2017 Kansas Mathematics Standards

Flip Book

5th Grade

This project used work created by the Departments of Education in Ohio, North Carolina, Georgia and resources created by Achieve the Core, EngageNY, Illustrative Mathematics, and NCTM.

About the Flip Books

This project attempts to organize some of the most valuable resources that help develop the intent, understanding and implementation of the 2017 Kansas Mathematics Standards. These documents provide a starting point for teachers and administrators to begin discussion and exploration into the standards. It is not the only resource to support implementation of the 2017 Kansas Mathematics Standards.

This project is built on the previous work started in the summer of 2012 from Melisa Hancock (Manhattan, KS), Debbie Thompson (Wichita, KS) and Patricia Hart (Wichita, KS) who provided the initial development of the "flip books." The "flip books" are based on a model that Kansas had for earlier standards; however, this edition specifically targets the Kansas Mathematics Standards that were adopted in the summer of 2017. These flip books incorporate the resources from other state departments of education, the mathematics learning progressions, and other reliable sources including The National Council of Teachers of Mathematics and the National Supervisors of Mathematics. In addition, mathematics educators across the country have suggested changes/additions that could or should be made to further enhance its effectiveness. The document is posted on the KSDE Mathematics website at http://community.ksde.org/Default.aspx?tabid=5646 and will continue to undergo changes periodically. When significant changes/additions are implemented, the modifications will be posted and dated.

Planning Advice - Focus on the Clusters

The (mathematics standards) call for a greater focus. Rather than racing to cover topics in today's mile-wide, inch-deep curriculum, we need to use the power of the eraser and significantly narrow and deepen how time and energy is spent in the mathematics classroom. There is a necessity to focus deeply on the major work of each grade to enable students to gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems both in and out of the mathematics classroom. (www.achievethecore.org)

Not all standards should have the same instructional emphasis. Some groups of standards require a greater emphasis than others. In order to be intentional and systematic, priorities need to be set for planning, instruction, and assessment. "Not everything in the Standards should have equal priority" (Zimba, 2011). Therefore, there is a need to elevate the content of some standards over that of others throughout the K-12 curriculum.

When the Standards were developed the following were considerations in the identification of priorities: 1) the need to be qualitative and well-articulated; 2) the understanding that some content will become more important than other; 3) the creation of a focus means that some essential content will get a greater share of the time and resources "while the remaining content is limited in scope." 4) a "lower" priority does not imply exclusion of content, but is usually intended to be taught in conjunction with or in support of one of the major clusters.

"The Standards are built on the progressions, so priorities have to be chosen with an eye to the arc of big ideas in the Standards. A prioritization scheme that respects progressions in the Standards will strike a balance between the journey and the endpoint. If the endpoint is everything, few will have enough wisdom to walk the path, if the endpoint is nothing, few will understand where the journey is headed. Beginnings and the endings both need particular care. … It would also be a mistake to identify such standard as a locus of emphasis. (Zimba, 2011)

The important question in planning instruction is: "What is the mathematics you want the student to walk away with?" In order to accomplish this, educators need to think about "grain size" when planning instruction. Grain size corresponds to the knowledge you want the student to know. Mathematics is simplest at the right grain size. According to Phil Daro (*Teaching Chapters, Not Lessons—Grain Size of Mathematics*), strands are too vague and too large a grain size, while lessons are too small a grain size. Units or chapters produce about the right "grain size". In the planning process educators should attend to the clusters, and think of the standards as the ingredients of a cluster. Coherence of mathematical ideas and concepts exists at the cluster level across grades.

A caution--Grain size is important but can result in conversations that do not advance the intent of this structure. Extended discussions among teachers where it is argued for "2 days" instead of "3 days" on a topic because it is a lower priority can detract from the overall intent of suggested priorities. The reverse is also true. As Daro indicates, focusing on lessons can provide too narrow a view which compromises the coherence value of closely related standards.

The video clip Teaching Chapters, Not Lessons—Grain Size of Mathematics presents Phil Daro further explaining grain size and the importance of it in the planning process. (Click on photo to view video.)

Along with "grain size", clusters have been given **priorities** which have important implications for instruction. These priorities should help guide the focus for teachers as they determine allocation of time for both planning and instruction. The priorities provided help guide the focus for teachers as they determine distribution of time for both planning and instruction, helping to assure that students really understand mathematics before moving on. Each cluster has been given a priority level. As professional educators begin planning, developing and writing units, these priorities provide guidance in assigning time for instruction and formative assessment within the classroom.

Each cluster within the standards has been given a priority level influenced by the work of Jason Zimba. The three levels are referred to as — **Major, Supporting** and **Additional**. Zimba suggests that about 70% of instruction should relate to the **Major** clusters. The lower two priorities (**Supporting** and **Additional**) can work together by supporting the **Major** priorities. You can find the grade Level Focus Documents for the 2017 Kansas Math Standards at: http://community.ksde.org/Default.aspx?tabid=6340.

Recommendations for Cluster Level Priorities

Appropriate Use:

- Use the priorities as guidance to inform instructional decisions regarding time and resources spent on clusters by varying the degrees of emphasis.
- Focus should be on the major work of the grade in order to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through sense-making, reasoning, arguing and critiquing, modeling, etc.
- Evaluate instructional materials by taking the cluster level priorities into account. The major work of the grade must be presented with the highest possible quality; the additional work of the grade should support the major priorities and not detract from them.
- Set priorities for other implementation efforts such as staff development, new curriculum development, and revision of existing formative or summative testing at the state, district or school level.

Things to Avoid:

- Neglecting any of the material in the standards. Seeing Supporting and Additional clusters as optional.
- Sorting clusters (from Major to Supporting to Additional) and then teaching the clusters in order. This would remove the coherence of mathematical ideas and create missed opportunities to enhance the major work of the grade with the other clusters.
- Using the cluster headings as a replacement for the actual standards. All features of the standards matter—from the practices to surrounding text, including the particular wording of the individual content standards. Guidance for priorities is given at the cluster level as a way of thinking about the content with the necessary specificity yet without going so far into detail as to comprise the coherence of the standards (grain size).

Mathematics Teaching Practices

(High Leverage Teacher Actions)

[National Council of Teachers of Mathematics. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: National Council of Teachers of Mathematics.]

The eight Mathematics Teaching Practices should be the foundation for mathematics instruction and learning. This framework was informed by over twenty years of research and presented in *Principles to Actions* by the National Council of Teachers of Mathematics (NCTM). If teachers are guided by this framework, they can move "toward improved instructional practice" and support "one another in becoming skilled at teaching in ways that matter for ensuring successful mathematics learning for all students" (NCTM, 2014, p. 12).

1. **Establish mathematics goals to focus learning.**
   Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
2. **Implement tasks that promote reasoning and problem solving.**
   Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
3. **Use and connect mathematical representations.**
   Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
4. **Facilitate meaningful mathematical discourse.**
   Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
5. **Pose purposeful questions.**
   Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
6. **Build procedural fluency from conceptual understanding.**
   Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
7. **Support productive struggle in learning mathematics.**
   Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
8. **Elicit and use evidence of student thinking.**
   Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Standards for Mathematical Practice in Grade 5

The Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that Grade 5 students complete.

Practices	Explanations and Examples
1) Make sense of problems and persevere in solving them.	Mathematically proficient students in Grade 5 solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. Fifth graders may consider different representations of the problem and different solution pathways, both their own and those of other students, in order to identify and analyze correspondences among approaches. When they find that their solution pathway does not make sense, they look for another pathway that does. They check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
2) Reason abstractly and quantitatively.	Mathematically proficient students in Grade 5 recognize that a number represents a specific quantity. They extend this understanding from whole numbers to their work with fractions and decimals. This involves two processes - decontextualizing and *contextualizing*. Grade 5 students *decontextualize* by taking a real-world problem and writing and solving equations based on the word problem. For example, consider the task, "There are $2\frac{2}{3}$ of a yard of rope in the shed. If a total of $4\frac{1}{6}$ yard is needed for a project, how much more rope is needed?" Students decontextualize the problem by writing the equation $4\frac{1}{6} - 2\frac{2}{3} = ?$ and then solving it. Further, students *contextualize* the problem after they find the answer, by reasoning that $1\frac{3}{6}\ or\ 1\frac{1}{2}$ yards of rope is the amount needed. Also, Grade 5 students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.
3) Construct viable arguments and critique the reasoning of others.	Mathematically proficient students in Grade 5 construct arguments using representations, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking through either discussions or written responses. In Grade 5, students return to their conjectures and arguments about whole numbers to determine whether they apply to fractions and decimals. For example, they might make an argument based on an area representation of multiplication to show that the distributive property applies to problems involving fractions.

4) Model with mathematics.	Mathematically proficient students in Grade 5 experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems. Example, when students encounter situations such as sharing a pan of brownies among 6 people, they might first show how to divide the brownies into 6 equal pieces using a picture of a rectangle. The rectangle divided into 6 equal pieces is a model of the essential mathematics elements of the situation. When the students write the name of each piece in relation to the whole pan as $\frac{1}{6}$, they are now modeling the situation with mathematical notation.
5) Use appropriate tools strategically.	Mathematically proficient students in Grade 5 consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data. *Estimation* is also seen as a tool. For example, in order to solve $\frac{4}{6}-\frac{1}{2}$, a 5th grader might recognize that knowledge of *equivalents* of $\frac{1}{2}$ is an appropriate tool: since $\frac{1}{2}$ is equivalent to $\frac{3}{6}$, the result can easily be found: $\frac{1}{6}$. This practices is also related to looking for structure (SMP 7), which often results in building mathematical tools that can then be used to solve problems.
6) Attend to precision.	Mathematically proficient students in grade 5 continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.
7) Look for and make use of structure.	Mathematically proficient students in Grade 5 look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation. For example, when 5th graders calculate 16×9, they might apply the structure of place value and the distributive property to find the product: $16 \times 9 = (10 + 9) \times 9 = (10 \times 9) + (6 \times 9)$.
8) Look for and express regularity in repeated reasoning.	Mathematically proficient students in Grade 5 use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations. For example, 5th graders might notice a pattern in the change to the product when a factor is increased by 1: $5 \times 7 = 35$ *and* $5 \times 8 = 40$---the product changes by 5; $9 \times 4 = 36$ *and* $10 \times 4 = 40$—the product changes by 4. Fifth graders might then express this regularity by saying something like, *"When you change one factor by 1, the product increases by the other factor."* As students practice articulating their observations, they learn to communicate with greater precisions (SMP 6). As they explain why these generalizations must be true, they construct, critique, and compare arguments (SMP 3).

Implementing Standards for Mathematical Practice

This guide was created to help educators implement these standards into their classroom instruction. These are the practices for the **students,** and the teacher can assist students in using them efficiently and effectively.

#1 – Make sense of problems and persevere in solving them.

Summary of this Practice:

- Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem.
- Plan a solution pathway instead of jumping to a solution.
- Monitor their progress and change the approach if necessary.
- See relationships between various representations.
- Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another.
- Continually ask themselves, "Does this make sense?"
- Understand various approaches to solutions.

Student Actions	Teacher Actions
• Actively engage in solving problems and thinking is visible (doing mathematics vs. following steps or procedures with no understanding). • Relate current "situation" to concepts or skills previously learned, and checking answers using different methods. • Monitor and evaluate their own progress and change course when necessary. • Always ask, "Does this make sense?" as they are solving problems.	• Allow students time to initiate a plan; using question prompts as needed to assist students in developing a pathway. • Constantly ask students if their plans and solutions make sense. • Question students to see connections to previous solution attempts and/or tasks to make sense of the current problem. • Consistently ask students to defend and justify their solution(s) by comparing solution paths.

What questions develop this Practice?

- How would you describe the problem in your own words? How would you describe what you are trying to find?
- What do you notice about...?
- What information is given in the problem? Describe the relationship between the quantities.
- Describe what you have already tried. What might you change? Talk me through the steps you've used to this point.
- What steps in the process are you most confident about? What are some other strategies you might try?
- What are some other problems that are similar to this one?
- How might you use one of your previous problems to help you begin? How else might you organize...represent...show...?

What are the characteristics of a good math task for this Practice?

- Requires students to engage with conceptual ideas that underlie the procedures to complete the task and develop understanding.
- Requires cognitive effort - while procedures may be followed, the approach or pathway is not explicitly suggested by the task, or task instructions and multiple entry points are available.
- Encourages multiple representations, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations to develop meaning.
- Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.

#2 – Reason abstractly and quantitatively.

Summary of this Practice:

- Make sense of quantities and their relationships.
- Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships.
- Understand the meaning of quantities and are flexible in the use of operations and their properties.
- Create a logical representation of the problem.
- Attend to the meaning of quantities, not just how to compute them.

Student Actions	Teacher Actions
• Use varied representations and approaches when solving problems. • Represent situations symbolically and manipulating those symbols easily. • Give meaning to quantities (not just computing them) and making sense of the relationships within problems.	• Ask students to explain the meaning of the symbols in the problem and in their solution. • Expect students to give meaning to all quantities in the task. • Question students so that understanding of the relationships between the quantities and/or the symbols in the problem and the solution are fully understood.

What questions develop this Practice?

- What do the numbers used in the problem represent? What is the relationship of the quantities?
- How is____related to_____?
- What is the relationship between ____ and ____?
- What does ______ mean to you? (e.g. symbol, quantity, diagram)
- What properties might you use to find a solution?
- How did you decide that you needed to use ____? Could we have used another operation or property to solve this task? Why or why not?

