(subitizing**Error! Bookmark not defined.**)**; given a number from 1 to 20, count out that many objects. **(K.CC.5)**

Compare numbers.

(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 5)

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, *(e.g. by using matching and counting strategies.)* Include groups with up to ten objects. **(K.CC.6)**
K.CC.7 Compare two numbers between 1 and 10 presented as written **numerals**. **(K.CC.7)**

Operations and Algebraic Thinking K.OA

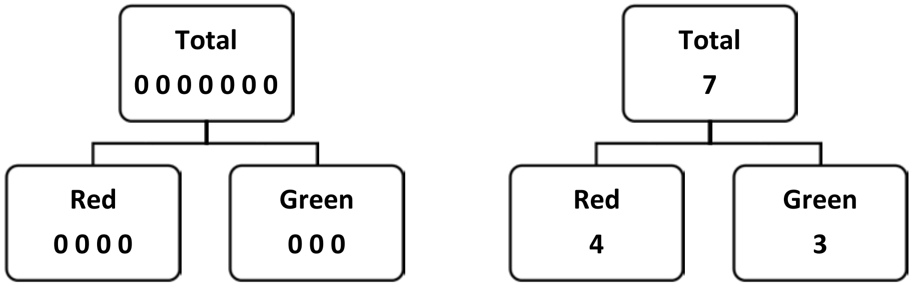
(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 5 last paragraph)

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (*e.g. claps*), acting out situations, verbal explanations, expressions, or equations. **(K.OA.1)**

K.OA.2. Solve addition and subtraction word problems, and add and subtract within 10, (*e.g. by using objects or drawings to represent the problem.)* Refer to shaded section of Table 1 for specific situation types. **(K.OA.2)**

K.OA.3. **Decompose** numbers less than or equal to 10 into pairs in more than one way, (*e.g. by using objects or drawings, and record each decomposition by a drawing or equation* ($e.g.\ 5 = 2 + 3\ and\ 5 = 4 + 1$)). **(K.OA.3)**



K.OA.4. For any number from 1 to 9, find the number that makes 10 when added to the given number, *(e.g. by using objects or drawings, and record the answer with a drawing or equation.).* **(K.OA.4)**

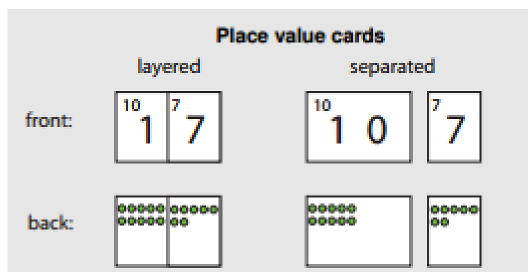
K.OA.5. Fluently (efficiently, accurately, and flexibly) add and subtract within 5. **(K.OA.5)**

Number and Operations in Base Ten K.NBT

(Numbers & Operations Base 10 Progression K-5 Pg. 5)

Work with numbers 11–19 to gain foundations for place value.

K.NBT.1. Compose and **decompose** numbers from 11 to 19 into ten ones and some further ones, *(e.g. by using objects or drawings, and record each composition or decomposition by a drawing or equation* *($e.g.\ 10 + 8 = 18\ and\ 19 = 10 + 9$);*



); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. **(K.NBT.1)**

Measurement and Data K.MD

Describe and compare measurable attributes.

(Measurement & Data progression – measurement part K-5 Pg. 6-7)

K.MD.1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. **(K.MD.1)**

K.MD.2. Directly compare two objects, with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.* **(K.MD.2)**

Classify objects and count the number of objects in each category.

(Measurement & Data Progression – data part K-5 Pg. 5)

K.MD.3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count (*Limit category counts to be less than or equal to 10*). **(K.MD.3)**

Geometry K.G

(Geometry Progression K-6 Pgs. 6-7)

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind, and next to*. **(K.G.1)**

K.G.2. Correctly gives most precise name of shapes regardless of their orientations (position and direction in space) or overall size. **(K.G.2)**

K.G.3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). **(K.G.3)**

Analyze, compare, create, and compose shapes.

K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations (position and direction in space), using informal language to describe their similarities, differences, parts *(e.g. number of sides and vertices/"corners"*) and other attributes *(e.g. having sides of equal length)*. **(K.G.4)**

K.G.5. Model shapes in the world by building shapes from components (*e.g. sticks and clay balls*) and drawing shapes. **(K.G.5)**

K.G.6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"* **(K.G.6)**

Grade One Content Standards Overview

Critical Areas for COHERENCE in Grade One

Operations and Algebraic Thinking (1.OA)

A. Represent and solve problems involving addition and subtraction.
   OA.1 OA.2
B. Understand & apply properties of operations and the relationship between addition & subtraction.
   OA.3 OA.4
C. Add and subtract within 20.
   OA.5 OA.6
D. Work with addition and subtraction equations.
   OA.7 OA.8

Number and Operations in Base Ten (1.NBT)

A. Extend the counting **sequence**.
   NBT.1
B. Understand place value.
   NBT.2 NBT.3
C. Use place value understanding and properties of operations to add and subtract.
   NBT.4 NBT.5 NBT.6

Measurement and Data (1.MD)

A. Measure lengths indirectly and by iterating length units.
   MD.1 MD.2
B. Tell and write time.
   MD.3
C. Represent and interpret data.
   MD.4

Geometry (1.G)

A. Reason with shapes and their attributes.
   G.1 G.2 G.3

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Click on the box to open specific details related to Grade One!

Operations and Algebraic Thinking 1.OA

(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 12)

Represent and solve problems involving addition and subtraction.
(Refer to shaded section of Table 1 for specific situation types.)

1.OA.1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, *(e.g. by using objects, drawings, and situation equations and/or solution equations with a symbol for the unknown number to represent the problem.)* **(1.OA.1)**

For Example: *A clown had 20 balloons. He sold some and has 12 left. How many did he sell?* Situation Equation: 20 – ? = 12 Solution Equation: 20 -12 = ?

1.OA.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, *(e.g. by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.)* **(1.OA.2)**

Understand and apply properties of operations and the relationship between addition and subtraction.

1.OA.3. Apply (not necessary to name) properties of operations as strategies to add and subtract. *Examples:* $8+3=11$ *is known, then* $3+8=11$ *is also known. (Commutative property of addition.) To add* $2+6+4$, *the second two numbers can be added to make a ten, so* $2+6+4=2+10=12$. *(Associative property of addition.) To add 0 to any number, the answer is that number* $7+0=7$ *(Additive identity property of 0).* Students need not use formal terms for these properties. **(1.OA.3)**

1.OA.4. Understand subtraction as an unknown-addend problem. *For example, subtract* $10-8$ *by finding the number that makes 10 when added to 8.* **(1.OA.4)**

Add and subtract within 20.

1.OA.5. Relate counting to addition and subtraction (*e.g. by counting on 2 to add 2, counting back 1 to subtract 1*). **(1.OA.5)**

1.OA.6. Add and subtract within 20, demonstrating fluency (efficiently, accurately, and flexibly) for addition and subtraction within 10. Use mental strategies such as counting on; making *ten (e.g.* $8+6=8+2+4=10+4=14$*)*; decomposing a number leading to a ten *(e.g.* $13-4=13-3-1=10-1=9$*)*; using the relationship between addition and subtraction *(e.g. knowing that* $8+4=12$, *one knows* $12-8=4$); and creating equivalent but easier or known sums *(e.g. adding* $6+7$ *by creating the known equivalent* $6+6+1=12+1=13$*)*. **(1.OA.6)**

Work with addition and subtraction equations.

1.OA.7. Understand the meaning of the equal sign (the value is the same on both sides of the equal sign), and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false?*
$6=6$; $7=8-1$; $5+2=2+5$; $4+1=3+2$; $7-1=4$; $5+4=7-2$ **(1.OA.7)**

1.OA.8. Using related equations, Determine the unknown whole number in an addition or subtraction equation. *For example, determine the unknown number that makes the equation true in each of the equations* $\blacksquare - 3 = 7$; $7 + 3 = \blacksquare$. **(1.OA.8)**

Number and Operations in Base Ten 1.NBT

(Numbers & Operations Base 10 Progression K-5 Pgs. 6-7)

Extend the counting sequence.

1.NBT.1. Count to 120 (recognizing growth and repeating patterns), starting at any number less than 120. In this range, read and write **numerals** and represent a number of objects with a written numeral. **(1.NBT.1)**

Understand place value.

1.NBT.2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
- 1.NBT.2a. 10 can be thought of as a grouping of ten ones—called a "ten." **(1.NBT.2a)**
- 1.NBT.2b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. **(1.NBT.2b)**
- 1.NBT.2c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **(1.NBT.2c)**
- 1.NBT.2d. Show flexibility in composing and decomposing tens and ones *(e.g. 20 can be composed from 2 tens or 1 ten and 10 ones, or 20 ones.)* **(2017)**

1.NBT.3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the relational symbols >, <, =, and ≠. **(1.NBT.3)**

Use place value understanding and properties of operations to add and subtract.

1.NBT.4. Add within 100 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used including: **(1.NBT.4)**
- 1.NBT.4a. Adding a two-digit number and a one-digit number **(1.NBT.4)**
- 1.NBT.4b. Adding a two-digit number and a multiple of 10 **(1.NBT.4)**
- 1.NBT.4c. Understanding that when adding two-digit numbers, combine like base-ten units such as tens and tens, ones and ones; and sometimes it is necessary to compose a ten. **(1.NBT.4)**

1.NBT.5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **(1.NBT.5)**

1.NBT.6. Subtract multiples of 10 in the range 10 to 90 from multiples of 10 in the range 10 to 90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **(1.NBT.6)**

Measurement and Data 1.MD

Measure lengths indirectly and by iterating length units.

1.MD.1. Order three objects by length; compare the lengths of two objects indirectly by using a third object. **(1.MD.1)**

(Measurement and Data (measurement part) Progression K–5 Pg. 8 Paragraph 1.)

1.MD.2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* **(1.MD.2)**
(Measurement and Data (measurement part) Progression K–5 Pg. 8, 3rd Section.)

Tell and write time.

1.MD.3. Tell and write time in hours and half-hours using analog and digital clocks. **(1.MD.3)**

Represent and interpret data.

1.MD.4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. **(1.MD.4)**
(Measurement and Data (data part) Progression K–5 Pg. 5).

Geometry 1.G

(Geometry Progression K-6 Pgs. 8-9)

Reason with shapes and their attributes.

1.G.1. Distinguish between defining attributes *(e.g. triangles are closed and three-sided)* versus non-defining attributes *(e.g. color, orientation, overall size);* build and draw shapes that possess defining attributes. **(1.G.1)**

1.G.2. Compose two-dimensional shapes (rectangles, squares, **trapezoids**, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. Students do not need to learn formal names such as "right rectangular prism." **(1.G.2)**

1.G.3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Note: fraction notation ($\frac{1}{2}, \frac{1}{4}$) is not expected at this grade level. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. **(1.G.3)**

Grade Two Content Standards Overview

Critical Areas for COHERENCE in Grade Two

Operations and Algebraic Thinking (2.OA)

A. Represents and solves problems involving addition and subtraction
   OA.1
B. Add and subtract within 20
   OA.2
C. Work with equal groups of objects to gain foundations for multiplication
   OA.3 OA.4

Number and Operations in Base Ten (2.NBT)

A. Understand place value.
   NBT.1 NBT.2 NBT.3 NBT.4
B. Use place value understanding and properties of operations to add and subtract.
   NBT.5 NBT.6 NBT.7 NBT.8
   NBT.9

Measurement and Data (2.MD)

A. Measure and estimate lengths in standard units
   MD.1 MD.2 MD.3 MD.4
B. Relate addition and subtraction to length
   MD.5 MD.6
C. Work with time and money
   MD.7 MD.8 MD.9
D. Represent and interpret data
   MD.10 MD.11

Geometry (2.G)

A. Reason with shapes and their attributes
   G.1 G.2 G.3

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Click on the box to open specific details related to Grade Two!

Operations and Algebraic Thinking 2.OA

(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 18)

Represent and solve problems involving addition and subtraction.

2.OA.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, *with unknowns in all positions, (e.g. by using drawings and situation equations and/or solution equations with a symbol for the unknown number to represent the problem.) Refer to shaded section of Table 1 for specific situation types.* **(2.OA.1)**

For Example:
A clown had 20 balloons. He sold 8. Another clown came by and gave him more. He now has 24 balloons. How many did the clown give him?
Situation Equation: 20 – 8 = ?
? + □ = 24
Solution Equation: 20 – 8 = ?
24 - ? = □

Add and subtract within 20.

2.OA.2. Fluently (efficiently, accurately, and flexibly) add and subtract within 20 using mental strategies (counting on, making a ten, decomposing a number, creating an equivalent but easier and known sum, and using the relationship between addition and subtraction) Work with equal groups of objects to gain foundations for multiplication. **(2.OA.2)**

Work with equal groups of objects to gain foundations for multiplication.

2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, *(e.g. by pairing objects or counting them by 2s)*; write an equation to express an even number as a sum of two equal addends. **(2.OA.3)**

2.OA.4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **(2.OA.4)**

Number and Operations in Base Ten 2.NBT

(Numbers & Operations Base 10 Progression K-5 Pg. 8)

Understand place value.

2.NBT.1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; *(e.g. 706 equals 7 hundreds, 0 tens, and 6 ones.)* Understand the following as special cases:

- 2.NBT.1a. 100 can be thought of as a bundle of ten tens—called a "hundred." **(2.NBT.1a)**
- 2.NBT.1b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds **(2.NBT.1b)**
- 2.NBT.1c. Show flexibility in composing and decomposing hundreds, tens and ones *(e.g. 207 can be composed from 2 hundreds 7 ones OR 20 tens 7 ones OR 207 ones OR 1 hundred 10 tens 7 ones OR 1 hundred 9 tens 17 ones, etc.)* **(2017)**

2.NBT.2. Count within 1000; skip-count by 2s, 5s, 10s, and 100s; explain and generalize the patterns. **(2.NBT.2)**

2.NBT.3. Read and write numbers within 1000 using base-ten **numerals**, number names, expanded form, and **unit form** unit form. **(2.NBT.3)**

2.NBT.4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, <, =, and ≠ relational symbols to record the results of comparisons. **(2.NBT.4)**

Use place value understanding and properties of operations to add and subtract.
(Numbers & Operations Base 10 Progression K-5 Pg. 8)

2.NBT.5. Fluently (efficiently, accurately, and flexibly) add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction *(e.g. composing/decomposing by like base-10 units, using friendly or benchmark numbers, using related equations, compensation, number line, etc.).* **(2.NBT.5)**

2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations. **(2.NBT.6)**

2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, like base-ten units such as hundreds and hundreds, tens and tens, ones and ones are used; and sometimes it is necessary to compose or **decompose** tens or hundreds. **(2.NBT.7)**

2.NBT.8. Mentally add 10 or 100 to a given number 100 – 900, and mentally subtract 10 or 100 from a given number 100 – 900. **(2.NBT.8)**

2.NBT.9. Explain why addition and subtraction strategies work using place value and the properties of operations. The explanations given may be supported by drawings or objects. **(2.NBT.9)**

Measurement and Data 2.MD

Measure and estimate lengths in standard units.

2.MD.1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **(2.MD.1)**

2.MD.2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **(2.MD.2)**
(Measurement and Data (measurement part) Progression K–5 Pg. 12.)

2.MD.3. Estimate lengths using whole units of inches, feet, centimeters, and meters. **(2.MD.3)**
(Measurement and Data (measurement part) Progression K–5 Pg. 14-15.)

2.MD.4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit (inches, feet, centimeters, and meters). **(2.MD.4)**

Relate addition and subtraction to length.

2.MD.5. Use addition and subtraction within 100 to solve one- and two-step word problems involving lengths that are given in the same units*, e.g. by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.* **(2.MD.5)**

2.MD.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram. **(2.MD.6)**

Work with time and money.

2.MD.7. Tell and write time from analog and digital clocks to the nearest five minutes. **(2.MD.7)**

2.MD.8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately (Do not use decimal point, if showing 25 cents, use the word cents or ¢). *For example: If you have 2 dimes and 3 pennies, how many cents do you have?* **(2.MD.8)**

2.MD.9. Identify coins and bills and their values. **(2017)**

Represent and interpret data.

2.MD.10. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object using different units. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. **(2.MD.9)**

2.MD.11. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph (See Table 1). **(2.MD.10)**

Geometry 2.G

Reason with shapes and their attributes
(Geometry Progression K-6 Pg. 10).

2.G.1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. **(2.G.1)**

2.G.2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. **(2.G.2)**

2.G.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. *Note: fraction notation* $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ *is not expected at this grade level.* Recognize that equal shares of identical wholes need not have the same shape. **(2.G.3)**

Grade Three Content Standards Overview

Critical Areas for COHERENCE in Grade Three

Operations and Algebraic Thinking (3.OA)

A. Represents and solves problems involving multiplication and division
   OA.1 OA.2 OA.3 OA.4
B. Understand properties of multiplication and the relationship between multiplication and division
   OA.5 OA.6
C. Multiply and divide within 100
   OA.7
D. Solve problems involving the four operations, and identify and explain patterns in arithmetic.
   OA.8 OA.9

Number and Operations in Base Ten (3.NBT)

A. Use place value understanding and properties of operations to perform multi-digit arithmetic.
   NBT.1 NBT.2 NBT.3

Number and Operations – Fractions (3.NF)

A. Develop understanding of fractions as numbers.
   NF.1 NF.2 NF.3

Measurement and Data (3.MD)

A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
   MD.1 MD.2 MD.3
B. Represent and interpret data.
   MD.4 MD.5
C. Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
   MD.6 MD.7 MD.8
D. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
   MD.9

Geometry (3.G)

A. Reason with shapes and their attributes
   G.1 G.2

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Click on the box to open specific details related to Grade Three!

Operations and Algebraic Thinking 3.OA

(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 22)

Represent and solve problems involving multiplication and division.
(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 22)

3.OA.1. Interpret products of whole numbers, *(e.g. interpret* $5 \cdot 7$ *as the total number of objects in 5 groups of 7 objects each*.) **(3.OA.1)**

3.OA.2. Interpret whole-number quotients of whole numbers, *(e.g. interpret* $56 \div 8$ *as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.)* **(3.OA.2)**

3.OA.3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, *(e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.)* Refer to shaded section of Table 2 for specific situation types. **(3.OA.3)**

3.OA.4. Determine the unknown whole number in a multiplication or division equation by using related equations. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \cdot ? = 48$; $5 = ■ \div 3$; $6 X 6 =$ ___ **(3.OA.4)**

Understand properties *of* multiplication and the relationship between multiplication and division.
(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 24)

3.OA.5. Apply properties of operations as strategies to multiply and divide. *Examples: If* $6 \cdot 4 = 24$ *is known, then* $4 \cdot 6 = 24$ *is also known. (Commutative property of multiplication.)* $3 \cdot 5 \cdot 2$ *can be found by* $3 \cdot 5 = 15$*, then* $15 \cdot 2 = 30$ *, or by* $5 \cdot 2 = 10$ *, then* $3 \cdot 10 = 30$ *. (Associative property of multiplication.) Knowing that* $8 \cdot 5 = 40$ *and* $8 \cdot 2 = 16$ *, one can find* $8 \cdot 7$ *as* $8 \cdot (5 + 2) = (8 \cdot 5) + (8 \cdot 2) = 40 + 16 = 56$*. (Distributive property.) Students need not use formal terms for these properties.* **(3.OA.5)**

3.OA.6. Understand division as an unknown-factor problem. *For example, find* $32 \div 8$ *by finding the number that makes 32 when multiplied by 8.* **(3.OA.6)**

Multiply and divide within 100 (basic facts up to 10 x 10).

3.OA.7. Fluently (efficiently, accurately, and flexibly) multiply and divide with single digit multiplications and related divisions using strategies *(e.g. relationship between multiplication and division, doubles, double and double again, half and then double, etc.)* or properties of operations. **(3.OA.7)**

Solve problems involving the four operations, and identify and explain patterns in arithmetic.
(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 27 Paragraph 2)

3.OA.8. Solve two-step word problems using any of the four operations. Represent these problems using both situation equations and/or solution equations with a letter or symbol standing for the unknown quantity (*refer to Table 1 and Table 2 and standard 3.OA.3*). Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole-number answers. **(3.OA.8)**

> For Example:
> *A clown had 20 balloons. He sold some and has 12 left. Each balloon costs $2. How much money did he make?*
> Situation Equation: $20 - n = 12$
> $n \times \$2 = \square$
> Solution Equation: $20 - 12 = n$
> $n \times \$2 = \square$

3.OA.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations (See Table 5). *For example, observe that 4 times a number is always even, and explain why 4 times a number can be* ***decomposed*** *into two equal addends.* **(3.OA.9)**

Number and Operations in Base Ten 3.NBT

(Numbers & Operations Base 10 Progression K-5 Pg. 12)

Use place value understanding and properties of operations to perform multi-digit arithmetic.

3.NBT.1. Use place value understanding to round whole numbers to the nearest 10 or 100. **(3.NBT.1)**

3.NBT.2. Fluently (efficiently, accurately, & flexibly) add and subtract within 1000 using strategies *(e.g. composing/decomposing by like base-10 units, using friendly or benchmark numbers, using related equations, compensation, number line, etc.)* and algorithms (including, but not limited to: traditional, partial-sums, etc.) based on place value, properties of operations, and/or the relationship between addition and subtraction. **(3.NBT.2)**

3.NBT.3. Multiply one-digit whole numbers by multiples of 10 in the range 10 to 90 (*e.g.* $9 \cdot 80$, $5 \cdot 60$) using strategies based on place value and properties of operations. **(3.NBT.3)**

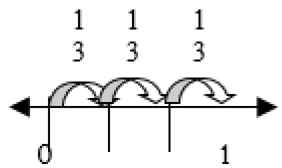
Number and Operations—Fractions 3.NF

Develop understanding of fractions as numbers.
(Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)
(Number and Operations – Fractions Progression Pg. 3-5)

3.NF.1. Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by *a* parts of size $\frac{1}{b}$. **(3.NF.1)**

3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

3.NF.2a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line. **(3.NF.2a)**

Ex: 

3.NF.2b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line (a is the countable units of $\frac{1}{b}$ that determines the place on the number line). **(3.NF.2b)**

3.NF.3. Explain **equivalence** of fractions, and compare fractions by reasoning about their size (it is a mathematical convention that when comparing fractions, the whole is the same size).

3.NF.3a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. **(3.NF.3a)**

3.NF.3b. Recognize and generate simple equivalent fractions, *(e.g.* $\frac{1}{2} = \frac{2}{4}, \frac{4}{6} = \frac{2}{3}$*.)* Explain why the fractions are equivalent, *e.g. by using a visual fraction model*. **(3.NF.3b)**

3.NF.3c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form* $3 = \frac{3}{1}$*; recognize that* $\frac{6}{1} = 6$*; locate* $\frac{4}{4}$ *and 1 at the same point of a number line diagram.* **(3.NF.3c)**

3.NF.3d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the relational symbols >, <, =, or ≠, and justify the conclusions, *(e.g. by using a visual fraction model.)* **(3.NF.3d)**

Measurement and Data 3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1. Tell and write time to the nearest minute using a.m. and p.m. and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, *(e.g. by representing the problem on a number line diagram.)* (See Table 1) **(3.MD.1)**

3.MD.2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l) (Excludes cubed units such as cm^3 and finding the geometric volume of a container). **(3.MD.2)**

3.MD.3. Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, *(e.g. by using drawings (such as a beaker with a measurement scale) to represent the problem.)* (Excludes multiplicative comparison problems) (See Table 1 and Table 2). **(3.MD.2)**

Represent and interpret data.

3.MD.4. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. (See Table 1). *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* **(3.MD.3)**
(Measurement and Data (data part) Progression K–5 Pg. 7)

3.MD.5. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. **(3.MD.4)**
(Measurement and Data (data part) Progression K–5 Pg. 10)

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.6. Recognize area as an attribute of plane figures and understand concepts of area measurement.

3.MD.6a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area (does not require standard square units). **(3.MD.5a)**

3.MD.6b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units (does not require standard square units). **(3.MD.5b)**

3.MD.7. Measure areas by counting unit squares (square cm, square m, square in, square ft, and non-standard square units). **(3.MD.6)**

3.MD.8. Relate area to the operations of multiplication and addition
(Measurement and Data (measurement part) Progression K–5 Pg. 16).

3.MD.8a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. **(3.MD.7a)**

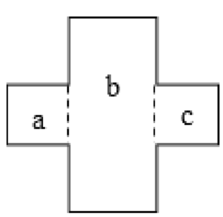
3.MD.8b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. **(3.MD.7b)**

3.MD.8c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \cdot b$ and $a \cdot c$. Use area models to represent the distributive property in mathematical reasoning (Supports 3.OA.5). **(3.MD.7c)**

(Measurement and Data (measurement part) Progression K–5 Pg. 18).

3.MD.8d. Recognize area as additive. Find areas of **rectilinear** figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. **(3.MD.7d)**

Example:



Students can find the total area of the shape by finding the areas of a, b, and c and adding them together.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.9. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. **(3.MD.8)** (Measurement and Data (measurement part) Progression K–5 Pg. 16)

Geometry 3.G

Reason with shapes and their attributes.

(Geometry Progression K-6 Pg. 13)

3.G.1. Understand that shapes in different categories *(e.g. rhombuses, rectangles,* **trapezoids***, kites and others)* may share attributes *(e.g. having four sides),* and that the shared attributes can define a larger category *(e.g. quadrilaterals).* Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. Refer to inclusive definitions noted in the glossary. **(3.G.1)**

3.G.2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as* $\frac{1}{4}$ *of the area of the shape.* **(3.G.2)**

Grade Four Content Standards Overview

Critical Areas for COHERENCE in Grade Four

Operations and Algebraic Thinking (4.OA)

A. Use the four operations with whole numbers to solve problems.
   OA.1 OA.2 OA.3
B. Gain familiarity with factors and multiples.
   OA.4
C. Generate and analyze patterns.
   OA.5

Number and Operations in Base Ten (4.NBT)

A. Generalize place value understanding for multi-digit whole numbers.
   NBT.1 NBT.2 NBT.3
B. Use place value understanding and properties of operations to perform multi-digit arithmetic.
   NBT.4 NBT.5 NBT.6

Number and Operations—Fractions (4.NF)

A. Extend understanding of fraction **equivalence** and ordering.
   NF.1 NF.2
B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
   NF.3 NF.4
C. Understand **decimal notation** for fractions, and compare decimal fractions.
   NF.5 NF.6 NF.7

Measurement and Data (4.MD)

A. Solve problems involving measurement and conversions of measurements from larger units to smaller units.
   MD.1 MD.2 MD.3
B. Represent and interpret data.
   MD.4

Geometry (4.G)

A. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
   G.1 G.2 G.3

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Click on the box to open specific details related to Grade Four!

Operations and Algebraic Thinking 4.OA

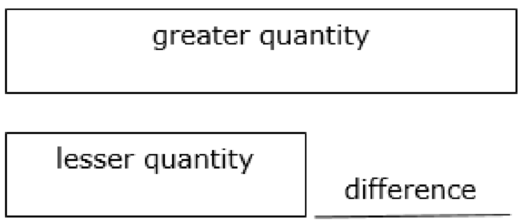
(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 29-31)

Use the four operations with whole numbers to solve problems.

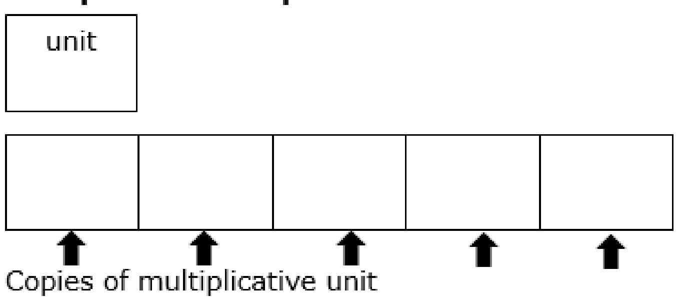
4.OA.1. Interpret a multiplication equation as a comparison, *(e.g. interpret* $35 = 5 \cdot 7$ *as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.)* Represent verbal statements of multiplicative comparisons as multiplication equations. **(4.OA.1)**

4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison*, (e.g. by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.)* **(4.OA.2)**

Additive Comparison



Multiplicative Comparison



4.OA.3. Solve multi-step word problem posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using situation equations and/or solution equations with a letter or symbol standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **(4.OA.3)**

For Example:
A clown had 20 balloons. He sold some and has 12 left. Each balloon costs \$2. How much money did he make?
Situation Equation: $20 - n = 12$
$n \times \$2 = \square$
Solution Equation: $20 - 12 = n$
$n \times \$2 = \square$

Gain familiarity with factors and multiples.

4.OA.4. Find all factor pairs for a whole number in the range 1 to 100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1 to 100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1 to 100 is prime or composite. **(4.OA.4)**

Generate and analyze patterns.

4.OA.5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting* **sequence** *and observe that the terms appear to alternate between odd and even numbers.* Explain informally why the numbers will continue to alternate in this way. **(4.OA.5)**

Number and Operations in Base Ten 4.NBT

(Numbers & Operations Base 10 Progression K-5 Pg. 13-17)

Generalize place value understanding for multi-digit whole numbers.

(Limited to whole numbers less than or equal to 1,000,000.)

4.NBT.1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that* $700 \div 70 = 10$ *by applying concepts of place value and division.* **(4.NBT.1)**

4.NBT.2. Read and write multi-digit whole numbers using base-ten **numerals**, number names, expanded form, and **unit form**. Compare two multi-digit numbers based on meanings of the digits in each place, using >, <, =, and ≠ symbols to record the results of comparisons. *(Note: Students should demonstrate understanding and application of place value decomposition. For example, 127 can be 1 hundred, 2 tens, 7 ones or 12 tens, 7 ones Refer to 2.NBT.1)* **(4.NBT.2)**

4.NBT.3. Use place value understanding to round multi-digit whole numbers to any place. **(4.NBT.3)**

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.4. Fluently (efficiently, accurately, and flexibly) add and subtract multi-digit whole numbers using an efficient algorithm (including, but not limited to: traditional, partial-sums, etc.), based on place value understanding and the properties of operations. **(4.NBT.4)**

4.NBT.5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **(4.NBT.5)**

4.NBT.6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **(4.NBT.6)**

Number and Operations—Fractions 4.NF

Extend understanding of fraction equivalence and ordering.
(Limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.)
(Number and Operations – Fractions Progression Pg. 3)

4.NF.1. Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \cdot a)}{(n \cdot b)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **(4.NF.1)**

(Number and Operations—Fractions Progression 3–5 Pg. 6)

4.NF.2. Compare two fractions with different numerators and different denominators, *(e.g. by creating common numerators or denominators, or by comparing to a benchmark fraction such as* $\frac{1}{2}$*.)* Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with relational symbols >, <, =, or ≠, and justify the conclusions, *(e.g. by using visual fraction models.).* **(4.NF.2)**

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
(Limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.)

4.NF.3. Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.

4.NF.3a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **(4.NF.3a)**

4.NF.3b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, *e.g. by using a visual fraction model.* **(4.NF.3b)**

Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.

4.NF.3c. Add and subtract mixed numbers with like denominators, *e.g. by replacing each mixed number with an equivalent fraction (simplest form is not an expectation), and/or by using properties of operations and the relationship between addition and subtraction.* **(4.NF.3c)**

4.NF.3d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, *e.g. by using visual fraction models and equations to represent the problem.* **(4.NF.3d)**

4.NF.4. Apply and extend previous understandings of multiplication (refer to 2.OA.3, 2.OA.4, 3.OA.1, 3.NF.1, 3.NF.2) to multiply a fraction by a whole number.

4.NF.4a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. *For example, use a visual fraction model to represent* $\frac{5}{4}$ *as 5 copies of* $\frac{1}{4}$*, recording the conclusion by the equation* $\frac{5}{4} = 5 \cdot \frac{1}{4}$. **(4.NF.4a)**

4.NF.4b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express* $3 \cdot \frac{2}{5}$ *as* $6 \cdot \frac{1}{5}$*, recognizing this product as* $\frac{6}{5}$*. (In general,* $n \cdot \frac{a}{b} = \frac{n \cdot a}{b}$*.).* **(4.NF.4b)**

4.NF.4c. Solve word problems involving multiplication of a fraction by a whole number, (See Table 2) *(e.g. by using visual fraction models and equations to represent the problem.) For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* **(4.NF.4c)**

Understand decimal notation for fractions, and compare decimal fractions.
(Students are expected to learn to add decimals by converting them to fractions with the same denominator, in preparation for general fraction addition in grade 5.)