What are the characteristics of a good math task for this Practice?

- Includes questions that require students to attend to the meaning of quantities and their relationships, not just how to compute them.
- Consistently expects students to convert situations into symbols in order to solve the problem; and then requires students to explain the solution within a meaningful situation.
- Contains relevant, realistic content.

#3 – Construct viable arguments and critique the reasoning of others.

Summary of this Practice:

- Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments.
- Justify conclusions with mathematical ideas.
- Listen to the arguments of others and ask useful questions to determine if an argument makes sense.
- Ask clarifying questions or suggest ideas to improve/revise the argument.
- Compare two arguments and determine correct or flawed logic.

Student Actions	Teacher Actions
• Make conjectures and exploring the truth of those conjectures. • Recognize and use counter examples. • Justify and defend all conclusions and using data within those conclusions. • Recognize and explain flaws in arguments, which may need to be demonstrated using objects, pictures, diagrams, or actions.	• Encourage students to use proven mathematical understandings, (definitions, properties, conventions, theorems etc.), to support their reasoning. • Question students so they can tell the difference between assumptions and logical conjectures. • Ask questions that require students to justify their solution and their solution pathway. • Prompt students to respectfully evaluate peer arguments when solutions are shared. • Ask students to compare and contrast various solution methods • Create various instructional opportunities for students to engage in mathematical discussions (whole group, small group, partners, etc.)

What questions develop this Practice?

- What mathematical evidence would support your solution? How can we be sure that...? How could you prove that...?
- Will it still work if...?
- What were you considering when...? How did you decide to try that strategy?
- How did you test whether your approach worked?
- How did you decide what the problem was asking you to find? (What was unknown?)
- Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not?
- What is the same and what is different about...? How could you demonstrate a counter-example?

What are the characteristics of a good math task for this Practice?

- Structured to bring out multiple representations, approaches, or error analysis.
- Embeds discussion and communication of reasoning and justification with others.
- Requires students to provide evidence to explain their thinking beyond merely using computational skills to find a solution.
- Expects students to give feedback and ask questions of others' solutions.

#4 – Model with mathematics.

Summary of this Practice:

- Understand reasoning quantitatively and abstractly (able to decontextualize and contextualize).
- Apply the math they know to solve problems in everyday life.
- Simplify a complex problem and identify important quantities to look at relationships.
- Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation.
- Reflect on whether the results make sense, possibly improving/revising the model.
- Ask themselves, "How can I represent this mathematically?"

Student Actions	Teacher Actions
• Apply mathematics to everyday life. • Write equations to describe situations. • Illustrate mathematical relationships using diagrams, data displays, and/or formulas. • Identify important quantities and analyzing relationships to draw conclusions.	• Demonstrate and provide students experiences with the use of various mathematical models. • Question students to justify their choice of model and the thinking behind the model. • Ask students about the appropriateness of the model chosen. • Assist students in seeing and making connections among models.

What questions develop this Practice?

- What number model could you construct to represent the problem?
- How can you represent the quantities?
- What is an equation or expression that matches the diagram..., number line..., chart..., table...?
- Where did you see one of the quantities in the task in your equation or expression?
- What math do you know that you could use to represent this situation?
- What assumptions do you have to make to solve the problem?
- What formula might apply in this situation?

What are the characteristics of a good math task for this Practice?

- Structures represent the problem situation and their solution symbolically, graphically, and/or pictorially (may include technological tools) appropriate to the context of the problem.
- Invites students to create a context (real-world situation) that explains numerical/symbolic representations.
- Asks students to take complex mathematics and make it simpler by creating a model that will represent the relationship between the quantities.

#5 – Use appropriate tools strategically.

Summary of this Practice:

- Use available tools recognizing the strengths and limitations of each.
- Use estimation and other mathematical knowledge to detect possible errors.
- Identify relevant external mathematical resources to pose and solve problems.
- Use technological tools to deepen their understanding of mathematics.
- Use mathematical models for visualize and analyze information

Student Actions	Teacher Actions
• Choose tools that are appropriate for the task. • Know when to use estimates and exact answers. • Use tools to pose or solve problems to be most effective and efficient.	• Demonstrate and provide students experiences with the use of various math tools. A variety of tools are within the environment and readily available. • Question students as to why they chose the tools they used to solve the problem. • Consistently model how and when to estimate effectively, and requiring students to use estimation strategies in a variety of situations. • Ask student to explain their mathematical thinking with the chosen tool. • Ask students to explore other options when some tools are not available.

What questions develop this practice?

- What mathematical tools could we use to visualize and represent the situation?
- What information do you have?
- What do you know that is not stated in the problem? What approach are you considering trying first?
- What estimate did you make for the solution?
- In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? Why was it helpful to use...?
- What can using a_____show us that _____may not?
- In what situations might it be more informative or helpful to use...?

What are the characteristics of a good math task for this Practice?

- Lends itself to multiple learning tools. (Tools may include; concrete models, measurement tools, graphs, diagrams, spreadsheets, statistical software, etc.)
- Requires students to determine and use appropriate tools to solve problems.
- Asks students to estimate in a variety of situations:
  - a task when there is no need to have an exact answer
  - a task when there is not enough information to get an exact answer
  - a task to check if the answer from a calculation is reasonable

#6 – Attend to precision.

Summary of this Practice:

- Communicate precisely with others and try to use clear mathematical language when discussing their reasoning.
- Understand meanings of symbols used in mathematics and can label quantities appropriately.
- Express numerical answers with a degree of precision appropriate for the problem context.
- Calculate efficiently and accurately.

Student Actions	Teacher Actions
• Use mathematical terms, both orally and in written form, appropriately. • Use and understanding the meanings of math symbols that are used in tasks. • Calculate accurately and efficiently. • Understand the importance of the unit in quantities.	• Consistently use and model correct content terminology. • Expect students to use precise mathematical vocabulary during mathematical conversations. • Question students to identify symbols, quantities and units in a clear manner.

What questions develop this Practice?

- What mathematical terms apply in this situation? How did you know your solution was reasonable?
- Explain how you might show that your solution answers the problem.
- Is there a more efficient strategy?
- How are you showing the meaning of the quantities?
- What symbols or mathematical notations are important in this problem?
- What mathematical language…, definitions…, properties can you use to explain…?
- How could you test your solution to see if it answers the problem?

What are the characteristics of a good math task for this Practice?

- Requires students to use precise vocabulary (in written and verbal responses) when communicating mathematical ideas.
- Expects students to use symbols appropriately.
- Embeds expectations of how precise the solution needs to be (some may more appropriately be estimates).

#7 – Look for and make use of structure.

Summary of this Practice:

- Apply general mathematical rules to specific situations.
- Look for the overall structure and patterns in mathematics.
- See complicated things as single objects or as being composed of several objects.

Student Actions	Teacher Actions
• Look closely at patterns in numbers and their relationships to solve problems. • Associate patterns with the properties of operations and their relationships. • Compose and decompose numbers and number sentences/expressions.	• Encourage students to look for something they recognize and having students apply the information in identifying solution paths (i.e. compose/decompose numbers and geometric figures, identify properties, operations, etc.) • Expect students to explain the overall structure of the problem and the big math idea used to solve the problem.

What questions develop this Practice?

- What observations do you make about...? What do you notice when...?
- What parts of the problem might you eliminate..., simplify...?
- What patterns do you find in...?
- How do you know if something is a pattern?
- What ideas that we have learned before were useful in solving this problem?
- What are some other problems that are similar to this one? How does this relate to...?
- In what ways does this problem connect to other mathematical concepts?

What are the characteristics of a good math task for this Practice?

- Requires students to look for the structure within mathematics in order to solve the problem. (i.e. – decomposing numbers by place value; working with properties; etc.)
- Asks students to take a complex idea and then identify and use the component parts to solve problems. i.e. Building on the structure of equal sharing, students connect the understanding to the traditional division algorithm. When "unit size" cannot be equally distributed, it is necessary to break down into a smaller "unit size". (example below)

4)351 -32	3 *hundreds* units cannot be distributed into 4 equal groups. Therefore, they must be broken down into *tens* units.
31	There are now 35 *tens* units to distribute into 4 groups. Each group gets 8 sets of tens, leaving 3 extra *tens* units that need to become *ones* units.
-28 3	This leaves 31 *ones* units to distribute into 4 groups. Each group gets 7 *ones* units, with 3 *ones* units remaining. The quotient means that each group has 87 with 3 left.

- Expects students to recognize and identify structures from previous experience(s) and apply this understanding in a new situation. i.e. $7 \times 8 = (7 \times 5) + (7 \times 3)$ OR $7 \times 8 = (7 \times 4) + (7 \times 4)$ new situations could be, distributive property, area of composite figures, multiplication fact strategies.

#8 – Look for and express regularity in repeated reasoning.

Summary of this Practice:

- See repeated calculations and look for generalizations and shortcuts.
- See the overall process of the problem and still attend to the details.
- Understand the broader application of patterns and see the structure in similar situations.
- Continually evaluate the reasonableness of their intermediate results.

Student Actions	Teacher Actions
• Notice if processes are repeated and look for both general methods and shortcuts. • Evaluate the reasonableness of intermediate results while solving. • Make generalizations based on discoveries and constructing formulas when appropriate.	• Ask what math relationships or patterns can be used to assist in making sense of the problem. • Ask for predictions about solutions at midpoints throughout the solution process. • Question students to assist them in creating generalizations based on repetition in thinking and procedures.

What questions develop this Practice?

- Will the same strategy work in other situations?
- Is this always true, sometimes true or never true? How would we prove that...?
- What do you notice about...?
- What is happening in this situation? What would happen if…?
- Is there a mathematical rule for...?
- What predictions or generalizations can this pattern support? What mathematical consistencies do you notice?

What are the characteristics of a good math task for this Practice?

- Present several opportunities to reveal patterns or repetition in thinking, so students can make a generalization or rule.
- Requires students to see patterns or relationships in order to develop a mathematical rule.
- Expects students to discover the underlying structure of the problem and come to a generalization.
- Connects to a previous task to extend learning of a mathematical concept.

Critical Areas for Mathematics in 5th Grade

In Grade 5, instructional time should focus on **three** critical areas:

1. **Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).**
   Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

2. **Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.**
   Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

3. **Developing understanding of volume.**
   Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Dynamic Learning Maps (DLM) and Essential Elements

The Dynamic Learning Maps and Essential Elements are knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities.

For more information please visit the Dynamic Learning Maps and Essential Elements website.

Growth Mindset

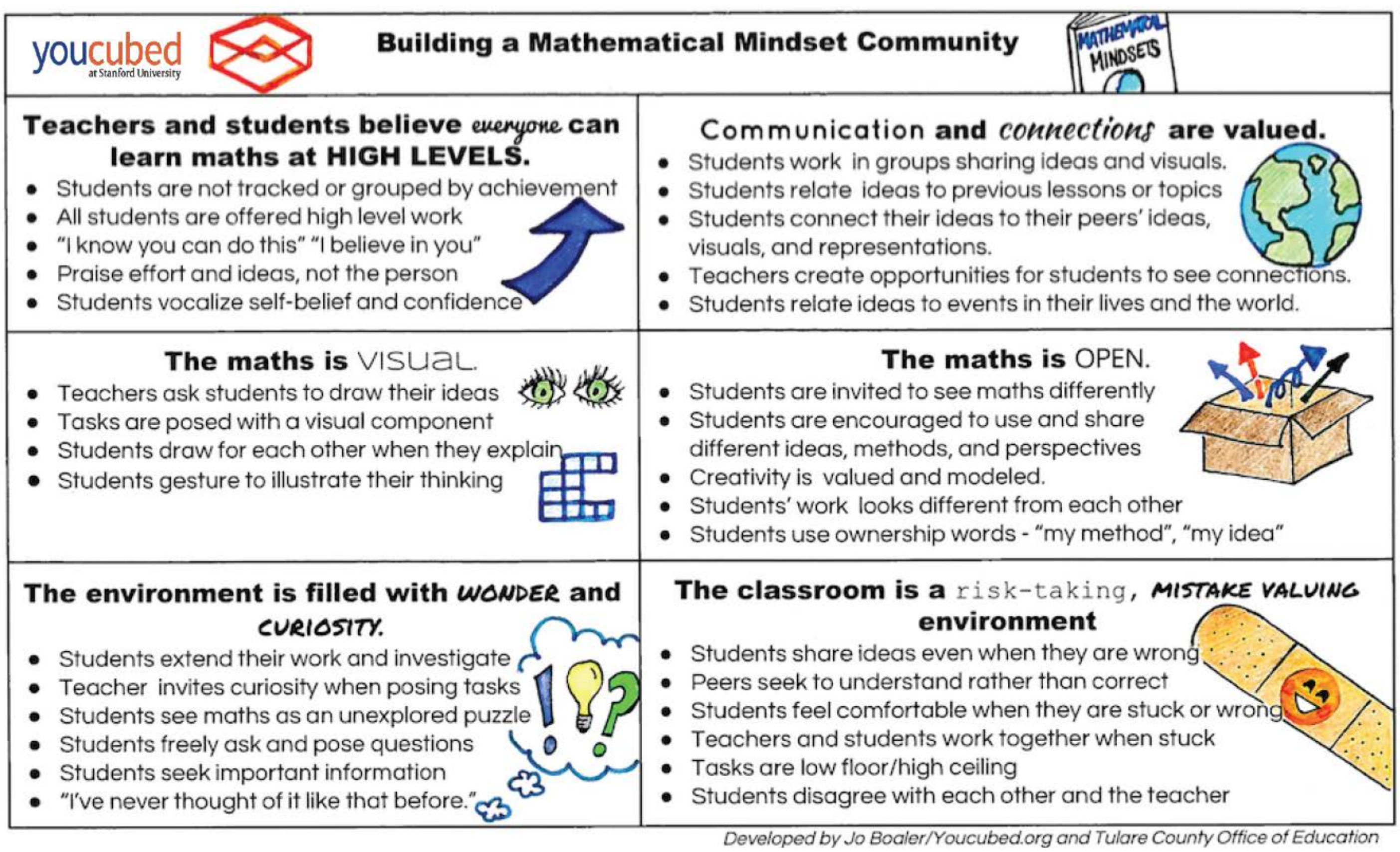
The term "growth mindset" comes from the groundbreaking work of Carol Dweck. She identified that everyone holds ideas about their own potential. Some people believe that their intelligence is more or less fixed in math – that you can do math or you can't, while others believe they can learn anything and that their intelligence can grow.