4.NF.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express $\frac{3}{10}$ as $\frac{30}{100}$, and add $\frac{3}{10}+\frac{4}{100}=\frac{34}{100}$.* **(4.NF.5)**

4.NF.6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **(4.NF.6)**

4.NF.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the relational symbols >, < =, or ≠, and justify the conclusions, *(e.g. by using a visual model.).* **(4.NF.7)**

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …* **(4.MD.1)**

(Measurement and Data (measurement part) Progression K–5 Pg. 20)

4.MD.2. Use the four operations to solve word problems (See Table 1 and Table 2) involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. **(4.MD.2)**

4.MD.3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems explaining and justifying the appropriate unit of measure. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* **(4.MD.3)**

Represent and interpret data.

4.MD.4. Make a data display (line plot, bar graph, pictograph) to show a set of measurements in fractions of a unit $\left(\frac{1}{2},\frac{1}{4},\frac{1}{8}\right)$. Solve problems involving addition and subtraction of fractions by using information presented in the data display. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.* **(4.MD.4)**

Geometry 4.G

(Geometry Progression K-6 Pg. 15-16)

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse, straight, reflex), and perpendicular and parallel lines. Identify these in two-dimensional figures. **(4.G.1)**

4.G.2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles (right, acute, obtuse, straight, reflex). Recognize and categorize triangles based on angles (right, acute, obtuse, and equiangular) and/or sides (scalene, isosceles, and equilateral). **(4.G.2)**

4.G.3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. **(4.G.3)**

Grade Five Content Standards Overview

Critical Areas for COHERENCE in Grade Five

Operations and Algebraic Thinking (5.OA)

A. Write and interpret numerical expressions.
OA.1 OA.2

Number and Operations in Base Ten 5.(NBT)

A. Understand the place value system.
NBT.1 NBT.2 NBT.3 NBT.4

B. Perform operations with multi-digit whole numbers and with decimals to hundredths.
NBT.5 NBT.6 NBT.7

Number and Operations—Fractions (5.NF)

A. Use equivalent fractions as a strategy to add and subtract fractions.
NF.1 NF.2

B. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
NF.3 NF.4 NF.5 NF.6
NF.7

Measurement and Data (5.MD)

A. Convert like measurement units within a given measurement system.
MD.1

B. Represent and interpret data.
MD.2

C. Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.
MD.3 MD.4 MD.5

Geometry (5.G)

A. Graph points on the coordinate plane to solve real world and mathematical problems.
G.1 G.2

B. Classify two-dimensional figures into categories based on their properties.
G.3 G.4

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Click on the box to open specific details related to Grade Five!

Operations and Algebraic Thinking 5.OA

(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 32)

Write and interpret numerical expressions.

5.OA.1. Use parentheses in numerical expressions and evaluate expressions with these symbols. **(5.OA.1)**

5.OA.2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "multiply the sum of 8 and 7 by 2" as* $2 \times (8 + 7)$ *because parenthetical information must be solved first. Recognize that* $3 \times (18932 + 921)$ *is three times as large as* $18932 + 921$*, without having to calculate the indicated sum or product.* **(5.OA.2)**

Number and Operations in Base Ten 5.NBT

(Numbers & Operations Base 10 Progression K-5 Pg. 18-20)

Understand the place value system.

5.NBT.1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left. **(5.NBT.1)**

5.NBT.2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **(5.NBT.2)**

5.NBT.3. Read, write, and compare decimals to thousandths.

5.NBT.3a. Read and write decimals to thousandths using base-ten **numerals**, number names, expanded form, and **unit form** *(e.g.*

$$expanded\ form\ 47.392 = 4 \cdot 10 + 7 \cdot 1 + 3 \cdot \frac{1}{10} + 9 \cdot \frac{1}{100} + 2 \cdot \frac{1}{1000}$$

$unit\ form$ 47.392 = 4 tens + 7 ones + 3 tenths + 9 hundredths + 2 thousandths). **(5.NBT.3a)**

5.NBT.3b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, <, =, and ≠relational symbols to record the results of comparisons. **(5.NBT.3b)**

5.NBT.4. Use place value understanding to round decimals to any place (Note: In fifth grade, decimals include whole numbers and decimal fractions to the hundredths place.) **(5.NBT.4)**

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.5. Fluently (efficiently, accurately, and flexibly) multiply multi-digit whole numbers using an efficient algorithm *(ex., traditional, partial products, etc.)* based on place value understanding and the properties of operations. **(5.NBT.5)**

5.NBT.6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **(5.NBT.6)**
(Number and Operations Base 10 Progression K-5 Pg. 16-17)

5.NBT.7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **(5.NBT.7)**
(Number and Operations Base 10 Progression K-5 Pg. 18-20)

Number and Operations—Fractions 5.NF

(Number and Operations – Fractions Progression Pg. 3)

Use equivalent fractions as a strategy to add and subtract fractions.

(Number and Operations – Fractions Progression Pg. 3-5)

5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$ *In general,* $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$ **(5.NF.1)**

5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, *(e.g. by using visual fraction models or equations to represent the problem.)* Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. (See Table 1 to view situation types). *For example, recognize an incorrect result* $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$ *by observing that* $\frac{3}{7}<\frac{1}{2}$. **(5.NF.2)**

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

(Number and Operations – Fractions Progression Pg. 12-14)

5.NF.3. Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b}=a\div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, *e.g. by using visual fraction models or equations to represent the problem. For example, interpret* $\frac{3}{4}$ *as the result of dividing 3 by 4, noting that* $\frac{3}{4}$ *multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size* $\frac{3}{4}$. *If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **(5.NF.3)**

5.NF.4. Apply and extend previous understandings of multiplication (refer to 2.OA.3, 2.OA.4, 3.OA.1, 3.NF.1, 3.NF.2, 4.NF.4) to multiply a fraction or whole number by a fraction. (Number and Operations—Fractions Progression 3–5 Pg. 12 - 13).

5.NF.4a. Interpret the product $\frac{a}{b}\cdot q$ as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a **sequence** of operations $a\cdot q\div b$. *For example, use a visual fraction model to show* $\frac{2}{3}\cdot 4=\frac{8}{3}$ *and create a story context for this equation. Do the same with* $\frac{2}{3}\cdot\frac{4}{5}=\frac{8}{15}$. *(In general,* $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$*).* **(5.NF.4a)**

5.NF.4b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **(5.NF.4b)**

5.NF.5. Interpret multiplication as scaling (resizing), by:

5.NF.5a. Comparing the size of a product to the size of one factor based on the size of the other factor, without performing the indicated multiplication *(e.g. They see* $\left(\frac{1}{2}\cdot 3\right)$ *as half the size of 3.).* **(5.NF.5a)**

5.NF.5b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction **equivalence** $\frac{a}{b} = \frac{na}{nb}$ to the effect of multiplying $\frac{a}{b}$ by 1. *(e.g. Students may have the misconception that multiplication always produces a larger result. They need to have the conceptual understanding with examples like;* $\frac{3}{4} \times one\ dozen\ eggs$ *will have a product that is less than 12.)* **(5.NF.5b)**

5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers, *(e.g. by using visual fraction models or equations to represent the problem)* (See Table 2 to view situation types). **(5.NF.6)**

5.NF.7. Apply and extend previous understandings of division (3.OA.2, 3.OA.5), to divide unit fractions by whole numbers and whole numbers by unit fractions. Division of a fraction by a fraction is not a requirement at this grade.

5.NF.7a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for* $\frac{1}{3} \div 4$*, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $\frac{1}{3} \div 4 = \frac{1}{12}$ *because* $\frac{1}{12} \cdot 4 = \frac{1}{3}$. **(5.NF.7a)**

5.NF.7b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for* $4 \div \frac{1}{5}$*, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $4 \div \frac{1}{5} = 20$ *because* $20 \cdot \frac{1}{5} = 4$. **(5.NF.7b)**

5.NF.7c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions*, e.g. by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{1}{3}$ *cup servings are in 2 cups of raisins?* **(5.NF.7c)**

Measurement and Data 5.MD

Convert like measurement units within a given measurement system.

5.MD.1. Convert among different-sized standard measurement units within a given measurement system *(e.g. convert 5 cm to 0.05 m),* and use these conversions in solving multi-step, real world problems. **(5.MD.1)** (Measurement and Data (measurement part) Progression K–5 Pg. 26)

Represent and interpret data.

5.MD.2. Make a data display (line plot, bar graph, pictograph) to show a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$). Use operations (add, subtract, multiply) on fractions for this grade to solve problems involving information presented in the data display. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. After lunch everyone measured how much milk they had left in their containers. Make a line plot showing data to the nearest* $\frac{1}{4}$ *cup. Which value has the greatest amount? What is the total?* **(5.MD.2)**

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

(Measurement and Data (measurement part) Progression K–5 Pg. 26 Section 2).

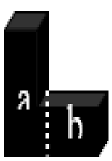
5.MD.3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

- 5.MD.3a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **(5.MD.3a)**
- 5.MD.3b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **(5.MD.3b)**

5.MD.4. Measure volumes by counting unit cubes such as cubic cm, cubic in, cubic ft. or non-standard cubic units. **(5.MD.4)**

5.MD.5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

- 5.MD.5a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent three-dimensional whole-number products as volumes, *(e.g. to represent the associative property of multiplication.)* **(5.MD.5a)**
- 5.MD.5b. Apply the formulas $V = l \cdot w \cdot h$ and $V = B \cdot h$ (*B represents the area of the base)* for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. **(5.MD.5b)**
- 5.MD.5c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. **(5.MD.5c)**



Students find the volume of each rectangular prism that is decomposed from the original figure and add the individual volumes to find the total volume.

Geometry 5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.

(Geometry Progression K-6 Pg. 17 and graphic from Pg. 17-18)

5.G.1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond *(e.g. x-axis and x-coordinate, y-axis and y-coordinate)*. **(5.G.1)**

5.G.2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. *(e.g. plotting the relationship between two positive quantities such as maps, coordinate grid games (such as Battleship), time/temperature, time/distance, cost/quantity, etc.).* **(5.G.2)**

K 1 2 3 4 5 6 7 8 HS

Classify two-dimensional figures into categories based on their properties
(Geometry Progression K-6 Pg. 17 and graphic from Pg. 18)

5.G.3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* **(5.G.3)**

5.G.4. Classify two-dimensional figures in a hierarchy based on properties. **(5.G.4)**

Grade Six Content Standards Overview

Critical Areas for COHERENCE in Grade Six

Ratios and Proportional Relationships (6.RP)

A. Understand ratio concepts and use ratio reasoning to solve problems.
6.RP.1 6.RP.2 6.RP.3

The Number System (6.NS)

A. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
6.NS.1

B. Compute fluently (efficiently, accurately, and flexibly) with multi-digit numbers and find common factors and multiples.
6.NS.2 6.NS.3 6.NS.4

C. Apply and extend previous understandings of numbers to the system of **rational numbers**
6.NS.5 6.NS.6 6.NS.7 6.NS.8

Expressions and Equations (6.EE)

A. Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.1 6.EE.2 6.EE.3

B. Reason about and solve one-variable equations and inequalities.
6.EE.4 6.EE.5 6.EE.6 6.EE.7

C. Represent and analyze quantitative relationships between dependent and independent variables.
6.EE.8

Geometry (6.G)

A. Solve real-world and mathematical problems involving area, surface area, and volume.
6.G.1 6.G.2 6.G.3 6.G.4

Statistics and Probability (6.SP)

A. Develop concepts of statistical measures of center and variability and an informal understanding of **outlier**.
6.SP.1 6.SP.2 6.SP.3

B. Summarize and describe distributions.
6.SP.4 6.SP.5

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Click on the box to open specific details related to Grade Six!

Ratios and Proportional Relationships 6.RP

(Ratios and Proportional Relationships Progression 6–7 Pg. 1)

Understand ratio concepts and use ratio reasoning to solve problems.
(Ratios and Proportional Relationships Progression 6–7 Pg. 3)

6.RP.1. Use ratio language to describe a relationship between two quantities. Distinguish between part-to-part and part-to-whole relationships. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **(6.RP.1)**

6.RP.2. Use unit rate language ("for each one", "for every one" and "per") and unit rate notation to demonstrate understanding the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a: b \textit{ with } b \neq 0,$ *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is* $\frac{3}{4}$ *cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* (Expectations for unit rates in this grade are limited to non-complex fractions.) **(6.RP.2)**

6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, *(e.g. by reasoning about tables of equivalent ratios,* ***tape diagrams***, ***double number line diagram***, *or using calculations.)*

6.RP.3a. Make tables of equivalent ratios relating quantities with whole-number measurements, find the missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* **(6.RP.3a) (6.RP.3b)**

6.RP.3b. Find a percent of a quantity as a rate per 100 *(e.g. 30% of a quantity means* $\frac{30}{100}$ *times the quantity*); solve problems involving finding the whole, given a part and the percent. **(6.RP.3c)**

6.RP.3c. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. **(6.RP.3d)**

The Number System 6.NS

(Number System 6–8 and High School Number Progression Pg. 1)

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, requiring multiple exposures connecting various concrete and abstract models. **(6.NS.1)** (Number System 6–8 and High School Number Progression Pg. 5 - 6.)

Compute fluently (efficiently, accurately, and flexibly) with multi-digit numbers and find common factors and multiples.

6.NS.2. Fluently (efficiently, accurately, and flexibly) divide multi-digit numbers using an efficient algorithm. **(6.NS.2)**

6.NS.3. Fluently (efficiently, accurately, and flexibly) add, subtract, multiply, and divide multi-digit decimals using an efficient algorithm for each operation. **(6.NS.3)**

6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express* $18 + 48 \textit{ as } 6(3 + 8)$. **(6.NS.4)**

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.5. Understand positive and negative numbers to describe quantities having opposite directions or values *(e.g. temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge);* **(6.NS.5)**

6.NS.5a. Use positive and negative numbers to represent quantities in real-world contexts, **(6.NS.5)**

6.NS.5b. Explaining the meaning of 0 in each situation. **(6.NS.5)**

6.NS.6. Understand a rational number as a point on the number line and a coordinate pair as a location on a coordinate plane.

6.NS.6a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, *(e.g.* $-(-3) = 3$,*)* and that 0 is its own opposite. **(6.NS.6a) (6.NS.6b)**

6.NS.6b. Recognize signs of numbers in ordered pairs indicate locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. **(6.NS.6c)**

6.NS.6c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **(6.NS.6d)**

6.NS.7. Understand ordering and absolute value of rational numbers.

6.NS.7a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret* $-3 > -7$ *as a statement that –3 is located to the right of –7 on a number line oriented from left to right.* **(6.NS.7a)**

6.NS.7b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write* $-3^{\circ}C > -7^{\circ}C$ *to express the fact that* $-3^{\circ}C$ *is warmer than* $-7^{\circ}C$. **(6.NS.7b)**

6.NS.7c. Explain the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write* $|-30| = 30$ *to describe the size of the debt in dollars.* **(6.NS.7c)**

6.NS.7d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.* **(6.NS.7d)**

6.NS.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **(6.NS.8)**

Expressions and Equations 6.EE

(Expressions and Equations Progression 6–8 Pg. 4)

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.1. Write and evaluate numerical expressions involving whole-number exponents. **(6.EE.1)**

6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers.

6.EE.2a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as* $5 - y$. **(6.EE.2a)**

6.EE.2b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* $2(8+7)$ *as a product of two factors; view* $(8+7)$ *as both a single entity and a sum of two terms.* **(6.EE.2b)**

6.EE.2c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas* $V = s^3$ *and* $A = 6s^2$ *to find the volume and surface area of a cube with sides of length* $s = \frac{1}{2}$. **(6.EE.2c)**

6.EE.3. Apply the properties of operations and combine like terms, with the conventions of algebraic notation, to identify and generate equivalent expressions. *For example, apply the distributive property to the expression* $3(2+x)$ *to produce the equivalent expression* $6+3x$*; apply properties of operations to* $y+y+y$ *to produce the equivalent expression 3y.* **(6.EE.3) (6.EE.4)**

Reason about and solve one-variable equations and inequalities.

6.EE.4. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **(6.EE.5)**

6.EE.5. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **(6.EE.6)**

6.EE.6. Write and solve one-step equations involving non-negative rational numbers using addition, subtraction, multiplication and division. **(6.EE.7)**

6.EE.7. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **(6.EE.8)**

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.8. Use variables to represent two quantities in a real-world problem that change in relationship to one another.

6.EE.8a. Identify the independent and dependent variable. **(6.EE.9)**

6.EE.8b. Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* $d = 65t$ *to represent the relationship between distance and time.* **(6.EE.9)**

6.EE.8c. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. **(6.EE.9)**

Geometry 6.G

(Geometry Progression K-6 Pg. 19)

Solve real-world and mathematical problems involving area, surface area, and volume.
(Geometry Progression K-6 Pg. 19 - 20)

6.G.1. Find the area of all triangles, special quadrilaterals (including parallelograms, kites and **trapezoids**), and polygons whose edges meet at right angles (**rectilinear figure** (*See Geometry Progression K-6 Pg. 19 Paragraph 4) polygons*) by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. **(6.G.1)**

6.G.2. Find the volume of a right rectangular prism with fractional edge lengths by applying the formulas $V = lwh \text{ and } V = Bh$ (*B* is the area of the base and *h* is the height) to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (Builds on the 5th grade concept of packing unit cubes to find the volume of a rectangular prism with whole number edge lengths.) **(6.G.2)**

6.G.3. Draw polygons whose edges meet at right angles (**rectilinear figure** polygons) in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **(6.G.3)**

6.G.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. **(6.G.4)**

Statistics and Probability 6.SP

(Statistics and Probability Progression 6–8 Pg. 4)

Develop concepts of statistical measures of center and variability and an informal understanding of outlier.
(Statistics and Probability Progression 6–8 Pg. 4)

6.SP.1. Recognize and generate a **statistical question** as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* **(6.SP.1)**

6.SP.2. Analyze a set of data collected to answer a **statistical question** with a distribution which can be described by its center (mean, median and/or mode), spread (range and/or **interquartile range**), and overall shape (**cluster**, **peak**, gap, symmetry, **skew (data)** and/or outlier). **(6.SP.2)**

6.SP.3. Recognize that a measure of center (mean, median and/or mode) for a numerical data set summarizes all of its values with a single number, while a measure of variation (range and/or **interquartile range**) describes how its values vary with a single number. **(6.SP.3)**

Summarize and describe distributions.

6.SP.4. Display numerical data on dot plots, histograms, stem-and-leaf plots, and box plots. **(6.SP.4)**

6.SP.5. Summarize numerical data sets in relation to their context, such as by:

- 6.SP.5a. Reporting the number of observations. **(6.SP.5a)**
- 6.SP.5b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. **(6.SP.5b)**

6.SP.5c. Giving quantitative measures of center (mean, median and/or mode) and variability (range and/or **interquartile range**), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. **(6.SP.5c)**

6.SP.5d. Relating the choice of measures of center and variability to the distribution of the data. **(6.SP.5d)**

Grade Seven Content Standards Overview

Critical Areas for COHERENCE in Grade Seven

Ratios and Proportional Relationships (7.RP)

A. Analyze proportional relationships and use them to solve real-world and mathematical problems.
7.RP.1 7.RP.2 7.RP.3

The Number System (7.NS)

A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
7.NS.1 7.NS.2 7.NS.3

Expressions and Equations (7.EE)

A. Use properties of operations to generate equivalent expressions.
7.EE.1 7.EE.2
B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
7.EE.3 7.EE.4

Geometry (7.G)

A. Draw, construct, and describe geometrical figures and describe the relationships between them.
7.G.1 7.G.2 7.G.3
B. Solve real-life and mathematical problems involving area, surface area, and volume.
7.G.4 7.G.5 7.G.6

Statistics and Probability (7.SP)

A. Use random sampling to draw inferences about a population.
7.SP.1 7.SP.2
B. Draw informal comparative inferences about two populations.
7.SP.3 7.SP.4
C. Investigate chance processes and develop, use, and evaluate probability models.
7.SP.5 7.SP.6 7.SP.7 7.SP.8

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Click on the box to open specific details related to Grade Seven!

Ratios and Proportional Relationships 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.
(Ratios and Proportional Relationships Progression 6–7 Pg. 8)

7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks* $\frac{1}{2}$ *mile in each* $\frac{1}{4}$ *hour, compute the unit rate as the* ***complex fraction*** $\frac{\frac{1}{2}}{\frac{1}{4}}$ *miles per hour (interpreting a complex fraction as division of fractions), equivalently 2 miles per hour.* **(7.RP.1)**
(Ratios and Proportional Relationships Progression 6–7 Pg. 9 Graphic)

7.RP.2. Recognize and represent proportional relationships between quantities:

- 7.RP.2a. Determine whether two quantities are in a proportional relationship*, e.g. by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.* **(7.RP.2a)**
- 7.RP.2b. Analyze a table or graph and recognize that, in a proportional relationship, every pair of numbers has the same unit rate (referred to as the “m”). **(7.RP.2b)**
- 7.RP.2c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$*.* **(7.RP.2c)**
- 7.RP.2d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. **(7.RP.2d)**

7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* **(7.RP.3)**

The Number System 7.NS

(Number System 6–8 and High School Number Progression Pg. 9)

Apply and extend previous understandings of operations with positive rational numbers to add, subtract, multiply, and divide all rational numbers.

7.NS.1. Represent addition and subtraction on a horizontal or vertical number line diagram.

- 7.NS.1a. Describe situations in which opposite quantities combine to make 0. Show that a number and its opposite have a sum of 0 (are additive inverses). *For example, show zero-pairs with two-color counters.* **(7.NS.1a)**
- 7.NS.1b. Show $p + q$ as the number located a distance $|q|$ from *p*, in the positive or negative direction depending on whether *q* is positive or negative. **(7.NS.1b)**
- 7.NS.1c. Model subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. **(7.NS.1c)**
- 7.NS.1d. Model subtraction as the distance between two rational numbers on the number line where the distance is the absolute value of their difference. **(7.NS.1c)**
- 7.NS.1e. Apply properties of operations as strategies to add and subtract rational numbers. **(7.NS.1d)**

7.NS.2. Apply and extend previous understandings of multiplication and division of positive rational numbers to multiply and divide all rational numbers.

7.NS.2a. Describe how multiplication is extended from positive rational numbers to all rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. **(7.NS.2a)**

7.NS.2b. Explain that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. Leading to situations such that if *p* and *q* are integers, then $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}$. **(7.NS.2b)**

7.NS.2c. Apply properties of operations as strategies to multiply and divide rational numbers. **(7.NS.2c)**

7.NS.2d. Convert a rational number in the form of a fraction to its decimal equivalent using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **(7.NS.2d)**

7.NS.3. Solve and interpret real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) **(7.NS.3)**

Expressions and Equations 7.EE

(Expressions and Equations Progression 6–8 Pg. 8)

Use properties of operations to generate equivalent expressions.

7.EE.1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. Note: factoring is limited to integer coefficients. *For example: apply the distributive property to the expression* $24x + 18y$ *to produce the equivalent expression* $6(4x + 3y)$. **(7.EE.1)**

7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,* $a + 0.05a = 1.05a$ *means that "increase by 5%" is the same as "multiply by 1.05."* **(7.EE.2)**

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3. Solve multi-step real-life and mathematical problems with rational numbers. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional* $\frac{1}{10}$ *of her salary an hour, or $2.50, for a new salary of $27.50.* **(7.EE.3)**

7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct two-step equations and inequalities to solve problems by reasoning about the quantities.

7.EE.4a. Solve word problems leading to equations of the form $px + q = r$, and $p(x + q) = r$ where *p, q,* and *r* are specific rational numbers. Solve equations of these forms fluently (efficiently, accurately, and flexibly). Compare an algebraic solution to an arithmetic solution, identifying the **sequence** of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* **(7.EE.4a)**

7.EE.4b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$ where *p*, *q*, and *r* are specific rational numbers and $p > 0$. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.* **(7.EE.4b)**

Geometry 7.G

(Geometry High School Progression Pg. 6)

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.1. Solve problems involving scale drawings of geometric figures, such as computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **(7.G.1)**

7.G.2. Identify three-dimensional objects generated by rotating a two-dimensional (rectangular or triangular) object around one edge. **(G.GMD.4)**

7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right cylinder. **(7.G.3)**

Solve real-life and mathematical problems involving area, surface area, and volume.

7.G.4. Use the formulas for the area and circumference of a circle and solve problems; give an **informal derivation** of the relationship between the circumference and area of a circle. **(7.G.4)**

7.G.5. Investigate the relationship between three-dimensional geometric shapes; **(2017)**

7.G.5a. Generalize the volume formula for prisms and cylinders ($V = Bh$ where *B* is the area of the base and *h* is the height). **(2017)**

7.G.5b. Generalize the surface area formula for prisms and cylinders ($SA = 2B + Ph$ where *B* is the area of the base, *P* is the perimeter of the base, and *h* is the height (in the case of a cylinder, perimeter is replaced by circumference)). **(2017)**

7.G.6. Solve real-world and mathematical problems involving area of two-dimensional objects and volume and surface area of three-dimensional objects including cylinders and right prisms. (Solutions should **not** require students to take square roots or cube roots. *For example, given the volume of a cylinder and the area of the base, students would identify the height*.) **(7.G.6)**

Statistics and Probability 7.SP

(Statistics and Probability Progression 6–8 Pg. 7)

Use random sampling to draw inferences about a population.

7.SP.1. Use statistics to gain information about a population by examining a sample of the population;

7.SP.1a. Know that generalizations about a population from a sample are valid only if the sample is representative of that population and generate a valid representative sample of a population. **(7.SP.1)**

7.SP.1b. Identify if a particular random sample would be representative of a population and justify your reasoning. **(7.SP.1)**

7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to informally gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* **(7.SP.2)**

Draw informal comparative inferences about two populations.

(Statistics and Probability Progression 6–8 Pg. 5 Paragraph 3)

7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability (requires introduction of **mean absolute deviation**). *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* **(7.SP.3)**

7.SP.4. Use measures of center (mean, median and/or mode) and measures of variability (range, **interquartile range** and/or **mean absolute deviation**) for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book. (NOTE: Students should not have to calculate mean absolute deviation but use it to interpret data).* **(7.SP.4)**

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5. Express the probability of a chance event as a number between 0 and 1 that represents the likelihood of the event occurring. (Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.) **(7.SP.5)**

7.SP.6. Collect data from a chance process (probability experiment). Approximate the probability by observing its long-run relative frequency. Recognize that as the number of trials increase, the experimental probability approaches the theoretical probability. Conversely, predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* **(7.SP.6)**

7.SP.7. Develop a **probability model** and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

7.SP.7a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* **(7.SP.7a)**

7.SP.7b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* **(7.SP.7b)**

7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

7.SP.8a. Know that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. **(7.SP.8a)**

7.SP.8b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language *(e.g. "rolling double sixes")*, identify the outcomes in the sample space which compose the event. **(7.SP.8b)**

7.SP.8c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?* **(7.SP.8c)**

Grade Eight Content Standards Overview

Critical Areas for COHERENCE in Grade Eight

The Number System (8.NS)

A. Know that there are numbers that are not rational, and approximate them by rational numbers.
NS.1 NS.2

Expressions and Equations (8.EE)

A. Work with radicals and integer exponents.
EE.1 EE.2 EE.3

B. Understand the connections between proportional relationships, lines, and linear equations.
EE.4 EE.5 EE.6

C. Analyze and solve linear equations and inequalities.
EE.7

Functions (8.F)

A. Define, evaluate, and compare functions.
F.1 F.2 F.3

B. Use functions to model relationships between quantities.
F.4 F.5

Geometry (8.G)

A. Geometric measurement: understand concepts of angle and measure angles.
G.1 G.2 G.3 G.4
G.5 G.6

B. Understand and apply the Pythagorean Theorem.
G.7 G.8 G.9

C. Solve real-world and mathematical problems involving measurement.
G.10 G.11 G.12

Statistics and Probability (8.SP)

A. Investigate patterns of association in **bivariate data**.
SP.1 SP.2 SP.3

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Click on the box to open specific details related to Grade Eight!

The Number System 8.NS

(Number System 6–8 and High School Number Progression Pg. 14)

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. **(8.NS.1)**

8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions *(e.g. π^2). For example, for the approximation of 68, show that $\sqrt{68}$ is between 8 and 9 and closer to 8.* **(8.NS.2)**

Expressions and Equations 8.EE

(Expressions and Equations Progression 6–8 Pg. 11)

Work with radicals and integer exponents.

8.EE.1. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where *p* is a positive rational number. Evaluate square roots of whole number perfect squares with solutions between 0 and 15 and cube roots of whole number perfect cubes with solutions between 0 and 5. Know that $\sqrt{2}$ is irrational. **(8.EE.2)**

8.EE.2. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.* **(8.EE.3)**

8.EE.3. Read and write numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities *(e.g. use millimeters per year for seafloor spreading)*. Interpret scientific notation that has been generated by technology. **(8.EE.4)**

Understand the connections between proportional relationships, lines, and linear equations.

8.EE.4. Graph proportional relationships, interpreting its unit rate as the slope (m) of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **(8.EE.5)**

8.EE.5. Use similar triangles to explain why the slope (m) is the same between any two distinct points on a non-vertical line in the coordinate plane and extend to include the use of the slope formula ($m = \frac{y_2-y_1}{x_2-x_1}$ when given two coordinate points (x_1, y_1) and (x_2, y_2)). Generate the equation $y = mx$ for a line through the origin (proportional) and the equation $y = mx + b$ for a line with slope *m* intercepting the vertical axis at y-intercept *b* (not proportional when $b \neq 0$). **(8.EE.6)**

8.EE.6. Describe the relationship between the proportional relationship expressed in $y = mx$ and the non-proportional linear relationship $y = mx + b$ as a result of a vertical translation. *Note: be clear with students that all linear relationships have a constant rate of change (slope), but only the special case of proportional relationships (line that goes through the origin) continue to have a* ***constant of proportionality****.* **(2017)**

Analyze and solve linear equations and inequalities.

8.EE.7. Fluently (efficiently, accurately, and flexibly) solve one-step, two-step, and multi-step linear equations and inequalities in one variable, including situations with the same variable appearing on both sides of the equal sign.

8.EE.7a. Give examples of linear equations in one variable with one solution ($x = a$), infinitely many solutions ($a = a$), or no solutions ($a = b$). Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a, a = a, or\ a = b$ results (where *a* and *b* are different numbers). **(8.EE.7a)**

8.EE.7b. Solve linear equations and inequalities with rational number coefficients, including equations/inequalities whose solutions require expanding and/or factoring expressions using the distributive property and collecting like terms. **(8.EE.7b)**

Functions 8.F

Define, evaluate, and compare functions.

8.F.1. Explain that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. *(Function notation is not required in Grade 8.)* **(8.F.1)**

8.F.2. Compare properties of two linear functions represented in a variety of ways (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change, the greater y-intercept, or the point of intersection.* **(8.F.2)**

8.F.3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.* **(8.F.3)**

Use functions to model relationships between quantities.

8.F.4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **(8.F.4)**

8.F.5. Describe qualitatively the functional relationship between two quantities by analyzing a graph *(e.g. where the function is increasing or decreasing, linear or nonlinear).* Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **(8.F.5)**

Geometry 8.G

(Geometry High School Progression Pg. 9)

Geometric measurement: understand concepts of angle and measure angles.

8.G.1. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

8.G.1a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the **circular arc** between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a “one-degree angle,” and can be used to measure angles. **(4.MD.5a)**

8.G.1b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **(4.MD.5b)**

8.G.2. Measure angles in whole-number degrees using a protractor. Draw angles of specified measure using a protractor and straight edge. **(4.MD.6)**

([Measurement and Data (measurement part) Progression K-5 Pg. 22-25])

8.G.3. Recognize angle measure as additive. When an angle is **decomposed** into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, *e.g. by using an equation with a symbol for the unknown angle measure.* **(4.MD.7)**

8.G.4. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and use them to solve simple equations for an unknown angle in a figure. **(7.G.5)**

8.G.5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* **(8.G.5)**

8.G.6. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on drawing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. **(7.G.2)**

Understand and apply the Pythagorean Theorem.

8.G.7. Explain a proof of the Pythagorean Theorem and its converse. **(8.G.6)**

8.G.8. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. *For example: Finding the slant height of pyramids and cones.* **(8.G.7)**

8.G.9. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **(8.G.8)**

Solve real-world and mathematical problems involving measurement.

8.G.10. Use the formulas or informal reasoning to find the arc length, areas of sectors, surface areas and volumes of pyramids, cones, and spheres. *For example, given a circle with a 60° central angle, students identify the arc length as* $\frac{1}{6}$ *of the total circumference* $\left(\frac{1}{6}=\frac{60}{360}\right)$. **(8.G.9)**

8.G.11. Investigate the relationship between the formulas of three dimensional geometric shapes;

8.G.11a. Generalize the volume formula for pyramids and cones ($V=\frac{1}{3}Bh$). **(G.GMD.3)**

8.G.11b. Generalize surface area formula of pyramids and cones ($SA=B+\frac{1}{2}Pl$). **(G.GMD.3)**

8.G.12. Solve real-world and mathematical problems involving arc length, area of two-dimensional shapes including sectors, volume and surface area of three-dimensional objects including pyramids, cones and spheres. **(8.G.9)**

Statistics and Probability 8.SP

(Statistics and Probability Progression 6–8 Pg. 11)

Investigate patterns of association in bivariate data.