In a fixed mindset, people believe their basic qualities, like their intelligence or talent, are simply fixed traits. They spend their time documenting their intelligence or talent instead of developing it. They also believe that talent alone creates success—without effort. Students with a fixed mindset are those who are more likely to give up easily.

In a **growth mindset**, people believe that their most basic abilities can be developed through dedication and hard work—brains and talent are just the starting point. This view creates a love of learning and a resilience that is essential for great accomplishment. Students with a growth mindset are those who keep going even when work is hard, and who are persistent.

It is possible to change mindsets and to shift students' mindsets from fixed to growth and cause higher mathematics achievement and success in life. Watch this short video to get a better understanding of what Growth Mindset is and the benefits it can bring our students.

You can find a variety of resources related to **Growth Mindset** at: http://community.ksde.org/Default.aspx?tabid=6383.



Grade 5 Content Standards Overview

Operations and Algebraic Thinking (5.OA)

A. Write and interpret numerical expressions.
   OA.1 OA.2

Number and Operations in Base Ten 5.(NBT)

A. Understand the place value system.
   NBT.1 NBT.2 NBT.3 NBT.4
B. Perform operations with multi-digit whole numbers and with decimals to hundredths.
   NBT.5 NBT.6 NBT.7

Number and Operations—Fractions (5.NF)

A. Use equivalent fractions as a strategy to add and subtract fractions.
   NF.1 NF.2
B. Apply and extend previous understandings of multiplication and division to and divide fractions.
   NF.3 NF.4 NF.5 NF.6 NF.7

Measurement and Data (5.MD)

A. Convert like measurement units within a given measurement system.
   MD.1
B. Represent and interpret data.
   MD.2
C. Geometric measurement: understand concepts of volume and related volume to multiplication and to addition.
   MD.3 MD.4 MD.5

Geometry (5.G)

A. Graph points on the coordinate plane to solve real world and mathematical problems.
   G.1 G.2
B. Classify two-dimensional figures into categories based on their properties.
   G.3 G.4

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Click on the box to open specific details related to Grade Five!

Major Clusters   Supporting Clusters   Additional Clusters

Domain: Operations and Algebraic Thinking (OA)

● ***Cluster A:*** *Write and interpret numerical expressions*.

Standard: 5.OA.1

Use parentheses in numerical expressions and evaluate expressions with these symbols. **(5.OA.1)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 - Make sense of problems and persevere in solving them.
- ✓ MP.5 - Use appropriate tools strategically.
- ✓ MP.8 - Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

- Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.
- Evaluating numerical Expressions with whole-number exponents (6.EE.1)

Explanation and Examples: 5.OA.1

This standard builds on the expectations of third grade where students are expected to start learning the conventional order of operations. Students need experiences with multiple expressions that use grouping symbols throughout the year to develop understanding of when and how to use them. First, students use these symbols with whole numbers. Then the symbols can be used as students add, subtract, multiply and divide decimals and fractions.

Examples:

Problem	Answer
$(26 + 18) + 4$	11
$2 \times (10 + 5)$	30
$12 - (0.4 \times 2)$	11.2
$(2 + 3) \times (1.5 - 0.5)$	5
$6 - \left(\frac{1}{2} + \frac{1}{3}\right)$	$5\frac{1}{6}$

To further develop students' understanding of grouping symbols and facility with operations, students place the parentheses in equations to make the equations true or they compare expressions that are grouped differently.

Examples:

$15 - 7 - 2 = 10 \rightarrow 15 - (7 - 2) = 10$
Compare $3 \times 2 + 5$ and $3 \times (2 + 5)$
Compare $15 - 6 + 7$ and $15 - (6 + 7)$

Instructional Strategies: 5.OA.1 through 5.OA.2
Students should be given ample opportunities to explore numerical expressions with mixed operations. This is the foundation for evaluating numerical and algebraic expressions that will include whole-number exponents in Grade 6.

There are conventions (rules) determined by mathematicians that must be learned with no conceptual basis. For example, multiplication and division are always done before addition and subtraction. Multiplication and division generally has a bigger impact on a problem than addition and subtraction.

Begin with expressions that have two operations without any grouping symbols (multiplication or division combined with addition or subtraction) before introducing expressions with multiple operations. Use the same digits, with the operations in a different order, and have students evaluate the expressions, then discuss why the value of the expression is different. For example, have students evaluate $5 \times 3 + 6$ and $5 + 3 \times 6$.

Discuss the rules (conventions) that must be followed. Have students insert parentheses around the multiplication or division section in an expression. A discussion should focus on the similarities and differences in the problems and the results. This should assist students in being able to solve problem situations which require an understanding of the order in which operations should take place.

After students have evaluated expressions without grouping symbols, present problems parentheses. If you students are ready, you can move into using brackets and braces, but these must be done so students have an understanding of why these grouping symbols are being used. Have students write numerical expressions in words without calculating the value. This is the foundation for writing algebraic expressions. Then, have students write numerical expressions from phrases without calculating them.

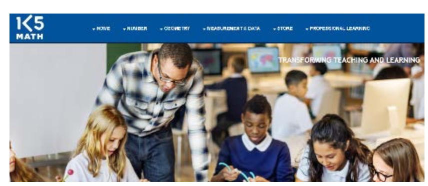
Resources/Tools
For detailed information, see the Learning Progression *Operations and Algebraic Thinking*.

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.OA.A
  - You Can Multiply Three Numbers in Any Order
- 5.OA.A.1
  - Watch Out for Parentheses 1
  - Bowling for Numbers
  - Using Operations and Parentheses

Video clip sharing the use of parentheses in mathematics – Using Parentheses in Math

Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.OA.1 to access resources specifically for this standard.



Common Misconceptions:

Students may believe the order in which a problem with mixed operations is written is the exact order to solve the problem. The use of mnemonic phrase "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally" to remember the order of operations (*Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction)* can mislead students to always perform multiplication before division and addition before subtraction. This is incorrect thinking. Multiplication and division are always performed first in the order that they appear in the problem – unless there are grouping symbols. To help correct students' thinking, they need to understand that addition and subtraction are inverse operations and multiplication and division are inverse operations, as in they have the same "impact". At this level, students need opportunities to explore the "impact" of the various operations on numbers and solve equations starting with the operation of greatest "impact".

Example:

$3 + 2 = 5, 5 - 2 = 3$ (generalize subtraction "undoes" addition – inverse operation)
$3 \times 2 = 6, 6 \div 2 = 3$ (generalize division "undoes" multiplication – inverse operation and multiplication and division have a greater "impact" on a number than addition and subtraction)
$3^2 = 9$ (generalize, exponents have a greater "impact" on a number)

Allow students to use calculators to determine the value of the expression and then discuss the order the calculator used to evaluate the expression. Do this with four-function and scientific calculators.

Students need lots of experience with writing multiplication in different ways. Multiplication can be indicated with a raised dot, such as $4 \cdot 5$, with a raised cross symbol, such as 4×5, or with parentheses, such as $4(5)$ *or* $(4)(5)$. Note that the raised cross symbol is *not the same as* the letter "x", and so care should be taken when writing or typing it. Students need to be exposed to all three notations and should be challenged to understand that all are useful. In instruction, teachers are encouraged to use a notation and stay consistent. Students also need help and practice remembering the convention that we write a rather than $1 \cdot a$ or $1a$, especially in expressions such as $a + 3a$.

Domain: Operations and Algebraic Thinking (OA)

● ***Cluster A:*** *Write and interpret numerical expressions***.**

Standard: 5.OA.2

Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "multiply the sum of 8 and 7 by 2" as* $2 \times (8 + 7)$ *because parenthetical information must be solved first. Recognize that* $3 \times (18932 + 921)$ *is three times as large as* $18932 + 921$*, without having to calculate the indicated sum or product.* **(5.OA.2)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 **-** Make sense of problems and persevere in solving them.
- ✓ MP.2 - Reason abstractly and quantitatively.
- ✓ MP.7 - Look for and make use of structure.
- ✓ MP.8 - Look for and express regularity in repeated reasoning.

Connections: See 5.OA.1

Explanation and Examples:

This standard refers to expressions. Expressions are a series of numbers and symbols $(+, -, \times, \div)$ without an equal sign.

Example:

$4(5 + 3)$ is an expression.

When we compute $4(5 + 3)$ we are evaluating the expression. The expression equals 32. $4(5 + 3) = 32$ is an equation.

This standard also expects students to verbally describe the relationship between expressions without actually calculating them. Students are to apply their reasoning of the four operations, as well as place value, while describing the relationship between the numbers. The standard does not include the use of variables, only numbers and relational and operational symbols.

Example:

Teacher: Write an expression for "double five and then add 26."

Student: $(2 \times 5) + 26$

Teacher: Tell me how the expression $5(10 \times 10)$ relates to (10×10).

Student: The expression $5(10 \times 10)$ is 5 times larger than the expression (10×10) since I know that $5(10 \times 10)$ means that I have 5 groups of (10×10). *Students use their understanding of operations and grouping symbols to write expressions and interpret the meaning of a numerical expression*.

Other Examples:

- Students write an expression for calculations given in words such as "divide 144 by 12 and then subtract $\frac{7}{8}$." The students would write $(144 \div 12) - \frac{7}{8}$.
- Students recognize that $0.5 \times (300 \div 15)$ is $\frac{1}{2}$ of $(300 \div 15)$ without having to calculate the quotient.

Instructional Strategies: See 5.OA.1

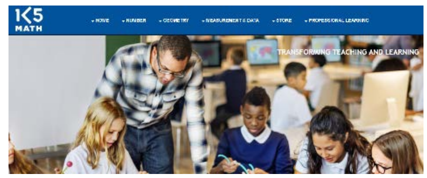
Resources/Tools

For detailed information, see Learning Progression Operations and Algebraic Thinking.

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.OA.A.2
  - Words to Expressions 1
  - Video Game Scores
  - Comparing Products
  - Seeing is Believing

Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.OA.2 to access resources specifically for this standard.



Common Misconceptions: See 5.OA.1

Students often do not use the correct terminology for the operations. Frequently students say "times" for multiplication. This is NOT the action for multiplication. Students need to say and think "groups of" (or "of" when using fractions and decimals) when explaining multiplication. For addition, students can explain that it is "joining", "combining", "putting together", or other appropriate words for addition. The same for the rest of the operational symbols.

Students need to realize that math symbols are just short cuts for using words but that ALL symbols represent words in mathematics.

Domain: Number and Operations Base Ten (NBT)

▶ ***Cluster A:*** *Understand the place value system*.

Standard: 5.NBT.1

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left. **(5.NBT.1)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: (5.NBT.1 through 5.NBT.4)

This cluster is connected to:

- Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.
- Understand decimal notation for fractions, and compare decimal fractions (4.NF.7).
- Students need to have a firm grasp of place value for future work with computing with numbers, exponents and scientific notation.

Explanation and Examples:

This standard expects students to reason about the magnitude of numbers. Students should work with the idea that the tens place is ten times as much as the ones place, and the ones place is $\frac{1}{10}$ the size of the tens place.

In fourth grade, students examined the relationships of the digits in numbers for whole numbers only. This standard extends this understanding to the relationship of decimal fractions. Students use decimal squares, base ten blocks, pictures of decimal squares or base ten blocks, and interactive images of decimal squares or base ten blocks to manipulate and investigate the place value relationships. They use their understanding of unit fractions to compare decimal places and fractional language to describe those comparisons.

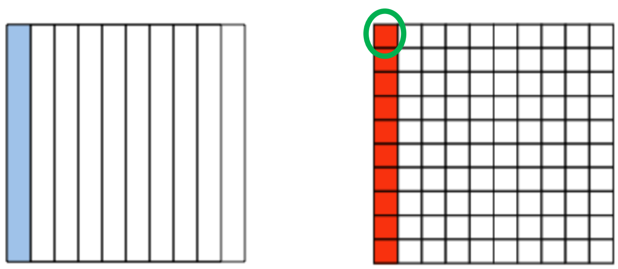
Before considering the relationship of decimal fractions, students express their understanding that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.

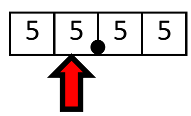
Example:

The 2 in the number 54**2** is different from the value of the 2 in 3**2**4. The 2 in 54**2** represents 2 ones or 2, while the 2 in 3**2**4 represents 2 tens or 20. Since the 2 in 3**2**4 is one place to the left of the 2 in 54**2** the value of the 2 is 10 times greater. Meanwhile, the 4 in 5**4**2 represents 4 tens or 40 and the 4 in 32**4** represents 4 ones or 4. Since the 4 in 32**4** is one place to the right of the 4 in 5**4**2 the value of the 4 in the number 32**4** is $\frac{1}{10}$ of its value in the number 5**4**2.

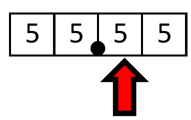
A student should reason and think, "I know that in the number 5555, the 5 in the tens place (5555) represents 50 and the 5 in the hundreds place (5555) represents 500. So a 5 in the hundreds place is ten times as much as a 5 in the tens place or a 5 in the tens place is $\frac{1}{10}$ of the value of a 5 in the hundreds place".

Example:
To extend this understanding of place value to their work with decimals, students use a model of one unit. They divide that unit into 10 equal pieces, then shade in or describe $\frac{1}{10}$ of that model using fractional language (*"This is 1 out of 10 equal parts. So it is* $\frac{1}{10}$*". I can write this using* $\frac{1}{10}$ *or 0.1*"). They repeat the process by finding $\frac{1}{10}$ of a $\frac{1}{10}$ (e.g., dividing $\frac{1}{10}$ into 10 equal parts to arrive at $\frac{1}{100}$ or 0.01) and can explain their reasoning, *"0.01 is* $\frac{1}{10}$ *of* $\frac{1}{10}$ *so it is* $\frac{1}{100}$ *of the whole unit."* In the number 55.55, each digit is 5, but the value of the digits is different because of the placement.

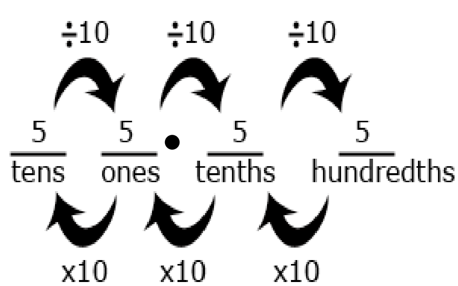




The 5 that the arrow points to is $\frac{1}{10}$ of the 5 to the left and 10 times the 5 to the right. The 5 in the ones place is $\frac{1}{10}$ of 50 and 10 times five tenths.



The 5 that the arrow points to is $\frac{1}{10}$ of the 5 to the left and 10 times the 5 to the right. The 5 in the tenths place is 10 times five hundredths.



Instructional Strategies: (5.NBT.1 through 5.NBT.4)
In Grade 5, the concept of place value is extended to include decimal values to thousandths. The strategies for Grades 3 and 4 should be drawn upon and extended for whole numbers and decimal numbers. For example, students need to continue to represent, write and state the value of numbers, including decimal numbers. For students who are not able to read, write and represent multi-digit numbers, working with decimals could potentially be challenging.

Money is a good medium to compare decimals (but keep in mind that 5th grade students are to extend their work into thousandths place so use other situations besides money). Present contextual situations that require the comparison of the cost of two items to determine which one is of a higher price or a lower price. Students should also be able to identify how many pennies, dimes, dollars and ten dollars, etc., are in a given value. Help students make connections between the number of each type of coin and the value of each coin, and the expanded form of the number. Build on the understanding that it always takes ten of the number to the right to make the number to the left.

Number cards, number cubes, spinners and other manipulatives can be used to generate decimal numbers. For example, have students roll three number cubes and have them create the largest and smallest decimal number to the thousandths place. Ask students to represent the number with numerals and words, including standard and unit forms.

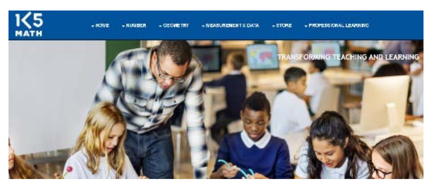
Instructional Resources/Tools:
For detailed information see: Learning Progressions for Numbers and Operations in Base Ten

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.NBT.A.1
  - Kipton's Scale
  - Tenths and Hundredths
  - Which number is it?
  - Millions and Billions of People

Video from Math Antics – Converting Base-Ten Fractions

Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**.
Scroll down to 5.NBT.1 to access resources specifically for this standard.



Common Misconceptions: (5.NBT.1 through 5.NBT.4)
A common misconception that students have when trying to extend their understanding of whole number place value to decimal place value is that as you move to the left of the decimal point, the number increases in value. Reinforcing the **concept of powers of ten** is essential for addressing this issue.

A second misconception that is directly related to comparing whole numbers is the idea that the longer the number the greater the number. With whole numbers, a 5-digit number is always greater than a 1, 2, 3, or 4-digit number. However, with decimals a number with one decimal place may be greater than a number with two or three decimal places. For example, 0.5 is greater than 0.12 or 0.009 or 0.499. Have students reason about the size of each decimal number.

Domain: Number and Operations in Base Ten (NBT)

▶ ***Cluster A:*** *Understand the place value system*.

Standard: 5.NBT.2

Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **(5.NBT.2)**

Standards for Mathematical Practice (MP) to be emphasized:

- ✓ MP.2. Reason abstractly and quantitatively.
- ✓ MP.6. Attend to precision.
- ✓ MP.7. Look for and make use of structure.

Connections: See 5.NBT.1

Explanation and Examples:

This standard includes multiplying by multiples of 10 and powers of 10 (such as 10^2 which is $10 \times 10 = 100$, and 10^3 which is $10 \times 10 \times 10 = 1{,}000$). Students should have experiences connecting the pattern of the number of zeros in the product when you multiply by powers of 10.

Example:

$2.5 \times 10^3 = 2.5(10 \times 10 \times 10) = 2.5 \times 1{,}000 = 2{,}500$. Students should reason (through experiences, not the teacher saying that this is a rule) that the exponent above the 10 indicates how many places the decimal point is moving because you are multiplying by powers of 10 (*not just that the decimal point is moving but that you are multiplying or making the number 10 times greater three times*). Since we are multiplying by a power of 10 the decimal point moves to the right because of this operation.

Experiences with division are also necessary to see the patterns.

$$350 \div 10^3 = 350 \div 1{,}000 = 0.350 = 0.35$$
$$350 \div 10 = 35, 35 \div 10 = 3.5$$
$$3.5 \div 10 = 0.35, or\ 350 \times \frac{1}{10}, 35 \times \frac{1}{10}, 3.5 \times \frac{1}{10},$$

This example shows that when we divide by powers of 10, the exponent above the 10 indicates how many places the decimal point is moving to the left because we are dividing (the number becomes ten times smaller). Since we are dividing by powers of 10, the decimal point moves to the left.

Provide students with many opportunities to explore this concept and come to this understanding; this should not be taught procedurally and just told that it is a rule.

Students might write:	**Students might think and/or say:**
• $36 \times 10 = 36 \times 10^1 = 360$ • $36 \times 10 \times 10 = 36 \times 10^2 = 3{,}600$ • $36 \times 10 \times 10 \times 10 = 36 \times 10^3 = 36{,}000$ • $36 \times 10 \times 10 \times 10 \times 10 = 36 \times 10^4 = 360{,}000$	**I noticed that every time I multiplied by 10 I added a zero to the end of the number. That makes sense because each digit's value became 10 times larger. To make a digit 10 times larger, I have to move it one place value to the left.* **When I multiplied 36 by 10, the 30 became 300. The 6 became 60 or the 36 became 360. So a zero is added at the end to have the 3 represent 3 one-hundreds (instead of 3 tens) and the 6 represents 6 tens (instead of 6 ones).*

Students should be able to use the same type of reasoning as above to **explain why** the following multiplication and division problem by powers of 10 make sense.

- $523 \times 10^3 = 523{,}000$ The place value of 523 is increased by 3 places.
- $5.223 \times 10^2 = 522.3$ The place value of 5.223 is increased by 2 places.
- $52.3 \times 10^1 = 5.23$ The place value of 52.3 is decreased by one place.

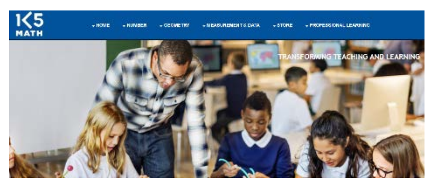
Instructional Strategies: See 5.NBT.1

Resources/Tools

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.NBT.A.2
    - o Marta's Multiplication Error
    - o Multiplying Decimals by 10

Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.NBT.2 to access resources specifically for this standard.



Common Misconceptions: See 5.NBT.1

Students memorize a rule of "adding zeros" to make the powers of 10 so they will misapply this "rule". Example – the students will look at 4.25 x 10^4 and believe that they need to just add 4 zeros to the end of the number and provide an incorrect answer of 4,250,000 instead of the correct number of 42,500. Students need lots of experience reasoning and showing WHY the decimal is moved.

Domain: Number and Operations in Base Ten (NBT)

▶ ***Cluster A:*** *Understand the place value system*.

Standard: 5.NBT.3

Read, write, and compare decimals to thousandths.

5.NBT.3a. Read and write decimals to thousandths using base-ten numerals, number names, expanded form, and unit form *(e.g.*

$$expanded\ form\ 47.392 = 4 \cdot 10 + 7 \cdot 1 + 3 \cdot \frac{1}{10} + 9 \cdot \frac{1}{100} + 2 \cdot \frac{1}{1000}$$

unit form 47.392 = 4 tens + 7 ones + 3 tenths + 9 hundredths + 2 thousandths). **(5.NBT.3a)**

5.NBT.3b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, <, =, and ≠ relational symbols to record the results of comparisons. **(5.NBT.3b)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See 5.NBT.1

Explanation and Examples: 5.NBT.3a

This standard references expanded form of decimals with fractions included. Students should build on their work from Fourth Grade, where they worked with both decimals and fractions interchangeably. Expanded form is included to build upon work in 5.NBT.2 and deepen students' understanding of place value.

Students should use concrete models and number lines to extend their understanding to decimals to the thousandths. Models may include decimal squares, base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based manipulatives, etc.

Students need to read decimals using fractional language and write decimals in fractional form and in expanded notation. This work supports their understanding of equivalence of decimals (0.8 = 0.80 = 0.800).

5.NBT.3b

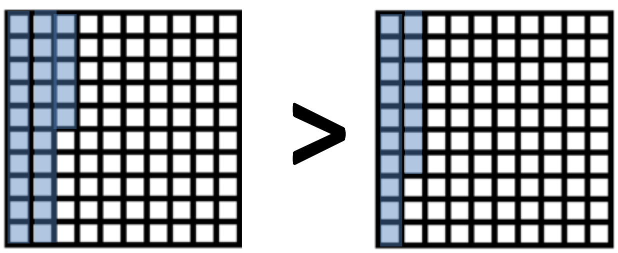
Comparing decimals builds on work from fourth grade when they were investigating equivalent forms of numbers.

Example:
Some equivalent forms of 0.72 are:

$\frac{72}{100}$	$\frac{7}{10}+\frac{2}{100}$
$7\times\left(\frac{1}{10}\right)+2\times\left(\frac{1}{100}\right)$	$0.70+0.02$
$\frac{70}{100}+\frac{2}{100}$	0.720
$7\times\frac{1}{10}+2\times\frac{1}{100}+0\times\frac{1}{1000}$	$\frac{720}{1000}$

Students need to develop a true understanding of the size of decimal numbers and relate them to **common benchmarks** such as 0, 0.5 (also 0.50 & 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals.

Example:
Comparing 0.25 and 0.17, a student might think, "25 hundredths is more than 17 hundredths". They may also think that it is 8 hundredths more. They may write this comparison as $0.25 > 0.17$ and recognize that $0.17 < 0.25$ is another way to express this comparison. To solidify this understanding and to share their reasoning, they may need to use decimal squares or other manipulatives to show the comparisons.



When comparing 0.207 to 0.26, a student might think, "*Both numbers have 2 tenths, so I need to compare the hundredths. The second number has 6 hundredths and the first number has no hundredths so the second number must be larger.*"

Another student might about these as decimal fractions, "*I know that 0.207 is 207 thousandths so the written fraction is* $\frac{207}{1000}$*. 0.26 is 26 hundredths so the written fraction is* $\frac{26}{100}$ *but I can also think of it as 260 thousandths which is written as a fraction to be* $\frac{260}{1000}$*. So, 260 thousandths is more than 207 thousandths.*"

Students build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They connect their prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100. They use concrete models and number lines to extend this understanding to decimals to the thousandths.

Models may include decimal squares, base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based manipulatives, etc. Students should read decimals using fractional language and write decimals in fractional form, as well as in expanded notation as show in the standard 5.NBT.3a.

Instructional Strategies: See 5.NBT.1

Resources/Tools:

See engageNY Module 1

Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.NBT.3 to access resources specifically for this standard.