8.SP.1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, **outliers,** positive or negative association, linear association, and nonlinear association. **(8.SP.1)**

8.SP.2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. **(8.SP.2)**

8.SP.3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height*. **(8.SP.3)**

High School Notation

The high school standards identify the mathematics that each and every student should study in order to be college and career ready. These standards are organized not by grade, but rather by conceptual category. The order of these categories are as follows:

- Modeling
- Number and Quantity
- Algebra
- Functions
- Geometry
- Statistics and Probability

Each conceptual category creates a coherent view of high school mathematics and the natural progression of the standards. For example, modeling is best interpreted not as a collection of isolated topics, but rather in relation to the other standards. For this reason, modeling is the first conceptual category and should be utilized throughout all other conceptual categories.

(★) **Modeling Standards:** Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol. The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

The standards themselves do not dictate curriculum, pedagogy, or delivery of content. Throughout the state, school districts have the opportunity to organize the standards into courses that will best benefit their students – whether that be through a traditional course sequence (Algebra I, Geometry, Algebra II) or an integrated course sequence (Mathematics 1, Mathematics 2, Mathematics 3).

Grade Level Classifications

To assist with the organization of high school mathematics courses, the standards have grade level classifications to identify the appropriate grade at which they should be taught. The classifications were designed with the following framework in mind:

Year of School	Traditional Course Sequence	Integrated Course Sequence
9^{th} Grade	Algebra I	Mathematics 1
10^{th} Grade	Geometry	Mathematics 2
11^{th} Grade	Algebra II	Mathematics 3

There will be variation with student placement in the courses listed above. At the present time, the "gateway" math class in Kansas for postsecondary schooling is College Algebra. The standards committee used this as a guide when identifying grade level classifications.

The grade level classifications are as follows:

(9/10)	These standards are required for all students by the end of their first two years of high school math courses.
(11)	These standards are required for all students by the end of their third year math course.
(9/10/11)	These standards are required for all students in their first three years of high school math courses. These standards are often further divided to (9/10) and (11) to identify specific concepts and their appropriate grade level. *(9/10) should primarily accomplish the standards described as linear, quadratic and absolute value while (11) should primarily accomplish the standards described as logarithmic, square root, cube root, and exponential .*
(all)	These standards should be taught throughout every high school math course, and often represent over-arching themes or key features of the mathematical concept. These standards should be taught in conjunction with the appropriate grade level standards.
(+)	These standards should be taught as **extensions** to grade level standards when possible, or in a 4th year math course. These standards prepare students to take advanced courses in high school such as college algebra, calculus, advanced statistics, or discrete mathematics.

Mathematics | High School—Modeling

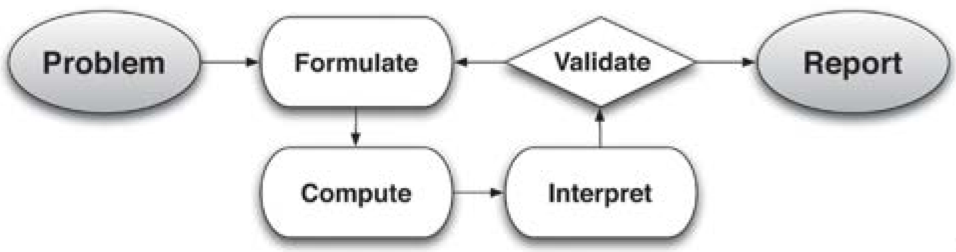
(High School Modeling Progression Pg. 1)

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, *e.g. applied to turnaround of an aircraft at an airport.*
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

There are different types of modeling. In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.
Analytic modeling seeks to explain data based on deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena *(e.g. the behavior of polynomials)* as well as physical phenomena.

Modeling Standards: Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

High School – Number and Quantity Content Standards Overview

The Real Number System (N.RN)

A. Use properties of rational and irrational numbers.
   N.RN.1 N.RN.2 N.RN.3

Quantities (★) (N.Q)

A. Reason quantitatively and use units to solve problems.
   N.Q.1 (★) N.Q.2 (★) N.Q.3 (★)

The Complex Number System (N.CN)

A. Perform arithmetic operations with complex numbers.
   N.CN.1 N.CN.2 N.CN.3

B. Represent complex numbers and their operations on the complex plane.
   N.CN.4 (+) N.CN.5 (+) N.CN.6 (+)

C. Use complex numbers in polynomial identities and equations.
   N.CN.7 (+) N.CN.8 N.CN.9 (+)
   N.CN.10 (+)

Vector and Matrix Quantities (N.VM)

A. Represent and model with vector quantities.
   N.VM.1 (+) N.VM.2 (+) N.VM.3 (+)

B. Perform operations on vectors.
   N.VM.4 (+) N.VM.5 (+)

C. Perform operations on matrices and use matrices in applications.
   N.VM.6 N.VM.7 N.VM.8
   N.VM.9 (+) N.VM.10 (+) N.VM.11 (+)
   N.VM.12 (+)

Standards for Mathematical Practices

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2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
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5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Click on the box to open specific details related to High School - Number and Quantity!

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The Real Number System N.RN

(Number System 6–8 and High School Number Progression Pg. 16)

Use properties of rational numbers and irrational numbers.

N.RN.1. **(9/10)** Know and apply the properties of integer exponents to generate equivalent numerical and algebraic expressions. **(8.EE.1)**

N.RN.2. **(11)** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $\left(5^{\frac{1}{3}}\right)^3 = 5^{\frac{1}{3}\cdot 3}$ to hold, so $\left(5^{\frac{1}{3}}\right)^3$ must equal 5.* **(N.RN.1)**

N.RN.3. **(11)** Rewrite expressions involving radicals and rational exponents using the properties of exponents. **(N.RN.2)**

Quantities★ N.Q

Reason quantitatively and use units to solve problems.

N.Q.1. **(all)** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★ **(N.Q.1)**

N.Q.2. **(all)** Define appropriate quantities for the purpose of descriptive modeling.★ **(N.Q.2)**

N.Q.3. **(all)** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★ **(N.Q.3)**

The Complex Number System N.CN

(Number System 6–8 and High School Number Progression Pg. 18)

Perform arithmetic operations with complex numbers.

N.CN.1. **(11)** Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real. **(N.CN.1)**

N.CN.2. **(11)** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **(N.CN.2)**

N.CN.3. **(11)** Find the conjugate of a complex number. **(N.CN.3)**

N.CN.4. **(+)** Use conjugates to find **moduli** and quotients of complex numbers. **(N.CN.3)**

Represent complex numbers and their operations on the complex plane.

N.CN.5. **(+)** Represent complex numbers on the complex plane in rectangular and **polar form** (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **(N.CN.4)**

N.CN.6. **(+)** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $\left(-1 + \sqrt{3} \cdot i\right)^3 = 8$ *because* $\left(-1 + \sqrt{3} \cdot i\right)$ *has modulus 2 and argument 120°.* **(N.CN.5)**

N.CN.7. **(+)** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. **(N.CN.6)**

Use complex numbers in polynomial identities and equations.

N.CN.8. **(11)** Solve quadratic equations with real coefficients that have complex solutions. **(N.CN.7)**

N.CN.9. **(+)** Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$. **(N.CN.8)**

N.CN.10. **(+)** Know the **Fundamental Theorem of Algebra**; show that it is true for quadratic polynomials. **(N.CN.9)**

Vector and Matrix Quantities N-VM

Represent and model with vector quantities.

N.VM.1. **(+)** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes $(e.g., v, |v|, \|v\|, v)$. **(N.VM.1)**

N.VM.2. **(+)** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **(N.VM.2)**

N.VM.3. **(+)** Solve problems involving velocity and other quantities that can be represented by vectors. **(N.VM.3)**

Perform operations on vectors.

N.VM.4. **(+)** Add and subtract vectors.

- N.VM.4a. **(+)** **Add vectors end-to-end**, component-wise, and by the **parallelogram rule** . Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. **(N.VM.4a)**
- N.VM.4b. **(+)** Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. **(N.VM.4b)**
- N.VM.4c. **(+)** Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of *w*, with the same magnitude as *w* and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. **(N.VM.4c)**

N.VM.5. **(+)** Multiply a vector by a **scalar**.

N.VM.5a. **(+)** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, *(e.g. as* $c(v_x, v_y) = (cv_x, cv_y)$*.)* **(N.VM.5a)**

N.VM.5b. **(+)** Compute the magnitude of a scalar multiple *cv* using $\|cv\| = |c|v$. Compute the direction of *cv* knowing that when $|c|v \neq 0$, the direction of *cv* is either along *v* (for $c > 0$) or against *v* (for $c < 0$). **(N.VM.5b)**

Perform operations on matrices and use matrices in applications.

N.VM.6. **(11)** Use matrices to represent and manipulate data, *(e.g. representing information in a linear programing problem as a matrix or rewriting a system of equations as a matrix.)* **(N.VM.6)**

N.VM.7. **(11)** Multiply matrices by scalars to produce new matrices, *(e.g. as when all of the payoffs in a game are doubled.)* **(N.VM.7)**

N.VM.8. **(11)** Add, subtract, and multiply matrices of appropriate dimensions; find determinants of 2×2 matrices. **(N.VM.8)**

N.VM.9. **(+)** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **(N.VM.9)**

N.VM.10. **(+)** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **(N.VM.10)**

N.VM.11. **(+)** Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. **(N.VM.11)**

N.VM.12. **(+)** Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. **(N.VM.12)**

High School – Algebra Content Standards Overview

Seeing Structure in Expressions (A.SSE)

A. Interpret the structure of expressions.

A.SSE.1 (★) A.SSE.2 (★)

B. Write expressions in equivalent forms to solve problems.

A.SSE.3 (★)

Arithmetic with Polynomials and Rational Expressions (A.APR)

A. Perform arithmetic operations on polynomials.

A.APR.1 A.APR.2 A.APR.3

B. Use polynomial identities to solve problems.

A.APR.4 A.APR.5 (+)

C. Rewrite rational expressions.

A.APR.6 (+) A.APR.7 (+)

Creating Equations (★) (A.CED)

A. Create equations that describe numbers or relationships.

A.CED.1 (★) A.CED.2 (★) A.CED.3 (★)

A.CED.4 (★)

Reasoning with Equations and Inequalities (A.REI)

A. Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.1

B. Solve equations and inequalities in one variable.

A.REI.2 A.REI.3 A.REI.4 A.REI.5

C. Solve systems of equations.

A.REI.6 A.REI.7 (+)

D. Represent and solve equations and inequalities graphically.

A.REI.8 A.REI.9 (★) A.REI.10

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Seeing Structure in Expressions A.SSE

(High School Algebra Progression Pg. 4)

Interpret the structure of expressions.

A.SSE.1. **(all)** Interpret expressions that represent a quantity in terms of its context. ★

A.SSE.1a. **(all)** Interpret parts of an expression, such as terms, factors, and coefficients. ★ **(A.SSE.1a)**

A.SSE.1b. **(all)** Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and* $(1+r)^n$. ★ **(A.SSE.1b)**

A.SSE.2. **(all)** Use the structure of an expression to identify ways to rewrite it. **(A.SSE.2)**

Write expressions in equivalent forms to solve problems.

A.SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

A.SSE.3a. **(9/10)** Factor a quadratic expression to reveal the zeros of the function it defines. ★ **(A.SSE.3a)**

A.SSE.3b. **(11)** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. ★ **(A.SSE.3b)**

A.SSE.3c. **(11)** Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 1.15^t *can be rewritten as* $\left(1.15^{\frac{1}{12}}\right)^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* ★ **(A.SSE.3c)**

Arithmetic with Polynomials and Rational Expressions A.APR

(High School Algebra Progression Pg. 7)

Perform arithmetic operations on polynomials.

A.APR.1. **(9/10)** Add, subtract, and multiply polynomials. **(A.APR.1)**

A.APR.2. **(11)** Factor higher degree polynomials; identifying that some polynomials are prime. **(2017)**

A.APR.3. **(11)** Know and apply the **Remainder Theorem**: For a polynomial *p*(*x*) and a number *c*, the remainder on division by $(x - c)$ is $p(c)$, so $p(c) = 0$ if and only if $(x - c)$ is a factor of $p(x)$. **(A.APR.2)**

Use polynomial identities to solve problems.

A.APR.4. **(9/10/11)** Generate polynomial identities from a pattern. *For example, difference of squares, perfect square trinomials, (emphasize sum and difference of cubes in grade **11**).* **(A.APR.4)**

A.APR.5. **(+)** Know and apply the **Binomial Theorem** for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle. The Binomial Theorem can be proven by mathematical **induction** or by a **combinatorial argument**. **(A.APR.5)**

Rewrite rational expressions.

A.APR.6. **(+)** Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where *a*(*x*), *b*(*x*), *q*(*x*), and *r*(*x*) are polynomials with the degree of *r*(*x*) less than the degree of *b*(*x*), using inspection, long division, or, for the more complicated examples, a computer algebra system. **(A.APR.6)**

A.APR.7. **(+)** Add, subtract, multiply, and divide rational expressions. **(A.APR.7)**

Creating Equations★ A.CED

(High School Algebra Progression Pg. 10)

Create equations that describe numbers or relationships.

A.CED.1. **(all)** Apply and extend previous understanding to create equations and inequalities in one variable and use them to solve problems.★ **(A.CED.1)**

A.CED.2. **(all)** Apply and extend previous understanding to create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ **(A.CED.2)**

A.CED.3. **(all)** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★ **(A.CED.3)**

A.CED.4. **(all)** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*★ **(A.CED.4)**

Reasoning with Equations and Inequalities A.REI

(High School Algebra Progression Pg. 13)

Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.1. **(all)** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **(A.REI.1)**

Solve equations and inequalities in one variable.

A.REI.2. **(all)** Apply and extend previous understanding to solve equations, inequalities, and compound inequalities in one variable, including literal equations and inequalities. **(A.REI.3)**

A.REI.3. Solve equations in one variable and give examples showing how extraneous solutions may arise.

A.REI.3a. **(9/10/11)** Solve rational, absolute value and square root equations. **(A.REI.2)**
(9/10) Limited to simple equations such as, $2\sqrt{x-3}+8=16$, $\frac{x+3}{2x-1}=5, x\neq\frac{1}{2}$.

A.REI.3b. **(+)** Solve exponential and logarithmic equations. **(2017)**

A.REI.4. **(11)** Solve radical and rational exponent equations and inequalities in one variable, and give examples showing how extraneous solutions may arise. **(A.REI.2)**

A.REI.5. Solve quadratic equations and inequalities

A.REI.5a. **(9/10)** Solve quadratic equations by inspection *(e.g. for $x^2=49$),* taking square roots, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives no real solutions. **(A.REI.4b)**

A.REI.5b. **(11)** Solve quadratic equations with complex solutions written in the form $a\pm bi$ for real numbers *a* and *b.* **(A.REI.4b)**

A.REI.5c. **(11)** Use the method of completing the square to transform and solve any quadratic equation in *x* into an equation of the form $(x-p)^2=q$ that has the same solutions. **(A.REI.4a)**

A.REI.5d. **(+)** Solve quadratic inequalities and identify the domain. **(2017)**

Solve systems of equations.

A.REI.6. **(9/10)** Analyze and solve pairs of simultaneous linear equations.

A.REI.6a. **(9/10)** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. **(8.EE.8a)**

A.REI.6b. **(9/10)** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x+2y=5$ *and* $3x+2y=6$ *have no solution because* $3x+2y$ *cannot simultaneously be 5 and 6.* **(8.EE.8b)**

A.REI.6c. **(9/10)** Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* **(8.EE.8c)**

A.REI.7. **(+)** Represent a system of linear equations as a single matrix equation and solve (incorporating technology) for matrices of dimension 3×3 or greater. **(A.REI.8) (A.REI.9)**

Represent and solve equations and inequalities graphically.

A.REI.8. **(all)** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **(A.REI.10)**

A.REI.9. **(9/10/11)** Solve an equation $f(x)=g(x)$ by graphing $y=f(x)$ and $y=g(x)$ and finding the *x*-value of the intersection point. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ For **(9/10)** focus on linear, quadratic, and absolute value. **(A.REI.11)**

A.REI.10. **(9/10)** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. **(A.REI.12)**

High School – Functions Content Standards Overview

Interpreting Functions (F.IF)

A. Understand the concept of a function and use function notation.
F.IF.1 F.IF.2 F.IF.3

B. Interpret functions that arise in applications in terms of the context.
F.IF.4 (★) F.IF.5 (★) F.IF.6 (★)

C. Analyze functions using different representations.
F.IF.7 (★) F.IF.8 F.IF.9

Building Functions (F.BF)

A. Build a function that models a relationship between two quantities.
F.BF.1 F.BF.2 (+)(★)

B. Build new functions from existing functions.
F.BF.3 F.BF.4 F.BF.5

Linear, Quadratic, and Exponential Models (★) (F.LQE)

A. Construct and compare linear, quadratic, and exponential models and solve problems.
F.LQE.1 (★) F.LQE.2 (★)

Trigonometric Functions (F.TF)

A. Extend the domain of trigonometric functions using the unit circle.
F.TF.1 (+) F.TF.2 (+) F.TF.3 (+)
F.TF.4 (+)

B. Model periodic phenomena with trigonometric functions.
F.TF.5 (+) (★) F.TF.6 (+) F.TF.7 (+) (★)

C. Prove and apply trigonometric identities.
F.TF.8 (+) F.TF.9 (+)

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Interpreting Functions F.IF

(High School Functions Progression Pg. 7)

Understand the concept of a function and use function notation.

F.IF.1. **(all)** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$. **(F.IF.1)**

F.IF.2. **(all)** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **(F.IF.2)**

F.IF.3. **(9/10/11)** Recognize patterns in order to write functions whose domain is a subset of the integers. **(9/10)** Limited to linear and quadratic. *For example, find the function given* $\{(-1,4),(0,7),(1,10),(2,13)\}$. **(F.IF.3)**

Interpret functions that arise in applications in terms of the context.

F.IF.4. **(all)** For a function that models a relationship between two quantities, interpret key features of expressions, graphs and tables in terms of the quantities, and sketch graphs showing key features given a description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★ **(F.IF.4)**

F.IF.5. **(all)** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★ **(F.IF.5)**

F.IF.6. **(9/10/11)** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **(9/10)** limited to linear functions. ★ **(F.IF.6)**

Analyze functions using different representations.

F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★

- F.IF.7a. **(9/10)** Graph linear, quadratic and absolute value functions and show intercepts, maxima, minima and end behavior. ★ **(F.IF.7a)**
- F.IF.7b. **(11)** Graph square root, cube root, and exponential functions. ★ **(F.IF.7b)**
- F.IF.7c. **(11)** Graph logarithmic functions, emphasizing the inverse relationship with exponentials and showing intercepts and end behavior. ★ **(F.IF.7e)**
- F.IF.7d. **(+)** Graph piecewise-defined functions, including step functions. ★ **(F.IF.7b)**
- F.IF.7e. **(11)** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. ★ **(F.IF.7c)**
- F.IF.7f. **(+)** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. ★ **(F.IF.7d)**
- F.IF.7g. **(+)** Graph trigonometric functions, showing period, midline, and amplitude. ★ **(F.IF.7e)**

F.IF.8. Write a function in different but equivalent forms to reveal and explain different properties of the function.

- F.IF.8a. **(9/10)** Use different forms of linear functions, such as slope-intercept, standard, and point-slope form to show rate of change and intercepts. **(2017)**
- F.IF.8b. **(11)** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **(F.IF.8a)**
- F.IF.8c. **(11)** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t, y = (0.97)^t, y = (1.01)^{12t}, y = (1.2)^{\frac{t}{10}}$, and classify them as representing exponential growth or decay. **(F.IF.8b)**

F.IF.9. **(all)** Compare properties of two functions using a variety of representations (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, a quantity increasing exponentially eventually exceeds a quantity increasing linearly.* **(F.IF.9)**

Building Functions F.BF

(High School Functions Progression Pg. 11)

Build a function that models a relationship between two quantities.

F.BF.1. Use functions to model real-world relationships.

- F.BF.1a. **(9/10)** Combine multiple functions to model complex relationships. *For example,* $p(x) = r(x) - c(x); (profit = revenue - cost)$. **(F.BF.1b)**

F.BF.1b. **(11)** Determine an **explicit expression** , a **recursive function**, or steps for calculation from a context. **(F.BF.1a)**

F.BF.1c. **(11)** Compose functions. *For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.* **(F.BF.1c)**

F.BF.2. **(+)** Write arithmetic and geometric **sequences** and series both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★**(F.BF.2) (A.SSE.4)**

Build new functions from existing functions.

F.BF.3. **(9/10/11)** Transform parent functions $(f(x))$ by replacing $f(x)$ with $f(x)+k,\ kf(x),\ f(kx)$, and $f(x+k)$ for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. For **(9/10)** focus on linear, quadratic, and absolute value functions. **(F.BF.3)**

F.BF.4. Find inverse functions.

F.BF.4a. **(11)** Write an expression for the inverse of a function. **(F.BF.4a)**

F.BF.4b. **(11)** Read values of an inverse function from a graph or a table, given that the function has an inverse. **(F.BF.4c)**

F.BF.4c. **(+)** Verify by composition that one function is the inverse of another. **(F.BF.4b)**

F.BF.4d. **(+)** Produce an invertible function from a non-invertible function by restricting the domain. **(F.BF.4d)**

F.BF.5. **(11)** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. **(F.BF.5)**

Linear, Quadratic, and Exponential Models★ F.LQE

(High School Functions Progression Pg. 16)

Construct and compare linear, quadratic, and exponential models and solve problems.

F.LQE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.★

F.LQE.1a. **(11)** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.★ **(F.LQE.1a)**

F.LQE.1b. **(11)** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.★ **(F.LQE.1b)**

F.LQE.1c. **(11)** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.★ **(F.LQE.1c)**

F.LQE.2. **(11)** Construct exponential functions, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★ **(F.LQE.2)**

Trigonometric Functions F-TF

(High School Functions Progression Pg. 18)

Extend the domain of trigonometric functions using the unit circle.

F.TF.1. **(+)** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **(F.TF.1)**

F.TF.2. **(+)** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **(F.TF.2)**

F.TF.3. **(+)** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\frac{\pi}{3}, \frac{\pi}{4}, and\ \frac{\pi}{6}$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x, \pi + x, and\ 2\pi - x$ in terms of their values for *x*, where *x* is any real number. **(F.TF.3)**

F.TF.4. **(+)** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **(F.TF.4)**

Model periodic phenomena with trigonometric functions.

F.TF.5. **(+)** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★ **(F.TF.5)**

F.TF.6. **(+)** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **(F.TF.6)**

F.TF.7. **(+)** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★ **(F.TF.7)**

Prove and apply trigonometric identities.

F.TF.8. **(+)** Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $\sin(\theta), \cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta), \cos(\theta)$, or $\tan(\theta)$ and the quadrant. **(F.TF.8)**

F.TF.9. **(+)** Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. **(F.TF.9)**

High School – Geometry Content Standards Overview

Congruence (G.CO)

A. Experiment with transformations in the plane.
G.CO.1 G.CO.2

B. Understand congruence in terms of rigid motions.
G.CO.3 G.CO.4 G.CO.5 (+)
G.CO.6 (+)

C. Construct arguments about geometric theorems using rigid transformations and/or logic.
G.CO.7 G.CO.8 G.CO.9
G.CO.10

D. Make geometric constructions.
G.CO.11 G.CO.12 (+)

Similarity, Right Triangles, and Trigonometry (G.SRT)

A. Understand similarity in terms of similarity transformations.
G.SRT.1 G.SRT.2 G.SRT.3
G.SRT.4

B. Construct arguments about theorems involving similarity.
G.SRT.5 G.SRT.6

C. Define trigonometric **ratios** and solve problems involving right triangles.
G.SRT.7 G.SRT.8 G.SRT.9 (★)

D. Apply trigonometry to general triangles.
G.SRT.10 (+) G.SRT.11 (+) G.SRT.12 (+)

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Click on the box to open specific details related to High School - Geometry!

Circles (G.C)

A. Understand and apply theorems about circles.
G.C.1 G.C.2 G.C.3 G.C.4 (+) G.C.5 (+)

B. Find arc lengths and areas of sectors of circles.
G.C.6 (+)

Expressing Geometric Properties with Equations (G.GPE)

A. Translate between the geometric description and the equation for a conic section.
G.GPE.1 G.GPE.2 (+) G.GPE.3 (+) G.GPE.4 (+) G.GPE.5 (+)

B. Use coordinates to prove simple geometric theorems algebraically.
G.GPE.6 G.GPE.7 G.GPE.8 (★)

Geometric Measurement and Dimensions (G.GMD)

A. Explain volume formulas and use them to solve problems.
G.GMD.1 (+) G.GMD.2 (+)

Modeling with Geometry (G.MG) (★)

A. Apply geometric concepts in modeling situations.
G.MG.1 (★) G.MG.2 (★) G.MG.3 (★)

The grade level classifications for the high school standards are as follows:

(9/10)	These standards are required for all students by the end of their first two years of high school math courses.
(11)	These standards are required for all students by the end of their third year math course.
(9/10/11)	These standards are required for all students in their first three years of high school math courses. These standards are often further divided to (9/10) and (11) to identify specific concepts and their appropriate grade level. *(9/10) should primarily accomplish the standards described as linear, quadratic and absolute value while (11) should primarily accomplish the standards described as logarithmic, square root, cube root, and exponential .*
(all)	These standards should be taught throughout every high school math course, and often represent over-arching themes or key features of the mathematical concept. These standards should be taught in conjunction with the appropriate grade level standards.
(+)	These standards should be taught as **extensions** to grade level standards when possible, or in a 4th year math course. These standards prepare students to take advanced courses such as college algebra, calculus, advanced statistics, or discrete mathematics.

(★) **Modeling Standards:** Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol. The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Congruence G.CO

(Geometry High School Progression Pg. 13)

Experiment with transformations in the plane.

G.CO.1. **(9/10)** Verify experimentally (for example, using patty paper or geometry software) the properties of rotations, reflections, translations, and symmetry:
- G.CO.1a. **(9/10)** Lines are taken to lines, and line segments to line segments of the same length. **(8.G.1a)**
- G.CO.1b. **(9/10)** Angles are taken to angles of the same measure. **(8.G.1b)**
- G.CO.1c. **(9/10)** Parallel lines are taken to parallel lines. **(8.G.1c)**
- G.CO.1d. **(9/10)** Identify any line of reflection and/or rotational symmetry within a figure. **(G.CO.3)**

G.CO.2. **(9/10)** Recognize transformations as functions that take points in the plane as inputs and give other points as outputs and describe the effect of translations, rotations, and reflections on two-dimensional figures. *For example,* (x, y) *maps to* $(x + 3, y - 5)$*; reflecting triangle ABC(input) across the line of reflection maps the triangle to exactly one location, A'B'C'(output).* **(G.CO.2)**

Understand congruence in terms of rigid motions.

G.CO.3. **(9/10)** Given two congruent figures, describe a **sequence** of rigid motions that exhibits the congruence (**isometry**) between them using coordinates and the non-coordinate plane. **(8.G.2)**

G.CO.4. **(9/10)** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **(G.CO.7)**

G.CO.5. **(+)** Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **(G.CO.6)**

G.CO.6. **(+)** Demonstrate triangle congruence using **rigid motion** (ASA, SAS, and SSS). **(G.CO.8)**

Construct arguments about geometric theorems using rigid transformations and/or logic.

G.CO.7. **(9/10)** Construct arguments about lines and angles using theorems. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (Building upon standard in 8th grade Geometry.) **(G.CO.9)**

G.CO.8. **(9/10)** Construct arguments about the relationships within one triangle using theorems. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point; angle sum and exterior angle of triangles. **(G.CO.10)**

G.CO.9. **(9/10)** Construct arguments about the relationships between two triangles using theorems. Theorems include: SSS, SAS, ASA, AAS, and HL. **(2017)**

G.CO.10. **(9/10)** Construct arguments about parallelograms using theorems. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (Building upon prior knowledge in elementary and middle school.) **(G.CO.11)**

Make geometric constructions.

G.CO.11. **(9/10)** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. **(G.CO.12)**

G.CO.12. **(+)** Construct an equilateral triangle, a square, and a regular hexagon **inscribed** in a circle. **(G.CO.13)**

Similarity, Right Triangles, and Trigonometry G.SRT

(Geometry High School Progression Pg. 16)

Understand similarity in terms of similarity transformations.

G.SRT.1. **(9/10)** Use geometric constructions to verify the properties of dilations given by a center and a scale factor:

- G.SRT.1a. **(9/10)** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. **(G.SRT.1a)**
- G.SRT.1b. **(9/10)** The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **(G.SRT.1b)**

G.SRT.2. **(9/10)** Recognize transformations as functions that take points in the plane as inputs and give other points as outputs and describe the effect of dilations on two-dimensional figures. **(8.G.3)**

G.SRT.3. **(9/10)** Given two similar figures, describe a **sequence** of transformations that exhibits the similarity between them using coordinates and the non-coordinate plane. **(8.G.4)**

G.SRT.4. **(9/10)** Understand the meaning of similarity for two-dimensional figures as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **(G.SRT.2)**

Construct arguments about theorems involving similarity.

G.SRT.5. **(9/10)** Construct arguments about triangles using theorems. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity, and AA. **(G.SRT.3) (G.SRT.4)**

G.SRT.6. **(9/10)** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **(G.SRT.5)**

Define trigonometric ratios and solve problems involving right triangles.

G.SRT.7. **(9/10)** Show that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **(G.SRT.6)**

G.SRT.8. **(9/10)** Explain and use the relationship between the sine and cosine of complementary angles. **(G.SRT.7)**

G.SRT.9. **(9/10)** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ★**(G.SRT.8)**

Apply trigonometry to general triangles

G.SRT.10. **(+)** Derive the formula $A = \frac{1}{2}ab\sin C$ for the area of a triangle by drawing an **auxiliary line** from a vertex perpendicular to the opposite side. **(G.SRT.9)**

G.SRT.11. **(+)** Prove the Laws of Sines and Cosines and use them to solve problems. **(G.SRT.10)**

G.SRT.12. **(+)** Understand and apply the **Law of Sines** and the **Law of Cosines** to find unknown measurements in right and non-right triangles (*e.g. surveying problems, resultant forces*). **(G.SRT.11)**

Circles G.C

Understand and apply theorems about circles.

G.C.1. **(9/10)** Construct arguments that all circles are similar. **(G.C.1)**

G.C.2. **(9/10)** Identify and describe relationships among **inscribed** angles, radii, and chords. Include the relationship between central, inscribed, and **circumscribed** angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. **(G.C.2)**

G.C.3. **(9/10)** Construct arguments using properties of polygons inscribed and circumscribed about circles. **(G.C.3)**

G.C.4. **(+)** Construct inscribed and circumscribed circles for triangles. **(G.C.3)**

G.C.5. **(+)** Construct inscribed and circumscribed circles for polygons and tangent lines from a point outside a given circle to the circle. **(G.C.4)**

Find arc lengths and areas of sectors of circles.

G.C.6. **(+)** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the **constant of proportionality**; derive the formula for the area of a sector. **(G.C.5)**

Expressing Geometric Properties with Equations G.GPE

Translate between the geometric description and the equation for a conic section.

G.GPE.1. **(9/10)** Write the equation of a circle given the center and radius or a graph of the circle; use the center and radius to graph the circle in the coordinate plane. **(G.GPE.1)**

G.GPE.2. **(+)** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; graph the circle in the coordinate plane; **(G.GPE.1)**

G.GPE.3. **(+)** Complete the square to find the center and radius of a circle given by an equation. **(G.GPE.1)**

G.GPE.4. **(+)** Derive the equation of a parabola given a **focus** and **directrix;** graph the parabola in the coordinate plane. **(G.GPE.2)**

G.GPE.5. **(+)** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant; graph the ellipse or hyperbola in the coordinate plane. **(G.GPE.3)**

Use coordinates to prove simple geometric theorems algebraically.

G.GPE.6. **(9/10)** Use coordinates to prove simple geometric theorems algebraically, including the use of slope, distance, and midpoint formulas *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle.* **(G.GPE.4)**

G.GPE.7. **(9/10)** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (*e.g. find the equation of a line parallel or perpendicular to a given line that passes through a given point*). **(G.GPE.5)**

G.GPE.8. **(9/10)** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, including the use of the distance and midpoint formulas. ★ **(G.GPE.7)**

Geometric Measurement and Dimension G.GMD

(Geometry High School Progression Pg. 19)

Explain volume formulas and use them to solve problems.

G.GMD.1. **(+)** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments and informal limit arguments. **(G.GMD.1)**

G.GMD.2. **(+)** Give an informal argument using **Cavalieri's principle** for the formulas for the volume of a solid figure. **(G.GMD.2)**

Modeling with Geometry G-MG

(Geometry High School Progression Pg. 19)

Apply geometric concepts in modeling situations.