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.NBT.A.3
  - Are these equivalent to 9.52?
  - Comparing Decimals on a Number Line
  - Placing Thousandths on the Number Line
- 5.NBT.A.3.b
  - Drawing Pictures to Illustrate Decimal Comparisons

NRICH mathematics – Spiraling Decimals

Videos comparing decimals – Compare Using Decimal Squares & Compare Using a Number Line

Online NCTM book chapter - Decimals

Common Misconceptions: See 5.NBT.1

A common misconception that is directly related to comparing whole numbers is the idea that the longer the number the greater the number. With whole numbers, a 5-digit number is always greater than a 1, 2, 3, or 4-digit number. However, with decimals a number with one decimal place may be greater than a number with two or three decimal places. For example, 0.5 is greater than 0.12 or 0.009 or 0.499. Have students reason about the size of each decimal number showing their reasoning with drawing or concrete models.

Domain: Number and Operations in Base Ten (NBT)

▶ ***Cluster A:*** *Understand the place value system*.

Standard: 5.NBT.4

Use place value understanding to round decimals to any place (Note: In fifth grade, decimals include whole numbers and decimal fractions to the hundredths place.) **(5.NBT.4)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See 5.NBT.1

Explanation and Examples:

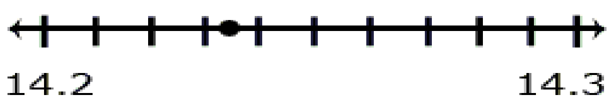
This standard refers to rounding. **Students should go beyond simply applying an algorithm or procedure for rounding**. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round.

Students should have numerous experiences using a number line to support their work with rounding. When rounding a decimal to a given place, students may identify the two possible answers, and use their understanding of place value to compare the given number to the possible answers.

Example:

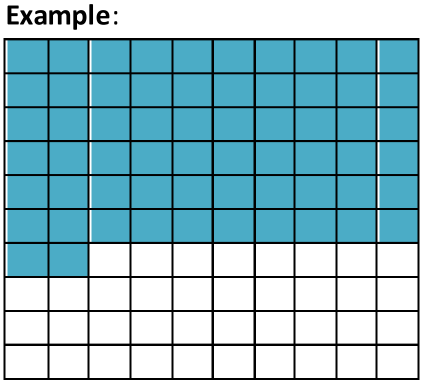
Round 14.235 to the nearest tenth.

Students recognize that the possible answer must be in tenths since they are rounding to the nearest tenth, so it is either 14.2 or 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30) by using the number line.



Students should use **benchmark** numbers to support this work. Benchmarks are convenient numbers for comparing and rounding numbers. 0., 0.5, 1, 1.5 are examples of benchmark numbers.

Example:



Which benchmark number is the best estimate of the shaded amount in the model to the left?

Explain your thinking.

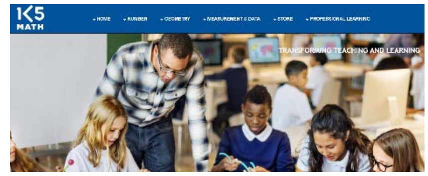
Instructional Strategies: See 5 NBT.1

Instructional Resources/Tools

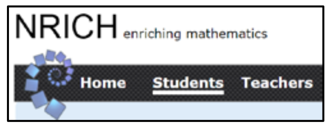
Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.NBT.A.4
    - Rounding to Tenths and Hundredths

Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.NBT.4 to access resources specifically for this standard.



NRICH mathematics – Round the Dice Decimals 2



Online NCTM book chapter - Decimals

Motion Math Zoom

Common Misconceptions: See 5.NBT.1

Domain: Number and Operations in Base Ten (NBT)

▶ ***Cluster B:*** *Perform operations with multi-digit whole numbers and with decimals to hundredths.*

Standard: 5.NBT.5

Fluently (efficiently, accurately, and flexibly) multiply multi-digit whole numbers using an efficient algorithm *(ex., traditional, partial products, etc.)* based on place value understanding and the properties of operations. **(5.NBT.5)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: (5.NBT.5 through 5.NBT.7)

This cluster is connected to:

- Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations. (5^{th} grade concepts)
- Use place value understanding and properties of operations to perform multi-digit arithmetic (4.NBT.5 & 4.NBT.6).

Explanation and Examples:

This standard refers to fluency which means students should feel comfortable in being able to select and use a variety of methods and tools to compute, including manipulatives, mental computation, estimation, diagrams, and calculators.

- They work flexibly with basic number combinations and use visual models, benchmarks, and equivalent forms.
- They are accurate, efficient, and flexible (able to use strategies such as the distributive property or breaking numbers apart [decomposing and recomposing]). The students also use strategies according to the properties of the numbers in the problem. {For example, 26×4 may lend itself to $(25 \times 4) + 4$ whereas another problem, such as 32 x 4, might lend itself to making an equivalent problem, $32 \times 4 = 64 \times 2$.}

This standard builds upon students' work with multiplying whole numbers in third and fourth grade. In fourth grade, students developed understanding of multiplication by using various strategies based in place value understanding and the properties of operations.

Efficiency in solving multiplication problems should be emphasized. When confronted with a problem such as 250 x 4, it can be very efficient to think about how factors of 4 can be doubled and then doubled again. So 250 doubled is 500 and 500 doubled again is 1000. Frequently teachers forget to reinforce fact strategies that can be used again with larger numbers. Challenging students to try to mentally solve problems will allow them to think about more efficient strategies instead of relying on one method only.

Example:

To find the product of 123×34, students can apply the standard algorithm, or they can decompose the factor of 34 into $30 + 4$. They can then multiply 123 by 30 (to get 3690) and then multiply 123 by 4 (double to 246 and double again to get 492) and add the two products (3690 + 492 = 4182). The ways in which students are taught to record this method may vary, but ALL should emphasize the place-value nature of the partial-products algorithm. For example, one student might write:

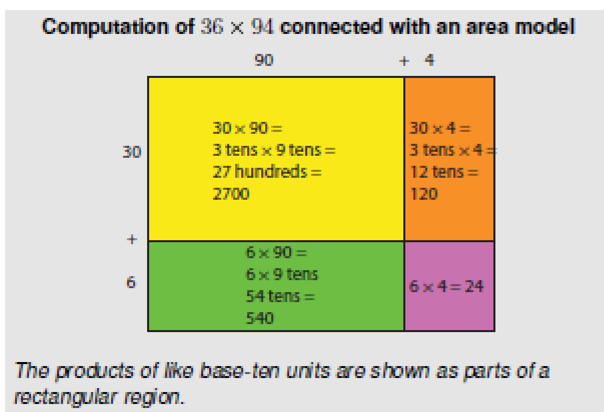
```
 123
×34
 492   ←   this is the product of 4 and 123
3690   ←   this is the product of 30 and 123
4182   ←   this is the produce of the two partial products
```

> Technically, the partial-products algorithm starts with the greatest place value but some students may start with the smallest. Either way works.

Note that the decomposition of 123 into 100 + 20 + 30 could also be used and recording of those partial products would be acceptable.

Draw an array model for 36 x 94:

$$36 \times 94 = (30 + 6) \times (90 + 4) = (30 + 6) \times 90 + (30 + 6) \times 4 = 30 \times 90 + 6 \times 90 + 30 \times 4 + 6 \times 4$$



Taken from Progression for the Common Core: K-5, Number and Operations in Base Ten
(Click picture of array to open complete document.)

Examples of alternative strategies:

There are 225 dozen cookies in the bakery. How many cookies are there?

Student 1

- 225×12
- *I broke 12 up into 10 and 2.*
- $225 \times 10 = 2{,}250$
- $225 \times 2 = 450$
- $2{,}250 + 450 = 2{,}700$

Student 2

- 225×12
- *I broke up 225 into 200 and 25.*
- $200 \times 12 = 2{,}400$
- *I decided to break 25 up into* 5×5, *so I had* $5 \times 5 \times 12$ *or* $5 \times 12 \times 5$.
- $5 \times 12 = 60, 60 \times 5 = 300$
- *I then added* - $2{,}400 + 300 = 2{,}700$

Student 3

- I doubled 225 and cut 12 in half to get 450×6.
- I then doubled 450 again and cut 6 in half to get 900×3.
- $900 \times 3 = 2{,}700$

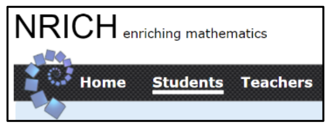
Instructional Strategies: (5.NBT.5 through 5.NBT.7)
Because students have used various models and strategies to solve problems involving multiplication with whole numbers, they should be able to transition to using standard algorithms with understanding effectively. With guidance from the teacher, they should understand the connection between the standard algorithm and other algorithms, methods, and strategies. Connections between the different algorithms for multiplying multi-digit whole numbers and strategies are **necessary for students' understanding.**

Instructional Resources/Tools:
Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.NBT.B.5
  - Elmer's Multiplication Error

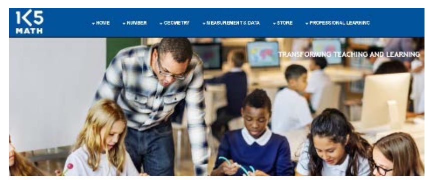
Nrich Mathematics

- Method in Multiplying Madness



Video showing partial products algorithm using the open array method – Using Partial Products

Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.NBT.5 to access resources specifically for this standard.



Common Misconceptions: 5.NBT.5 through 5.NBT.7
Frequently, students who only memorize steps for algorithms without understanding will confuse the "steps" in the addition algorithm with the "steps" in the multiplication algorithm. So they will multiply just the ones with the ones and the tens with the tens and the hundreds with the hundreds. For example:

$$\begin{array}{r} 45 \\ \times\ \ 21 \\ \hline 85 \end{array}$$

To help students think about their answer before computing, have them **estimate** what the answer should be close to when multiplying. Even just thinking about 45 x 20 (the first factor times the greatest place value of the second factor) will get them to reason more effectively about the reasonableness of their results.

Domain: Number and Operations in Base Ten (NBT)

▶ ***Cluster B:*** *Perform operations with multi-digit whole numbers and with decimals to hundredths.*

Standard: 5.NBT.6

Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **(5.NBT.6)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See 5.NBT.5

Explanation and Examples:

This standard references various strategies for division. Make sure students are gaining a foundation based in place value understanding and the properties of operations with the strategies. In fourth grade, students' experiences with division were limited to dividing by one-digit divisors so build on this understanding to get up to 2-digit divisors.

This standard extends students' prior experiences with strategies, illustrations, and explanations. A student might decompose the dividend using place value to make it easier to reason and think through the problem.

Example:

There are 1,716 students participating in Field Day. They are put into teams of 16 for the competition. How many teams are created? If you have leftover students, what could be done with them?

Student 1

1,716 divided by 16

There are one hundred 16's in 1,716.

$$1{,}716 - 1{,}600 = 116$$

I know there are at least six 16's now.

$$116 - 96 = 20$$

I can take out one more 16.

$$20 - 16 = 4$$

Then I add up all the 16's I took out, so there were 107 teams with 4 students left over. We can put the extra students with teams that already exist so 4 teams will have 17 students instead of 16.

Student 2

1,716 divided by 16

There are one hundred 16's in 1,716.

Ten groups of 16 is 160.

That's too big.

Half of that is 80, which is five groups.

I know that two groups of 16's is 32.

I have 4 students left over.

$1{,}716 - 1{,}600$	100
$116 - 80$	5
36-32	2
4	

Student 3

$$1{,}716 \div 16 = ?$$

I want to get to 1,716
I know that one hundred 16's equals 1,600
I know that five 16's equals 80

$$1{,}600 + 80 = 1{,}680$$

Two more groups of 16's equals 32, which gets us to 1,712.
I am 4 away from 1,716.
So we had $100 + 6 + 1 = 107$, so that makes 107 teams.
I guess those other 4 students can just hang out.

Student 4

How many 16's are in 1,716?
We have an area of 1,716.
I know that one side of my array is 16 units long.
I used 16 as the height.
I am trying to answer the question what is the width of my rectangle if the area is 1,716 and the height is 16.

$$100 + 7 = 107\ R4$$

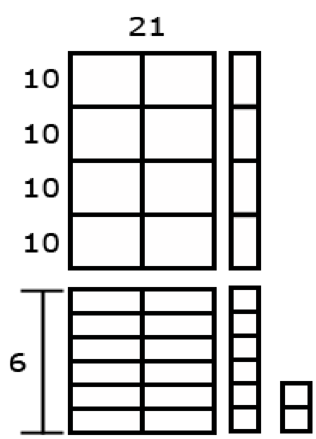
	100	7
16	$100 \times 16 = 1{,}600$	$7 \times 16 = 112$

$$1{,}716 - 1{,}600 = 116$$
$$116 - 112 = 4$$

Example:

$968 \div 21 = ?$

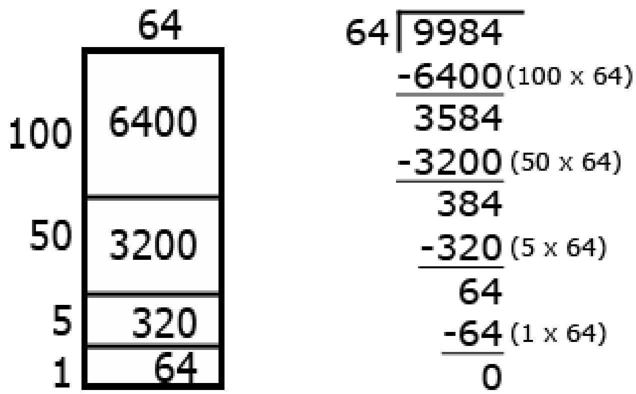
Students can use base ten models. Here a student can represent 968 and use the models to make an array with one dimension of 21. The student continues to make the array until no more groups of 21 can be made. Remainders are not part of the array with this model.