G.MG.1. **(9/10)** Use geometric shapes, their measures, and their properties to describe objects *(e.g. modeling a tree trunk or a human torso as a cylinder).*★ **(G.MG.1)**

G.MG.2. **(9/10)** Apply concepts of density and displacement based on area and volume in modeling situations *(e.g. persons per square mile, BTUs per cubic foot)*.★ **(G.MG.2)**

G.MG.3. **(9/10)** Apply geometric methods to solve design problems *(e.g. designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on* ***ratios****)*. ★**(G.MG.3)**

High School – Statistics & Probability Content Standards Overview

Interpreting Categorical and Quantitative Data (S.ID)

A. Summarize, represent, and interpret data on a single count or measurement variable.
S.ID.1 S.ID.2 S.ID.3 (+)

B. Summarize, represent, and interpret data on two categorical and quantitative variables.
S.ID.4 S.ID.5

C. Interpret linear models.
S.ID.6 S.ID.7 S.ID.8

Making Inferences and Justifying Conclusions (+) (S.IC)

A. Understand and evaluate random processes underlying statistical experiments.
S.IC.1 (+) S.IC.2 (+)

B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
S.IC.3 (+) S.IC.4 (+) S.IC.5 (+)
S.IC.6 (+)

Conditional Probability and the Rules of Probability (S.CP)

A. Understand independence and conditional probability and use them to interpret data.
S.CP.1 (+) S.CP.2 (+) S.CP.3 (+)
S.CP.4 (+) S.CP.5 (+)

B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.
S.CP.6 (+) S.CP.7 (+) S.CP.8 (+)
S.CP.9 (+)

Using Probability to Make Decisions (+) (★) (S.MD)

A. Calculate expected values and use them to solve problems.
S.MD.1 (+)(★) S.MD.2 (+)(★) S.MD.3 (+)(★)
S.MD.4 (+)(★)

B. Use probability to evaluate outcomes of decisions.
S.MD.5 (+)(★) S.MD.6 (+)(★) S.MD.7 (+)(★)

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Click on the box to open specific details related to High School – Statistics & Probability!

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(all)	These standards should be taught throughout every high school math course, and often represent over-arching themes or key features of the mathematical concept. These standards should be taught in conjunction with the appropriate grade level standards.
(+)	These standards should be taught as **extensions** to grade level standards when possible, or in a 4th year math course. These standards prepare students to take advanced courses such as college algebra, calculus, advanced statistics, or discrete mathematics.

(★) **Modeling Standards:** Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol. The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Interpreting Categorical and Quantitative Data S.ID

(High School Statistics and Probability Progression Pg. 3)

Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.1. **(9/10)** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (**interquartile range**, standard deviation) of two or more different data sets. **(S.ID.2)**

S.ID.2. **(9/10)** Interpret differences in shape, center, and spread in the context of the data sets using dot plots, histograms, and box plots, accounting for possible effects of extreme data points (**outliers**). **(S.ID.3)**

S.ID.3. **(+)** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **(S.ID.4)**

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.4. **(9/10)** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. **(S.ID.5)**

S.ID.5. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

S.ID.5a. **(9/10)** Use a given linear function to solve problems in the context of data. **(S.ID.6a)**

S.ID.5b. **(9/10)** Fit a linear function to data and use it to solve problems in the context of the data. **(S.ID.6a)**

S.ID.5c. **(+)** Assess the fit of a function by plotting and analyzing **residuals**. **(S.ID.6b)**

S.ID.5d. **(+)** Fit quadratic and exponential functions to the data. Use functions fitted to data to solve problems in the context of the data. **(S.ID.6a)**

Interpret linear models.

S.ID.6. **(9/10)** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **(S.ID.7)**
S.ID.7. **(11)** Compute (using technology) and interpret the correlation coefficient of a linear fit. **(S.ID.8)**
S.ID.8. **(11)** Distinguish between correlation and causation. **(S.ID.9)**

Making Inferences and Justifying Conclusions S.IC

(High School Statistics and Probability Progression Pg. 8)

Understand and evaluate random processes underlying statistical experiments.

S.IC.1. **(+)** Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population. **(S.IC.1)**
S.IC.2. **(+)** Decide if a specified model is consistent with results from a given data-generating process, *e.g. using simulation*. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* **(S.IC.2)**

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S.IC.3. **(+)** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **(S.IC.3)**
S.IC.4. **(+)** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error, *(e.g. through the use of simulation models for random sampling.)* **(S.IC.4)**
S.IC.5. **(+)** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. **(S.IC.5)**
S.IC.6. **(+)** Evaluate reports based on data. **(S.IC.6)**

Conditional Probability and the Rules of Probability S.CP

(High School Statistics and Probability Progression Pg. 13)

Understand independent and conditional probability and use them to interpret data.

S.CP.1. **(+)** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). **(S.CP.1)**
S.CP.2. **(+)** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. **(S.CP.2)**
S.CP.3. **(+)** Understand the conditional probability of *A* given *B* as $\frac{P(A \text{ and } B)}{P(B)}$, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. **(S.CP.3)**
S.CP.4. **(+)** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* **(S.CP.4)**

S.CP.5. **(+)** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* **(S.CP.5)**

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S.CP.6. **(+)** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. **(S.CP.6)**

S.CP.7. **(+)** Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. **(S.CP.7)**

S.CP.8. **(+)** Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. **(S.CP.8)**

S.CP.9. **(+)** Use permutations and combinations to compute probabilities of compound events and solve problems. **(S.CP.9)**

Using Probability to Make Decisions S.MD

(High School Statistics and Probability Progression Pg. 18)

Calculate expected values and use them to solve problems.

S.MD.1. **(+)** Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.★ **(S.MD.1)**

S.MD.2. **(+)** Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.★ **(S.MD.2)**

S.MD.3. **(+)** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*★ **(S.MD.3)**

S.MD.4. **(+)** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*★ **(S.MD.4)**

Use probability to evaluate outcomes of decisions.

S.MD.5. **(+)** Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.★

S.MD.5a. **(+)** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*★ **(S.MD.5a)**

S.MD.5b. **(+)** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*★ **(S.MD.5b)**

S.MD.6. **(+)** Use probabilities to make fair decisions *(e.g. drawing by lots, using a random number generator)*.★ **(S.MD.6)**

S.MD.7. **(+)** Analyze decisions and strategies using probability concepts *(e.g. product testing, medical testing, pulling a hockey goalie at the end of a game)*.★ **(S.MD.7)**

Click to open the Student Glossary

Teacher Glossary

Acute angle. An angle with a measure less than 90°

Acute triangle. A triangle with each of the interior angles measuring less than 90°

Addition. An operation that combines two or more numbers or groups of objects (component parts: addend + addend = sum)

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, 0-100, or 0-1,000 respectively. *For example,* $8 + 2 = 10$ *is an addition within 10,* $14 - 5 = 9$ *is a subtraction within 20, and* $55 - 18 = 37$ *is a subtraction within 100*

Additive identity (property of zero). Adding 0 to any number with the result of that number

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. *Example:* $\frac{3}{4}$ *and* $\left(-\frac{3}{4}\right)$ *are additive inverses of one another because* $\frac{3}{4} + \left(-\frac{3}{4}\right) = \left(-\frac{3}{4}\right) + \frac{3}{4} = 0$

Algorithm. A step-by-step method for computing or solving a problem

Angle. Two rays or line segments that share an endpoint

Area. The number of square units needed to cover a given surface

Array. A rectangular arrangement of objects with equal amounts in each row

Associative property. The associative property states that numbers in an addition expression can be grouped in different ways without changing the sum OR the numbers in a multiplication expression can be grouped in different ways without changing the product. By "grouped" we mean where the parenthesis are placed in the expression

Associative property of multiplication. See Table 3

Attribute. A defining characteristic of a number, geometric figure, mathematical operation, equation, or inequality

Auxiliary line. An auxiliary line (or helping line) is an extra line needed to complete a proof in plane geometry

Bar graph. A display that uses horizontal or vertical bars to represent data (categorical data)

Binomial theorem: Formula for finding any power of a binomial without multiplying at length. $(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set

Cardinality. Understands last number word said when counting, tells how many

Cavalieri's principle. If two solids have the same height and the same cross-sectional area at every level, then they have the same volume

Chance processes. A probability experiment. *For example, flipping a coin, drawing a card, tossing a number cube*

Chart (table). Information organized in columns and rows

Circle. A closed curve with all its points the same distance from the center

Circular arc. The arc of a circle is a portion of the circumference. It can be measured by its central angle or the length of the arc

Circumference. The distance around the outside (perimeter) of a circle

Cluster. Numbers which tend to crowd around a particular point in a set of values

Combinatorial argument. In mathematics, the term combinatorial proof is often used to mean either of two types of mathematical proof

Commutative property. Numbers may be added or multiplied together in any order without changing the answer. See Table 3

Compensation. Understanding that decreasing from one part and increasing it to another leaves the quantity unchanged

Complex fraction. A fraction $\frac{A}{B}$ where *A* and/or *B* are fractions (*B cannot equal* zero)

Component-wise vector addition. The component method of addition can be summarized this way:

C. Using trigonometry, find the x-component and the y-component for each vector. Refer to a diagram of each vector to correctly reason the sign, (+ or -), for each component
D. Add up both x-components, (one from each vector), to get the x-component of the total
E. Add up both y-components, (one from each vector), to get the y-component of the total
F. Add the x-component of the total to the y-component of the total, and then use the Pythagorean theorem and trigonometry to get the size and direction of the total

Composite number. A number that has more than two factors

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also: computation strategy*

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also: computation algorithm*

Cone. A 3-dimensional figure with a curved surface, a flat circular base, and a vertex

Congruent. Having exactly the same size and shape

Congruent figures. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations)

Conservation. Understands quantity stays the same when physical space is changed

Constant of proportionality: A fixed value of the ratio of two proportional quantities

Coordinate grid/plane. The plane formed by two perpendicular number lines intersecting at their zero points used for displaying the location of coordinates

Coordinates. An ordered pair of numbers that gives the location of a point on a coordinate grid

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. *For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again; one can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."*

Cube. A 3-dimensional figure with six congruent square faces

Cylinder. A 3-dimensional figure with one curved surface and two parallel, congruent circular bases

Data. Information that is collected by counting, measuring, asking questions, or observing that is usually organized for analysis

Data display. A way to visually organize data

Decagon. A polygon with 10 sides

Decimal. A number in a number system based on 10 (also known as base-ten system or Hindu-Arabic system)

Decimal fraction. A number written in standard base-10 notation

Decimal notation. Representation of a fraction or other real number using the base ten and consisting of any of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and a decimal point

Decimal point. A demarcation between whole numbers and numbers less than one

Decompose. The process of separating into smaller parts

Denominator. The number of equal parts making up a whole (the bottom number in a fraction)

Diameter. A line segment that passes through the center of a circle and has endpoints on the circle

Difference. The space between the value of two numbers on a number line (the result of subtracting one number from another)

Digit. Any one of the ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Digital root. The result of adding digits in a number until only one digit remains

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor

Directrix. A fixed line used in the description of a curve or surface

Distributive property of multiplication. A property indicating a special way in which multiplication is applied to addition or subtraction of two or more numbers in which each term inside a set of parentheses can be multiplied by a factor outside the parentheses. *For example,*
$4(2+3) = 4 \cdot 2 + 4 \cdot 3$ $\qquad$ $5(9-3) = 5 \cdot 9 - 5 \cdot 3$

Division. The operation of making equal groups to find out how many in each group or how many groups (component parts: dividend ÷ divisor = quotient)

Double number line diagram. Two number lines with different scales intended to organize and compare values

Edge. It is the line segment that joins two vertices where two faces of a solid shape intersect.

Ellipse. A curved line forming a closed loop, where the sum of the distances from two points (foci) to every point on the line is constant

End-to-end vector addition. Added or subtracted graphically by laying them end to end on a set of axes

Equal. Having the same value

Equation. A mathematical sentence where the left side of the equal sign has the same value as the right side of the equal sign

Equilateral. A polygon with all sides congruent

Equilateral triangle. A triangle whose sides are all the same length

Equivalence. The condition of being equal or equivalent in value, worth, function, etc.

Equivalent. Having the same value

Estimate. To find a number close to an exact amount

Even number. Whole numbers that are divisible by 2; even numbers have 0, 2, 4, 6, or 8 in the ones place

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. *For example,* $643 = 600 + 40 + 3$.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities

Experimental probability. The ratio of the number of times an event occurs to the total number of trials in a chance process

Explicit function. A function in which the dependent variable can be written explicitly in terms of the independent variable

Exponent. A numeral telling how many times a factor is to be multiplied

Expression. A mathematical phrase made up of numbers, variables, operational symbols, and/or parentheses

Face. The flat surface of a solid figure

First quartile. For a data set with median *M*, the first (lower) quartile is the median of the data values less than *M*. *See also: median, third quartile, interquartile range*

Fluency. Performing a skill flexibly, accurately, and efficiently

Focus. The locus of all points that are equidistant from a given point

Fraction. A number expressible in the form $\frac{a}{b}$ where *a* is the number of equal parts being referenced and *b* is the number of equal parts in the whole. *Note: There is no need to introduce "proper fractions" and "improper fractions" (i.e.* $\frac{5}{3}$ *is the quantity you get by combining 5 parts together when the whole is divided into 3 equal parts) rather student* recognize that fractions can be between 0 and 1 or more than 1

Frequency table. A table that shows how often that data point occurred (tally marks are commonly used)

Fundamental Theorem of Algebra. The theorem that establishes that, using complex numbers, all polynomials can be factored. A generalization of the theorem asserts that any polynomial of degree n has exactly n zeroes, counting multiplicity

Growth pattern. A type of pattern made by following a rule using operations

Height. The distance from the base to the top of an object or shape

Heptagon (septagon). A polygon with 7 sides

Hexagon. A polygon with 6 sides

Hierarchical inclusion. Numbers build by exactly one each time—smaller numbers are part of bigger numbers *(For example, 3 is "nested" in 4)*

Identity property of 0. See Table 3

Improper fraction. A fraction with a numerator that is greater than or equal to its denominator

Incidence relationships in a network. Shows the relationship between two classes of objects

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair

Induction. A means of proving a theorem by showing that if it is true of any particular case, it is true of the next case in a series, and then showing that it is indeed true in one particular case

Inequality. A number sentence comparing the size, amount, or value using one of the following symbols: $<$, $>$, $\leq$, $\geq$, $\neq$. Also used to define sets of numbers

Informal derivation: An informal development of a theorem

Inscribe. Draw (a figure) within another so that their boundaries touch but do not intersect

Integers. The set of whole numbers and their opposites: . . . ,-2, -1, 0, 1, 2, . . .

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the upper (third) quartile and the lower (first) quartile of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also: first quartile, third quartile*

Intersecting lines. Lines that meet or cross

Interval (linear). Space between numbers on a number line or the grid lines of a graph

Interval (time). A space of time between events

Inscribed. Draw (a figure) within another so that their boundaries do not intersect

Irrational number. A number that cannot be expressed as a ratio between two integers and is not an imaginary number. If written in decimal notation, an irrational number would have an infinite number of digits to the right of the decimal point, without repetition

Irregular polygon. A polygon whose sides are not all the same length

Isometry. A distance-preserving transformation

Isosceles triangle. A triangle with exactly two sides of equal length (exclusive); a triangle with at least two sides of equal length (inclusive)

Iteration. Repeating the same unit

Kite. A quadrilateral with two distinct pairs of equal adjacent sides (exclusive); a quadrilateral with two pairs of equal adjacent sides (inclusive)

Law of cosines. The law of cosines (also referred to as cosine law, cosine formula, cosine rule) is used to calculate one side of a triangle when the angle opposite and the other two sides are known $c^2 = a^2 + b^2 - 2ab\cos C$

Law of sines. The law of sines (also referred to as sine law, sine formula, sine rule) states that the ratio of the length of a side of a triangle to the sine of the angle opposite that side is the same for all sides and angles in a given triangle. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Length. the distance from one end of something to the other end

Line. An infinite set of points forming a straight path extending in opposite directions (Although mathematically undefined, we will use this description for line)

Line plot. A method of visually displaying a frequency of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot

Line segment. A part of a line defined by two endpoints

Line symmetry. *See symmetry*

Linear measure. A measure of a distance or length that is one- dimensional

Liquid volume (capacity). The amount that a container can hold (common units of measure: cup, pint, gallon, liter, etc.)