Example:

9,984 ÷ 64 =?

An area model for division is shown below. As the student uses the area model, s/he keeps track of how much of the 9984 is left to divide.



Instructional Strategies: See 5.NBT.5

Resources/Tools:

Video – Partial Quotients algorithm

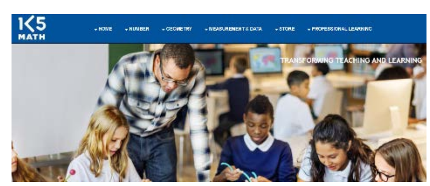
Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.NBT.B.6
  - Minutes and Days

NCTM Illuminations – NCTM has many great resources available to educators, some of these resources (i.e. interactives) are open to any educator while others (i.e. lessons) require an individual or institutional membership. If you find that a resource requires membership access check with your school/district to see if they have an institutional membership which would grant you access all NCTM documents. If they do not have a membership this would be a valuable resource to request.

- With this activity, you can visually explore the concept of factors by creating rectangular arrays. The length and width of the array are the factors in your number.

Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.NBT.6 to access resources specifically for this standard.



Learning Progressions for Numbers and Operations in Base Ten

Common Misconceptions: See 5.NBT.5

Domain: Number and Operations in Base Ten (NBT)

▶ ***Cluster B:*** *Perform operations with multi-digit whole numbers and with decimals to the hundredths.*

Standard: 5.NBT.7

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **(5.NBT.7)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.

Connections: See 5.NBT.5

Explanation and Examples:

This standard builds on the work from fourth grade where students were introduced to decimals using models, such as decimals squares, to develop decimal number sense and also work on comparing decimals to the hundredths place. In fifth grade, students will begin adding, subtracting, multiplying and dividing decimals. As stated in the standard, this work should focus on concrete models and pictorial representations, rather than relying on an algorithm; this will come later after establishing a deep understanding of how the operations work with decimals. The use of symbolic notations involves having students record the answers to computations ($2.25 \times 3 = 6.75$), but should not be done without models or pictures to accompany the work so understanding can be determined.

This standard includes students' reasoning and explanations of how they use models, pictures, and strategies. It requires students to extend the models and strategies they developed for whole numbers so far to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers. Developing number sense with decimals is essential.

Just as students developed efficient strategies for whole number operations, they should also develop efficient strategies with decimal operations. Students should learn to **estimate** decimal computations **before** they compute with pencil and paper. The focus on **estimation** should be on the meaning of the numbers and the operations, not on how many decimal places are involved. For example, to estimate the product of 32.84×4.6, the estimate would be more than 120, closer to 150. Students should consider that 32.84 is closer to 30 and 4.6 is closer to 5. The product of 30 and 5 is 150. Therefore, the product of 32.84×4.6 should be close to 150. *(Writing equations horizontally encourages using mental math).*

Have students use estimation to find the product of an expression that is similar by using exactly the same digits in one of the factors but with the decimal point in a different position each time (the second factor should remain the same each time). For example, have students estimate the product of 275×3.8 *and then* 27.5×3.8 *and then* 2.75×3.8. Lead a discussion about **why** the estimates should or should not be the same.

Examples:

$$3.6 + 1.7 = ?$$

A student might estimate the sum to be larger than 5 because 3.6 is more than $3\frac{1}{2}$ and 1.7 is more than $1\frac{1}{2}$. Using the magnitude of the decimals and developing number sense is critical.

$$5.4 - 0.8 = ?$$

A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted. Again using decimal number sense.

$$6 \times 2.4 = ?$$

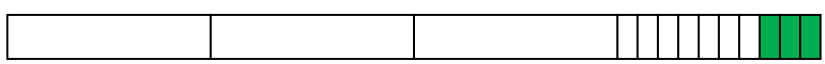
A student might estimate an answer between 12 and 18 since 6×2 is 12 and 6×3 is 18. Another student might give an estimate of a little less than 15 because s/he figures the answer to be very close, but smaller than $6 \times 2\frac{1}{2}$ and think of $2\frac{1}{2}$ groups of 6 as 12 (2 groups of 6) + 3 ($\frac{1}{2}$ of a group of 6).

Students should be able to express that when they add decimals they add tenths to tenths and hundredths to hundredths. So, when they are adding in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. This understanding can be reinforced by connecting addition of decimals to their understanding of addition of fractions. Adding fractions with denominators of 10 and 100 is a standard in fourth grade.

Example:

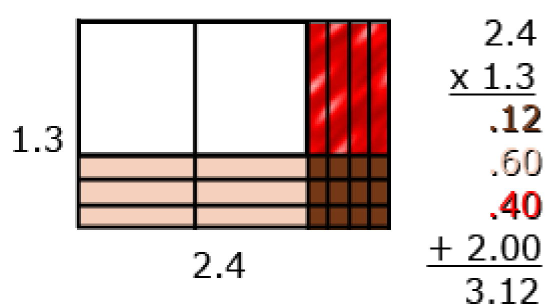
$$4 - 0.3 = ?$$

3 tenths subtracted from 4 wholes. The wholes must be divided into tenths. In this instance, just one of the wholes needs to be partitioned into tenths.



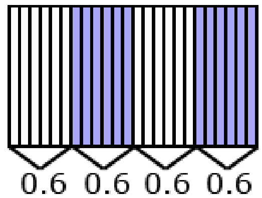
The answer is 3 and $\frac{7}{10}$ or 3.7.

Example of multiplication using the area model to illustrate partial products:



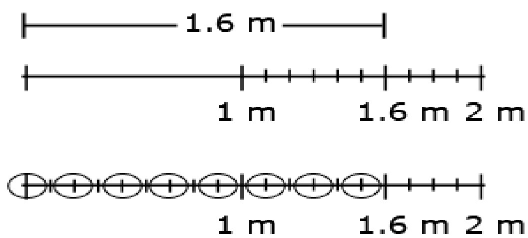
Students should be able to **describe** the partial products displayed by the area model. For example, "$\frac{3}{10}$ times $\frac{4}{10}$ is $\frac{12}{100}$. $\frac{3}{10}$ times 2 is $\frac{6}{10}$ or $\frac{60}{100}$. 1 group of $\frac{4}{10}$ is $\frac{4}{10}$ or $\frac{40}{100}$. 1 group of 2 is 2. So the total is 3 and $\frac{12}{100}$.

Example of division by finding the number in each group or share:
Students should be encouraged to apply a fair sharing model separating decimal values into equal parts such as shown in the example below.



Example of division by finding the number of groups:
Joe has 1.6 meters of rope. He has to cut pieces of rope that are 0.2 meters long. How many can he cut?

- To divide to find the number of groups, a student might:
  - Draw a segment to represent 1.6 meters. In doing so, s/he would count in tenths to identify the 6 tenths, and be able identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths.



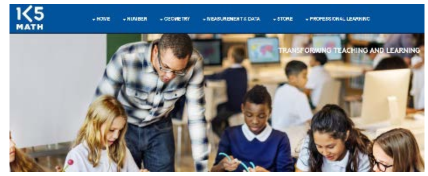
  - Count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as $\frac{10}{10}$, a student might think of 1.6 as 16 tenths. Counting 2 tenths, 4 tenths, 6 tenths, . . . 16 tenths, a student can count 8 groups of 2 tenths.
  - Use their understanding of multiplication and think, "8 groups of 2 is 16, so 8 groups of $\frac{2}{10}$ is $\frac{16}{10}$ or $1\frac{6}{10}$."

Instructional Strategies: See 5.NBT.5
Decimals are an extension of the whole number system so making the connections to the methods and strategies that made sense for whole numbers should be utilized when working with decimals. Continually emphasizing estimation before computation builds decimal number sense and operational sense.

Resources/Tools:

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.NBT.B.7
  - What is 23 ÷ 5?
  - The Value of Education



Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.NBT.7 to access resources specifically for this standard.

Thinking Blocks:

- Addition and Subtraction of Decimals
- More Than One Operation

Video clips:

- Multiplying Decimals Using Models
- Multiplying Decimals with the Area Model

Common Misconceptions: See 5.NBT.5

Students might compute the sum or difference of decimals by lining up the right-hand digits as they would whole number. For example, in computing the sum of 15.34 + 12.9, students will write the problem in this manner:

$$\begin{array}{r} 15.34 \\ +\ \ 12.9 \\ \hline 16.63 \end{array}$$

To help students add and subtract decimals correctly, have them **first estimate** the sum or difference. Providing students with a decimal-place value chart will enable them to place the digits in the proper place.

Domain: Number and Operations—Fractions (NF)

▶ ***Cluster A:*** *Uses equivalent fractions as a strategy to add and subtract fractions.*

Standard: 5.NF.1

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$ *in general,* $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$ **(5.NF.1)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2. Reason abstractly and quantitatively.
- ✓ MP.4. Model with mathematics.
- ✓ MP.7. Look for and make use of structure.

Connections:

This Cluster is connected to:

- Developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).
- Develop an understanding of fractions as numbers (3.NF.1 through 3.NF.3).

Explanations and Examples:

This standard builds on the work in fourth grade where students added fractions with like denominators. In fifth grade, the example provided in the standard has students finding a like denominator and replacing both of the fractions with equivalent fractions by finding the product of both denominators. For another example, $\frac{1}{3}+\frac{1}{6}$, a like, or common, denominator is 18 (the product of 3 and 6). This process should be introduced using visual fraction models (area models, number lines, etc.) to build understanding before moving into an algorithm.

Students should apply their understanding of equivalent fractions developed in fourth grade and their ability to rewrite fractions in an equivalent form to find like denominators. They should know that multiplying the denominators will always give a common denominator, but may not result in the smallest denominator.

The standards **do not** expect students to find least common multiples or to make improper fractions into mixed numbers BUT they do expect students to know equivalent fractions, including knowing the mixed number that is equivalent to the improper fraction and the improper fraction that is equivalent to the mixed number. So when solving problems, students **do not need to simplify**. It is enough that they are learning the process of adding and subtracting with fractions.

Examples:

1. $\frac{2}{5}+\frac{7}{8}=\frac{16}{40}+\frac{35}{40}=\frac{51}{40}$ (multiplying to find like denominators)
2. $3\frac{1}{4}-\frac{1}{6}=3\frac{3}{12}-\frac{2}{12}=3\frac{1}{12}$ (knowing equivalent fractions that have denominators that are the same)

Instructional Strategies: (5.NF.1 through 5.NF.2)
To add or subtract fractions with unlike denominators, students use their understanding of equivalent fractions to create fractions with the same denominators. Start with problems that require the changing of one of the fractions and progress to changing both fractions. Allow students to add and subtract fractions using different strategies, such as number lines, area models, fraction bars or strips. Have students share their strategies and discuss commonalities among the strategies to begin making generalizations.

Students need to develop an understanding that when adding or subtracting fractions, the fractions must refer to the **same whole**. All models used must refer to the same whole. Students may find that a circular model might not be the best model when adding or subtracting fractions because of the difficulty in partitioning the pieces so they are equal.

It is essential that instruction regularly includes word problems involving addition or subtraction of fractions. The concept of adding or subtracting fractions with unlike denominators will develop through solving problems where the students make sense of the fractional pieces.

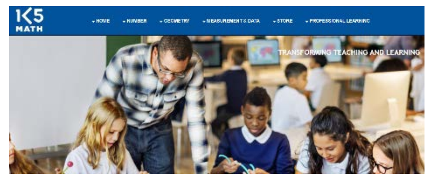
Mental computations and estimation strategies should be used to determine the reasonableness of answers. Students need to prove or disprove whether an answer provided for a problem is reasonable. Estimation is about getting useful answers, it is not about getting the right answer. Students need to be able to explain their reasoning.

Instructional Resources/Tools:

For detailed information, see: Learning Progressions for Numbers and Operations - Fractions

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.NF.A
  - To Multiply or not to multiply?
  - Measuring Cups
- 5.NF.A.1
  - Egyptian Fractions
  - Mixed Numbers with Unlike Denominators
  - Jog-A-Thon
  - Making S'Mores
  - Fractions on a Line Plot



Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.NF.1 to access resources specifically for this standard.

Thinking Blocks:

- Fractions – Addition and Subtraction Part A
- Addition and Subtraction B

NCTM Illuminations – NCTM has many great resources available to educators, some of these resources (i.e. interactives) are open to any educator while others (i.e. lessons) require an individual or institutional membership. If you find that a resource requires membership access check with your school/district to see if they have an institutional membership which would grant you access all NCTM documents. If they do not have a membership this would be a valuable resource to request.

- Equivalent Fractions (if students need support when adding and subtracting)
- Fraction Models

Video clip:

- Adding Fractions with Unlike Denominators Using an Area Model

Common Misconceptions:

Students often mix models when adding, subtracting or comparing fractions. Students will use a circle for thirds and a rectangle for fourths when comparing fractions with thirds and fourths. Remind students that the representations need to be from the same whole models with the same shape and same size.

These models of fractions are difficult to compare because the size of the whole is not the same for all representations

These models of fractions use the same size rectangle to represent the whole unit and are therefore much easier to compare fractions.