Literal equations. Equations with several variables often solved in terms of a single variable

Magnitude. Distance of a number from zero

Magnitude of a vector. Size of a mathematical object, a property by which the object can be compared as larger or smaller than other objects of the same kind

Mass. The amount of matter

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Also known as MAD. *For example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolution deviation is 19.96*

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values

Midline. In the graph of a trigonometric function, the horizontal line half-way between its maximum and minimum values

Mixed number. A quantity written with an integer and a fraction ($3\frac{1}{2}$)

Moduli/modulus. The modulus of a complex number is the square root of the product of a complex number and its conjugate

Multiple. The product of any number and a counting number

Multiplication. The operation of repeated addition (component parts: factor x factor = product)

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. *For example,* $(72|8) = 9$

Multiplicative comparison. Comparing the difference between values using multiplication

Multiplicative identity property of one. When a number is multiplied by 1, the product is that number

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. *For example,* $\frac{3}{4}$ *and* $\frac{4}{3}$ *are multiplicative inverses of one another because* $\frac{3}{4} \cdot \frac{4}{3} = \frac{4}{3} \cdot \frac{3}{4} = 1$

Negative numbers. Numbers less than zero

Nonagon. A polygon with 9 sides

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Number model. A mathematical representation of a situation

Number sentence. An equation and/or algebraic expressions (=) or inequality (<, >, …) with numbers

Number systems. The different subgroups of numbers *For example, natural numbers, whole numbers, integers, rational numbers, etc.*

Numeral. A symbol or group of symbols that stand for a number. *For example, the numeral symbol for twenty- four is 24*

Numerator. Tells how many equal parts of a whole are being described (the top number of a fraction)

Obtuse angle. An angle with a measure greater than 90° and less than 180°

Obtuse triangle. A triangle with a single angle measuring more than 90°

Octagon. A polygon with 8 sides

Odd number. Whole numbers that cannot be divided into 2 equal groups of whole numbers; odd numbers have 1, 3, 5, 7, or 9 in the ones place

One-to-one correspondence. Counting objects by saying one number for each object, when counting in sequential order

Operational symbols. Symbols used to indicate computation (+, -, x, ●, ÷, etc.)

Ordered pair. A pair of numbers that gives the coordinates of a point on a grid in this order: (horizontal coordinate, vertical coordinate)

Ordinality. A number indicating a series or specific order (1^{st}, 2^{nd}, etc.)

Orientation. Position and direction in space (usually around a fixed point)

Outcome. A possible result of a chance process

Outlier. A value in a data set that lies outside the overall pattern of a distribution or relationship

Parallel lines. Lines that are always the same distance apart

Parallelogram. A quadrilateral with two pairs of parallel sides

Parallelogram rule vector addition. When two vectors are represented by two adjacent sides of a parallelogram by direction and magnitude then the resultant of these vectors is represented in magnitude and direction by the diagonal of the parallelogram starting from the same point

Pattern. A logical sequence of numbers, pictures, shapes, or symbols

Peak. A data value that is greater than its neighboring values

Pentagon. A polygon with 5 sides

Percent rate of change. A rate of change expressed as a percent. *For example: if a population grows from 50 to 55 in a year, it grows by* $\frac{5}{50} = 10\%$ *per year*

Perimeter. The distance around a figure

Perpendicular lines. Two lines that form a right angle where they intersect

Pictograph. A display that uses pictures or symbols to represent data

Place value. The value of a digit depending on its place in a number

Plane figures (2-D). Any 2-dimensional shape that lays in a single plane

Plot. To place (points or other figures) on a graph by means of coordinates

Point. An exact location in space

Polar form. $z = r(\cos\theta + i\sin\theta)$

Polygon. A closed plane figure made from line segments that meet at endpoints and do not cross

Positive numbers. Numbers that are greater than zero

Prime number. A counting number greater than 1 that has exactly two factors, itself and 1

Prism. A 3-dimensional figure with two identical, parallel faces (bases) that are polygons; the remaining faces are parallelograms ***A prism is named by its base*

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, testing for a medical condition)

Probability distribution. The set of possible values of a random variable with a probability assigned to each

Probability model. A mathematical representation (such as tree diagram or table) used to assign probabilities to all outcomes in the sample space in which the probabilities sum to 1. *See also: uniform probability model*

Properties of equality. See Table 4

Properties of inequality. See Table 5

Properties of operations. See Table 3

Pyramid. A 3-dimensional figure whose base is a polygon and whose other faces are triangles that share a common vertex *(A pyramid is named by its base)*

Quadrilateral. A polygon with four sides

Radius. The distance from the center of a circle to any point on a circle

Random variable. An assignment of a numerical value to each outcome in a sample space

Ratio. The quantitative relation between two amounts

Rational expression. A quotient of two polynomials with a non-zero denominator

Rational number. A number expressible in the form $\frac{a}{b}$, where *a* and *b* are both integers and *b* cannot equal zero

Ray. A part of a line that has one endpoint and extends forever in one direction

Rectangle. A parallelogram with four right angles

Rectilinear figure. A polygon where all angles are right angles

Recursive function. Relating to or involving the repeated application of a rule, definition, or procedure to create successive results

Reflex angle. An angle that measures greater than 180°

Regular polygon. A polygon with all sides the same length and all angles the same measure

Related equations (used to be known as fact families). A set of equations that all communicate the same relationship between three values, but in different ways (there are eight ways to show a relationship between addition/subtraction and multiplication/division)

Relational symbols. Symbols used to show relationships between quantities, values, and figures ($=, \neq, <, >, \leq, \geq \sim, \approx, \cong$)

Remainder. When dividing, the part of a number or quantity that is left over

Remainder Theorem. The assertion that $P(c)$ is the remainder when polynomial $P(x)$ is divided by $(x - c)$

Repeating decimal. The decimal form of a rational number. *See also: terminating decimal*

Residuals. Difference between the observed y-value (from scatter plot) and the predicted y-value (from regression equation line). It is the vertical distance from the actual plotted point to the point on the regression line

Rhombus. A parallelogram with equal sides and opposite angles equal

Right angle. An angle that measures 90°

Right triangle. A triangle that has one 90° angle

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures

Rounding. Replacing a numerical value by another value that is approximately equal but has a shorter, simpler, or more explicit representation

Sample space. The set of all possible outcomes in the context of probability

Scalar. (of a quantity) having only magnitude, not direction

Scale. Ordered marks at fixed intervals (graphing or measurement)

Scalene triangle. A triangle having no equal sides

Scaling (resizing). Expressing the amount of the enlargement or reduction to the original

Scatter plot. A graph in the coordinate plane representing a set of bivariate data.

Septagon. *See heptagon*

Sequence. A particular order in which relative events, movements, or things follow each other

Similarity transformation. A rigid transformation (reflection, rotation, translation) followed by a dilation

Situation equation. An equation that **models** the situation in a real-life and/or word problem (*For example: A boy had some balloons and his dad gave him 2 more so he has 8. How many balloons did he start with? Situation equation* $-?+2=8$. *Solution equation* $-8-2=?$*)*

Skew (data). The asymmetry from the mean of a data distribution. A distribution is skewed if one tail is longer than another. When data has a long tail on the left side of the peak (in the negative direction on the number line), it is left-skewed. If it has a long tail on the right side of the peak (in the positive direction on the number line), it is right-skewed

Solid figures (3-D). A geometric figure with three dimensions (length, width, and height)

Solution equation. An equation that models how the situation in a real-life and/or word problem can be **solved** (the situation equation does not always allow for an easy solution path) [*For example: A boy had some balloons and his dad gave him 2 more so he has 8. How many balloons did he start with? Situation equation* $-?+2=8$. *Solution equation* $-8-2=?$]

Sphere. A closed three-dimensional figure with every point of its surface the same distance from the center

Square. A parallelogram with equal sides and four right angles

Standard form. A number written with one digit for each place value in a base ten numeric system

Statistical question. A question that can be answered by collecting data where there will be variability in the data

Straight angle. An angle with a measure of 180°

Strategies. System of finding and developing solutions when followed consistently

Subitizing. Instantly seeing how many

Subset. A set within a larger set. One unique subset of the whole set is the whole set itself

Subtraction. An operation that gives the difference or comparison between two numbers (component parts: minuend - subtrahend = difference)

Symmetric property of equality. The answer to an equation can be on either side of the equal sign

Symmetry (line symmetry). A line that divides a figure into two congruent halves that are mirror images of each other

Table. *See chart*

Tape diagram. A visual model using rectangles that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model

Terminating decimal. A decimal is called terminating if its repeating digit is 0

Theoretical probability. The ratio of the number of ways the event can occur to the total number of possible outcomes based on a probability model

Third quartile. For a data set with median *M*, the third (upper) quartile is the median of the data values greater than *M*. *See also: median, first quartile, interquartile range*

Time. The way we measure years, days, minutes, etc.

Transitivity principle. Indirect comparison of two objects by the use of a third object

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well

Trapezoid. A quadrilateral with exactly one pair of parallel sides (exclusive); a quadrilateral with at least one pair of parallel sides (inclusive)

Triangle. A polygon with three sides

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model

Unit (unit size). A single object or any group of things or persons regarded as an entity that can be iterated

Unit form. A way to write numbers showing the place value of each digit by using the name of the place (Ex: 3045 = 3 thousands + 4 tens + 5 ones)

Unit fraction. When a whole is divided into equal parts, a unit fraction is one of those parts (a unit fraction has a numerator of one)

Variable. A letter or symbol that represents a number

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Vertex. The point at which two line segments, lines, or rays meet to form an angle

Visual fraction model. A tape diagram, number line diagram, or area model.

Volume. The number of cubic units it takes to fill a three-dimensional figure

Weight. The measure of how heavy something is; the force of gravity on an object

Whole numbers. The numbers 0, 1, 2, 3, ….

Width. The measure of one side of an object.

Word form. Numbers written with only words (Ex: 3045 = three thousand forty-five)

Zero property (multiplication property of zero). When a number is multiplied by 0, the product is always 0

TABLE 1: Common Addition and Subtraction Situations

Shading taken from OA progression

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Taken from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and variants (blue and green). Yellow indicates problems that are the difficult four problem subtypes or variants that students in Grade 1 work with but do not need to master until Grade 2. [1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

TABLE 2: Common Multiplication and Division Situations

Grade level identification of introduction of problems taken from OA progression

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18, \quad 18 \div 3 = ?$	$? \times 6 = 18, \quad 18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[4], Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p \;\; and \;\; p \div a = ?$	$? \times b = p \;\; and \;\; p \div b = ?$

Multiplicative compare problems appear first in Grade 4 (green), with whole number values and with the "times as much" language from the table. In Grade 5, unit fractions language such as "one third as much" may be used. Multiplying and unit language change the subject of the comparing sentence ("A red hat costs n times as much as the blue hat" results in the same comparison as "A blue hat is $\frac{1}{n}$ times as much as the red hat" but has a different subject.)

TABLE 3: Fundamental Properties of Number and Operations

Name of Property	Representation of Property	Example of Property, Using Real Numbers
Properties of Addition		
Associative	$(a+b)+c=a+(b+c)$	$(78+25)+75=78+(25+75)$
Commutative	$a+b=b+a$	$2+98=98+2$
Additive Identity	$a+0=a$ and $0+a=a$	$9875+0=9875$
Additive Inverse	For every real number *a*, there is a real number $-a$ such that $a+-a=-a+a=0$	$-47+47=0$
Properties of Multiplication		
Associative	$(a\times b)\times c=a\times(b\times c)$	$(32\times 5)\times 2=32\times(5\times 2)$
Commutative	$a\times b=b\times a$	$10\times 38=38\times 10$
Multiplicative Identity	$a\times 1=a$ and $1\times a=a$	$387\times 1=387$
Multiplicative Inverse	*For every real number a,* $a\neq 0$, *there is a real number* $\frac{1}{a}$ *such that* $a\times\frac{1}{a}=\frac{1}{a}\times a=1$	$\frac{8}{3}\times\frac{3}{8}=1$
Distributive Property of Multiplication over Addition		
Distributive	$a\times(b+c)=a\times b+a\times c$	$7\times(50+2)=7\times 50+7\times 2$

(Variables a, b, and c represent real numbers.)
Excerpt from Developing Essential Understanding of Algebraic Thinking, grades 3-5 p. 16-17

TABLE 4: Properties of Equality

Name of Property	Representation of Property	Example of property
Reflexive Property of Equality	$a = a$	$3{,}245 = 3{,}245$
Symmetric Property of Equality	$If\ a = b, then\ b = a$	$2 + 98 = 90 + 10, then\ 90 + 10 = 2 + 98$
Transitive Property of Equality	$If\ a = b\ and\ b = c, then\ a = c$	$If\ 2 + 98 = 90 + 10\ and\ 90 + 10 = 52 + 48$ *then* $2 + 98 = 52 + 48$
Addition Property of Equality	$If\ a = b, then\ a + c = b + c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} + \frac{3}{5} = \frac{2}{4} + \frac{3}{5}$
Subtraction Property of Equality	$If\ a = b, then\ a - c = b - c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} - \frac{1}{5} = \frac{2}{4} - \frac{1}{5}$
Multiplication Property of Equality	$If\ a = b, then\ a \times c = b \times c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} \times \frac{1}{5} = \frac{2}{4} \times \frac{1}{5}$
Division Property of Equality	$If\ a = b\ and\ c \neq 0, then\ a \div c = b \div c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} \div \frac{1}{5} = \frac{2}{4} \div \frac{1}{5}$
Substitution Property of Equality	If $a = b$, then *b* may be substituted for *a* in any expression containing *a*.	$If\ 20 = 10 + 10$ *then* $90 + 20 = 90 + (10 + 10)$

(Variables a, b, and c can represent any number in the rational, real, or complex number systems.)

TABLE 5: Properties of Inequality

Exactly one of the following is true: $a < b, a = b, a > b$.
$If\ a > b\ and\ b > c\ then\ a > c.$
$If\ a > b, then\ b < a.$
$If\ a > b, then - a < -b.$
$If\ a > b, then\ a \pm c > b \pm c.$
$If\ a > b\ and\ c > 0, then\ a \times c > b \times c.$
$If\ a > b\ and\ c < 0, then\ a \times c < b \times c.$
$If\ a > b\ and\ c > 0, then\ a \div c > b \div c.$
$If\ a > b\ and\ c < 0, then\ a \div c < b \div c.$

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

Sample of Works Consulted

Existing Kansas College and Career Ready Standards for Mathematics.

Kansas Social, Emotional, and Character Development Model Standards, 2012.

Kansas Standards Science Core Idea Arrangement, 2013.

Kansas Standards Science Topic Arrangement, 2013.

Rose Standard Capacities, 2014.

ACT College Readiness Standards™

ACT National Curriculum Survey™

ACT Job Skill Comparison Charts

ACT WorkKeys Occupational Profiles™

ACT College Readiness Benchmarks™

Principles to Actions: Ensuring mathematical success for all. Reston, VA: National Council of Teachers of Mathematics, 2014.

National Assessment Governing Board, *Mathematics Framework for the 2013 National Assessment of Educational Progress*. U.S. Department of Education, 2013.

Clements, D. H., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach*. New York: Routledge.

Clements, D. H., Sarama, J., & DiBiase, A.-M. (2004). Mahwah, NJ: Lawrence Erlbaum Associates.

Real Math Standards April 2016 & May 2016.

Zimba, J., *Observations on CCSSM Standards for Mathematical Content What Content Is Visibly Emphasized*

McCallum, W., Daro, P., Zimba, J., Askey, R., Beckmann, S., Clements, D., . . . Wu, H. (2011). Progressions documents. Retrieved from http://math.arizona.edu/~ime/progressions/

Mathematics Standards of Learning for Virginia Public Schools, 2009.

Arizona's College & Career Ready Standards Mathematics – High School Algebra I, 2014.

Arizona's College & Career Ready Standards Mathematics – High School Algebra II, 2014.

Arizona's College & Career Ready Standards Mathematics – High School Geometry, 2014.

Nebraska Mathematics Standards, 2015.

Oklahoma C3 PASS Kindergarten

Oklahoma C3 Pass Pre K

Oklahoma C3mathPASS

English Language Proficiency (ELP) Standards. Council of Chief State School Officers, 2014.

SAT Suite of Assessments: Alignment to Common Core Standards, 2015.

Van de Walle, J. A., Karp, K., & Bay-Williams, J. M. (2010). *Elementary and Middle School Mathematics: Teaching Developmentally* (Seventh ed.). Boston: Allyn and Bacon.

Van de Walle, J.A., Karp, K.S., Lovin, L.H., & Bay-Williams, J.M. (2014). Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades PreK-2 (2nd ed.). Upper Saddle River, NJ; Pearson Education, Inc.

Van de Walle, J.A., Karp, K.S., Lovin, L.H., & Bay-Williams, J.M. (2014). Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 3 - 5 (2nd ed.). Upper Saddle River, NJ; Pearson Education, Inc.

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