Domain: Number and Operations – Fractions (NF)

▶ ***Cluster A:*** *Use equivalent fractions as a strategy to add and subtract fractions.*

Standard: 5.NF.2

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, *(e.g. by using visual fraction models or equations to represent the problem.)* Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. (See Table 1 to view situation types). *For example, recognize an incorrect result* $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$ *by observing that* $\frac{3}{7}<\frac{1}{2}$. **(5.NF.2)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1. Make sense of problems and persevere in solving them.
- ✓ MP.2. Reason abstractly and quantitatively.
- ✓ MP.3. Construct viable arguments and critique the reasoning of others.
- ✓ MP.4. Model with mathematics.
- ✓ MP.5. Use appropriate tools strategically.
- ✓ MP.6. Attend to precision.
- ✓ MP.7. Look for and make use of structure.
- ✓ MP.8. Look for and express regularity in repeated reasoning.

Connections: See 5.NF.1

Explanation and Examples:

This standard refers to number sense, which means students' **understanding of fractions** as numbers that lie between whole numbers on a number line. Number sense in fractions includes moving between decimals and fractions to find equivalents. Students should also be able to use the magnitude of a fraction to reason about the comparison between two fractions, such as $\frac{7}{8}$ is greater than $\frac{3}{4}$ because $\frac{7}{8}$ is only $\frac{1}{8}$ away from one and $\frac{3}{4}$ is $\frac{1}{4}$ away from one, so $\frac{7}{8}$ is closer to the whole which makes it greater. Also, students should use **benchmark fractions** to estimate and examine the reasonableness of their answers. Example, $\frac{5}{8}$ is greater than $\frac{6}{10}$ because $\frac{5}{8}$ is $\frac{1}{8}$ larger than $\frac{1}{2}\left(\frac{4}{8}\right)$ and $\frac{6}{10}$ is only $\frac{1}{10}$ larger than $\frac{1}{2}\left(\frac{5}{10}\right)$.

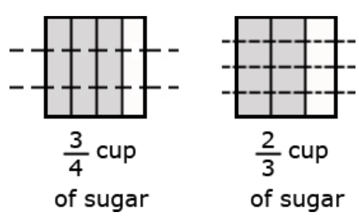
Example:
Your teacher gave you $\frac{1}{7}$ of the bag of candy. She gave your friend $\frac{1}{3}$ of the bag of candy. If you and your friend combined your candy, what fraction of the bag would you both have? Estimate your answer and then calculate the exact answer. How reasonable was your estimate?

Student 1:
$\frac{1}{7}$ is really close to 0. $\frac{1}{3}$ is larger than $\frac{1}{7}$, but still less than $\frac{1}{2}$. If we put them together we might get close to $\frac{1}{2}$.
$\frac{1}{7}+\frac{1}{3}=\frac{3}{21}+\frac{7}{21}=\frac{10}{21}$. I know that 10 is half of 20, so $\frac{10}{21}$ is a little less than $\frac{1}{2}$. So the estimate was really close.

Student 2: $\frac{1}{7}$ is close to $\frac{1}{6}$ but less than $\frac{1}{6}$, and $\frac{1}{3}$ is equivalent to $\frac{2}{6}$, so I have a little less than $\frac{3}{6}$ or $\frac{1}{2}$.

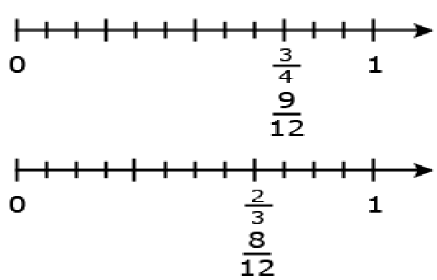
Example:
Jerry was making two different types of cookies. One recipe needed $\frac{3}{4}$ cup of sugar and the other needed $\frac{2}{3}$ cup of sugar. How much sugar did he need to make both recipes?

- Mental estimation (this is always recommended to be completed before actual computation):
  - A student may say that Jerry needs more than 1 cup of sugar but less than 2 cups. An explanation may compare both fractions to $\frac{1}{2}$ and state that both are larger than $\frac{1}{2}$ so the total must be more than 1. Also, when you put part of one of the $\frac{1}{3}$ with the $\frac{3}{4}$ to make 1 that leaves $\frac{1}{3}$ plus a little more, so it is probably close to $1\frac{1}{2}$.
- Area model

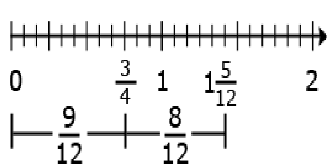


$\frac{3}{4}=\frac{9}{12}$ $\qquad$ $\frac{2}{3}=\frac{8}{12}$ $\qquad$ $\frac{3}{4}+\frac{2}{3}=\frac{17}{12}=\frac{12}{12}+\frac{5}{12}=1\frac{5}{12}$

- Linear model

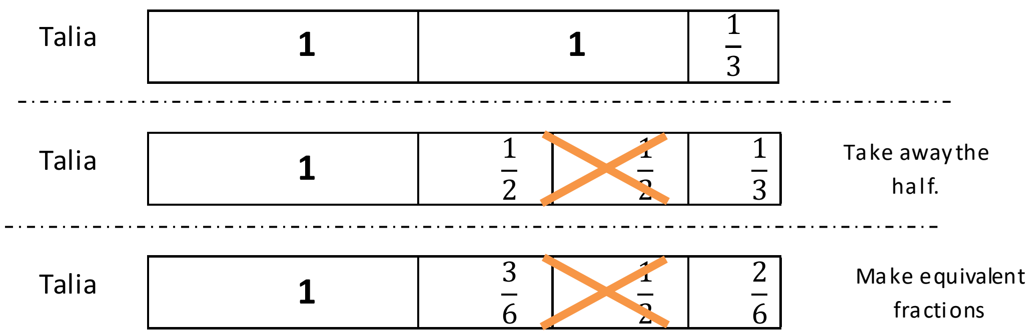


Solution:



Example: *Using a bar diagram*

Talia had $2\frac{1}{3}$ candy bars. She promised her brother that she would give him $\frac{1}{2}$ of a candy bar. How much will she have left after she gives her brother the amount she promised?



Talia will have $1\frac{5}{6}$ candy bars left.

Example:

Elli drank $\frac{3}{5}$ quart of milk and Javier drank $\frac{1}{10}$ of a quart less than Ellie. How much milk did they drink all together?

Solution:

$\frac{3}{5} - \frac{1}{10} = \frac{6}{10} - \frac{1}{10} = \frac{5}{10}$ This is how much milk Javier drank.

$\frac{3}{5} + \frac{5}{10} = \frac{6}{10} + \frac{5}{10} = \frac{11}{10}$ Together they drank $1\frac{1}{10}$ quarts of milk.

This solution is reasonable because Ellie drank more than $\frac{1}{2}$ quart and Javier drank $\frac{1}{2}$ quart so together they drank slightly more than one quart.

Instructional Strategies: See 5.NF.1

As with standard 5.NF.1, these standards **do not** expect students to find least common multiples or to make improper fractions into mixed numbers BUT they do expect students to know equivalent fractions, including knowing the mixed number that is equivalent to the improper fraction and the improper fraction that is equivalent to the mixed number. So when solving problems, students **do not need to simplify**. It is enough that they are learning the process of adding and subtracting with fractions.

Stress the reasoning over the use of algorithms with no meaning. After reasoning and making generalizations by seeing the patterns in the mathematics, the students will become more fluent and understand which procedures make more sense to use in particular situations.

Instructional Tools/Resources:

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.NF.A.2
  - Do These Add Up?
  - Salad Dressing
  - Sharing Lunches

Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.NF.2 to access resources specifically for this standard.

Thinking Blocks:

- Fractions – Addition and Subtraction Part A
- Addition and Subtraction B

Common Misconceptions: See 5.NF.1

Domain: Number and Operations – Fractions (NF)

▶ ***Cluster B:*** *Apply and extend previous understandings of multiplication and division to multiply and divide fractions.*

Standard: 5.NF.3

Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, *e.g. by using visual fraction models or equations to represent the problem. For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **(5.NF.3)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1. Make sense of problems and persevere in solving them.
- ✓ MP.2. Reason abstractly and quantitatively.
- ✓ MP.3. Construct viable arguments and critique the reasoning of others.
- ✓ MP.4. Model with mathematics.
- ✓ MP.5. Use appropriate tools strategically.
- ✓ MP.7. Look for and make use of structure.

Connections: 5.NF.3 through 5.NF.7

This cluster is connected to:

- Developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions). (5^{th} grade)
- Foundation for Learning in Grade 6: The Number System, Ratios and Proportional Relationships (6.RP.1).

Explanation and Examples:

This standard expects students to extend their work of partitioning a number line from third and fourth grades. Students need ample experiences to explore the concept that a fraction is a way to represent the division of two quantities.

Students are expected to demonstrate their understanding using concrete materials and drawing models; and explaining their thinking when working with fractions in multiple contexts. They read $\frac{3}{5}$ as "three-fifths" and after many experiences with sharing problems, learn that $\frac{3}{5}$ can also be interpreted as "3 divided by 5."

Examples:

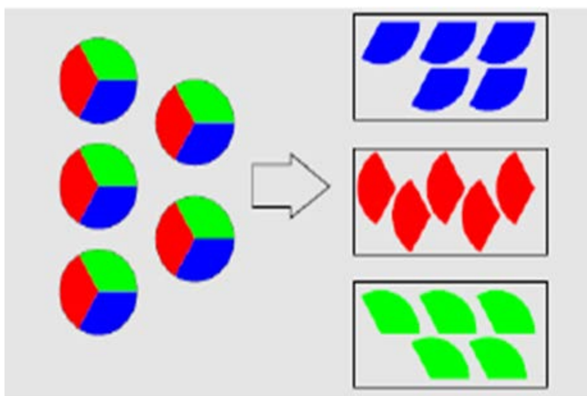
- *Ten team members are sharing 3 boxes of cookies. How much of a box will each student get?*

When working this problem a student should recognize that the 3 boxes are being divided into 10 groups, so s/he is seeing the solution to the following equation, $10 \times n = 3$ (10 groups of some amount is 3 boxes) which can also be written as $n = 3 \div 10$. Using models or diagram, they divide each box into 10 groups, resulting in each team member getting $\frac{3}{10}$ of a box. There should be <u>many</u> discussions about the results and the relationships between the numbers and representations.

- *The six 5th grade classrooms have a total of 27 boxes of pencils. How many boxes will each classroom receive?*

Students may recognize this as a whole number division problem but should also express this equal-sharing problem as $\frac{27}{6}$. They explain that each classroom gets $\frac{27}{6}$ boxes of pencils and can further determine that each classroom should get $4\frac{3}{6}$ boxes of pencils. (It is <u>not expected</u> that students "simplify" the fraction, but they should recognize that $4\frac{3}{6}$ and $4\frac{1}{2}$ are equivalent.)

Example: Sharing 5 objects equally into three sections.



If you divide 5 objects equally among 3, each of the 5 objects should contribute $\frac{1}{3}$ of itself to each share. Thus each share consists of 5 pieces, each of which is $\frac{1}{3}$ of an object, so each share is five $\frac{1}{3}$ or $\frac{5}{3}$.

Please see Progressions for the Common Core State Standards in Mathematics: Number & Operations – Fractions, 3-5.

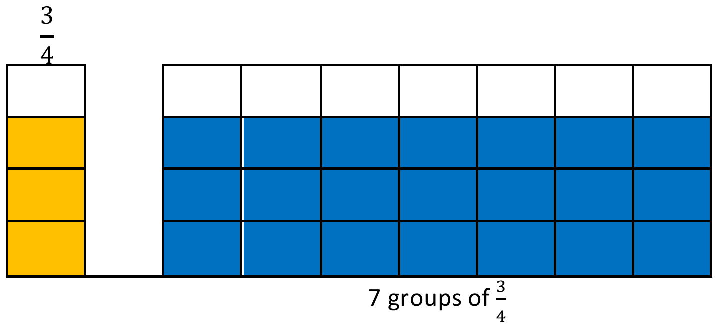
Instructional Strategies: (5.NF.3 through 5.NF.7)

Connect the meaning of multiplication and division of fractions with whole-number multiplication and division. Consider area models of multiplication; and both sharing and measuring models for division.

When connecting the understanding students already have of whole number multiplication and division, ask questions such as, "What does 2×3 mean?" and "What does $12 \div 3$ mean?" Then, follow those with questions for multiplication of fractions, such as, "What does $\frac{3}{4} \times \frac{1}{3}$ mean?" ($\frac{3}{4}$ of $\frac{1}{3}$); "What does $\frac{3}{4} \times 7$ mean?" ($\frac{3}{4}$ of 7); and "What does $\frac{3}{4}$ x 7 mean?" ($\frac{3}{4}$ of a set of 7)

When examining the meaning of division, ask "What does $4 \div \frac{1}{2}$ mean?" (How many $\frac{1}{2}$ are in 4?) and "What does $\frac{1}{2} \div 4$ mean?? (How many groups of 4 are in $\frac{1}{2}$?).

Students should use drawings and models to show their understanding of multiplication and division of fractions.



Encourage students to use models or drawings to multiply or divide with fractions. Begin with students modeling multiplication and division with whole numbers. Have them explain how they used the model or drawing to arrive at the solution.

Models to consider when multiplying or dividing fractions include, but are not limited to: area models (rectangles), linear models (fraction strips/bars, Cuisenaire rods, and number lines) and set models (counters).

Use calculators or models to explain what happens to the result when dividing a unit fraction by a non-zero whole number $\left(\frac{1}{8} \div 4, \frac{1}{8} \div 8,\ \frac{1}{8} \div \frac{1}{6}, \dots\right)$ and what happens to the result when dividing a whole number by a unit fraction $\left(4 \div \frac{1}{4},\ 8 \div \frac{1}{4},\ 12 \div \frac{1}{4}, \dots\right)$.

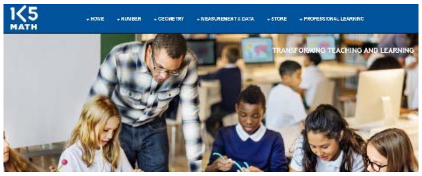
Present problem situations and have students use models and equations to solve the problem. <u>It is important for students to develop understanding of multiplication and division of fractions through contextual situations</u>.

Instructional Resources/Tools:

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.NF.B
  - Painting a Wall
- 5.NF.B.3
  - Converting Fractions of a Unit into a Smaller Unit
  - How Much Pie?
  - What is 23 ÷ 5?

Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.NF.3 to access resources specifically for this standard.



Common Misconceptions: (5.NF.3 through 5.NF.7)

Students may believe that multiplication always results in a larger number. Using models when multiplying with fractions will enable students to see that the results and begin to make generalizations that are based on understanding.

Additionally, students may believe that division always results in a smaller number. Using models will help students develop the understanding needed for computation with fractions.

Domain: Number and Operations –Fractions (NF)

▶ ***Cluster B:*** ***Apply and extend the previous understandings of multiplication and division to multiply and divide fractions.***

Standard: 5.NF.4

Apply and extend previous understandings of multiplication (refer to 2.OA.3, 2.OA.4, 3.OA.1, 3.NF.1, 3.NF.2, 4.NF.4) to multiply a fraction or whole number by a fraction.
(Number and Operations—Fractions Progression 3–5 Pg. 12 - 13).

5.NF.4a. Interpret the product $\frac{a}{b} \cdot q$ as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a **sequence** of operations $a \cdot q \div b$. *For example, use a visual fraction model to show* $\frac{2}{3} \cdot 4 = \frac{8}{3}$ *and create a story context for this equation. Do the same with* $\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$*. (In general,* $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$*).* **(5.NF.4a)**

5.NF.4b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **(5.NF.4b)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1. Make sense of problems and persevere in solving them.
- ✓ MP.2. Reason abstractly and quantitatively.
- ✓ MP.3. Construct viable arguments and critique the reasoning of others.
- ✓ MP.4. Model with mathematics.
- ✓ MP.5. Use appropriate tools strategically.
- ✓ MP.6. Attend to precision.
- ✓ MP.7. Look for and make use of structure.
- ✓ MP.8. Look for and express regularity in repeated reasoning.

Connections: See 5.NF.3

Explanation and Examples:

This standard extends student's work of multiplication from earlier grades. In fourth grade, students worked with recognizing that a fraction, such as $\frac{3}{5}$, could be represented as 3 pieces that are each one-fifth (3 pieces of size $\frac{1}{5}$).
In fifth grade, students are expected to multiply fractions including proper fractions, improper fractions, and mixed numbers. They are to multiply fractions efficiently and accurately as well as solve problems in both contextual and non-contextual situations.

This standard references both the multiplication of a fraction by a whole number and the multiplication of two fractions. Visual fraction models (*area models, tape diagrams, number lines*) should be used and created by students during their work with this standard.

As students multiply fractions such as $\frac{3}{5} \times 6$, they can think of the operation in more than one way.

$$3 \times (6 \div 5) \; or \; \left(3 \times \frac{6}{5}\right)$$
$$(3 \times 6) \div 5 \; or \; 18 \div 5$$

Students create a story problem for $\frac{3}{5} \times 6$ such as,

- Isabel had 6 feet of wrapping paper. She used $\frac{3}{5}$ of the paper to wrap some presents. How much does she have left?
- Every day Tim ran $\frac{3}{5}$ of mile. How far did he run after 6 days? (Interpreting this as 6 groups of $\frac{3}{5}$.)

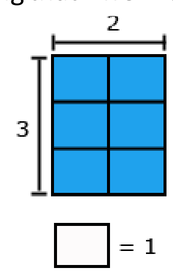
Example:

Three-fourths of the class are boys. Two-thirds of the boys are wearing tennis shoes. What fraction of the class are boys with tennis shoes?

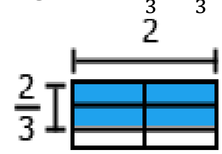
This question is asking what $\frac{2}{3}$ of $\frac{3}{4}$ is, or what is $\frac{2}{3} \times \frac{3}{4}$. In this case you have $\frac{2}{3}$ groups of size $\frac{3}{4}$. (Remember: with whole numbers a problem that is structurally similar is 4 x 5, so you have 4 groups of size 5.)

Examples: *Building on previous understandings of multiplication*

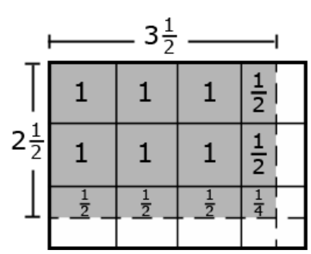
- Rectangle with dimensions of 2 and 3 showing that 2 x 3 = 6.



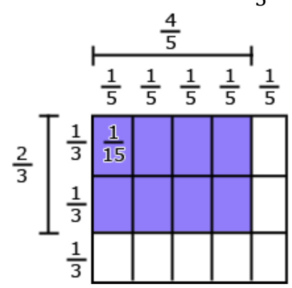
- Rectangle with dimensions of 2 and $\frac{2}{3}$ showing that 2 x $\frac{2}{3} = \frac{4}{3}$



- $2\frac{1}{2}$ groups of $3\frac{1}{2}$:

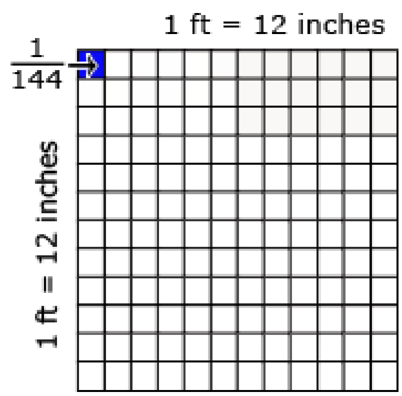


- In solving the problem $\frac{2}{3} \times \frac{4}{5}$, students use an area model to visualize it as an array of small rectangles (each small rectangle has side lengths of $\frac{1}{3}$ and $\frac{1}{5}$). They reason that $\frac{1}{3} \times \frac{1}{5} = \frac{1}{3 \times 5}$ by counting squares in the entire rectangle, so the area of the shaded area is (2×4) *of the* $\frac{1}{3\times5} = \frac{2\times4}{3\times5}$. They can explain that the product is less than $\frac{4}{5}$ because they are finding $\frac{2}{3}$ of $\frac{4}{5}$. They can further estimate that the answer must be between $\frac{2}{5}$ and $\frac{4}{5}$ because $\frac{2}{3}$ of $\frac{4}{5}$ is more than $\frac{1}{2}$ of $\frac{4}{5}$ and less than one group of $\frac{4}{5}$.



The area model and the line segments show that the area is the same quantity as the product of the side lengths.

- Larry knows that $\frac{1}{12} \times \frac{1}{12}$ is $\frac{1}{144}$. To prove this he makes the following array.



Students need to represent problems using various fraction models: Area (rectangle, circle, etc.), Linear (number line), and Set model. Explaining their thinking and reasoning with these models is critical.

Instructional Strategies: See 5.NF.3

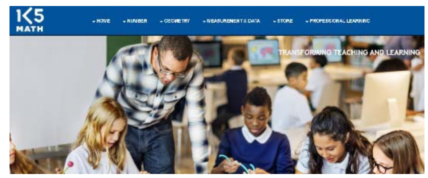
Tools/Resources

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.NF.B.4
  - Connor and Makayla Discuss Multiplication
  - Folding Strips of Paper
  - Cornbread Fundraiser
  - Mr. Gray's Homework Assignment
  - Cross Country Training
- 5.NF.B.4.a
  - Connecting the Area Model to Context
- 5.NF.B.4.b
  - Chavone's Bathroom Tiles
  - New Park

See EngageNY Modules 4 and 5 for 5th grade

Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.NF.4 to access resources specifically for this standard.



Common Misconceptions: See 5.NF.3

Domain: Number and Operations – Fractions (NF)

▶ ***Cluster B:*** *Apply and extend previous understandings of multiplication and division to multiply and divide fractions.*

Standard: 5.NF.5

Interpret multiplication as scaling (resizing), by:

5.NF.5a. Comparing the size of a product to the size of one factor based on the size of the other factor, without performing the indicated multiplication *(e.g. They see* $\left(\frac{1}{2}\cdot 3\right)$ *as half the size of 3.)*. **(5.NF.5a)**

5.NF.5b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction **equivalence** $\frac{a}{b}=\frac{na}{nb}$ to the effect of multiplying $\frac{a}{b}$ by 1. *(e.g. Students may have the misconception that multiplication always produces a larger result. They need to have the conceptual understanding with examples like;* $\frac{3}{4}\times \textit{one dozen eggs}$ *will have a product that is less than 12.)* **(5.NF.5b)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2. Reason abstractly and quantitatively.
- ✓ MP.4. Model with mathematics.
- ✓ MP.6. Attend to precision.
- ✓ MP.7. Look for and make use of structure.

Connections: See 5.NF.3

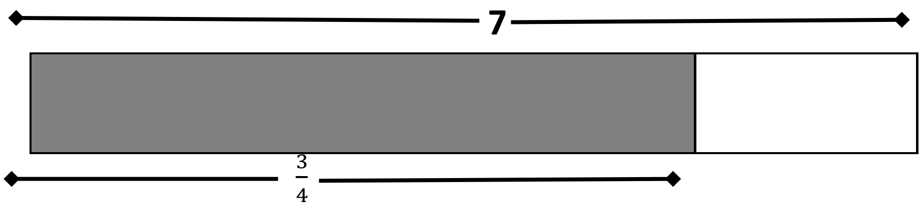
Explanation and Examples: 5.NF.5a

This standard expects students to examine the magnitude of products in terms of the relationship between the two factors in the problem. This extends the work students completed with whole numbers.

Example 1:	Example 2:
Mrs. Jones teaches in a room that is 60 feet wide and 40 feet long. Mr. Thomas teaches in a room that is half as wide, but has the same length. How do the dimensions and area of Mr. Thomas' classroom compare to Mrs. Jones' room? Draw a picture to prove your answer.	How does the product of 225×60 compare to the product of 225×30? How do you know? Since 30 is half of 60, the product of 225×60 will be double or twice as large as the product of 225×30.

Examples:

- $\frac{3}{4}$ • 7 is less than 7 because 7 is multiplied by a factor less than 1 so the product must be less than 7. The expression is read "three-fourths of seven", so we are taking just a part of the quantity of 7.



5.NF.5b
This standard expects students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases illustrated in the standard:
a) when multiplying by a fraction greater than 1, the number increases and
b) when multiplying by a fraction less the one, the number decreases.
This standard should be explored and discussed while students are working with 5.NF.4, and **should not** be taught in isolation.

Example:
Mrs. Bennett is planting two flower beds. The first flower bed is 5 meters long and $\frac{6}{5}$ meters wide. The second flower bed is 5 meters long and $\frac{5}{6}$ meters wide. How do the areas of these two flower beds compare? Is the value of the area larger or smaller than 5 square meters? Use pictures or words to prove your answer.

Example:
$2\frac{2}{3}$ x 8 must be more than 16 because 2 groups of 8 is 16 and $2\frac{2}{3}$ is almost 3 groups of 8 which is 24. So the answer must be close to, but less than, 24.

Instructional Strategies: See 5.NF.3
Encourage students to use models or drawings to multiply or divide with fractions. Begin with students modeling multiplication and division with whole numbers. Have them explain how they used the model or drawing to arrive at the solution.

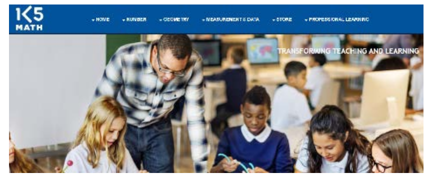
Models to consider when multiplying or dividing fractions include, but are not limited to: area models (rectangles), linear models (fraction strips/bars, Cuisenaire rods, and number lines) and set models (counters).

Resources/Tools

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.NF.B.4
    - Mr. Gray's Homework Assignment
- 5.NF.B.5
    - Running a Mile
    - Fundraising
    - Calculator Trouble
    - Comparing a Number and a Product
    - Grass Seedlings
    - Reasoning about Multiplication
    - Comparing Heights of Buildings
    - Scaling Up and Down

Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.NF.5 to access resources specifically for this standard.



Common Misconceptions:

Students may believe that multiplication always results in a larger number. Use models when multiplying with fractions so students can see and experience the results and begin to make generalizations that are based on understanding. They will begin to understand that multiplying by a fraction less than one will result in a lesser product, but when multiplying by a fraction greater than one will result in a greater product.

Domain: Number and Operations – Fractions (NF)

▶ ***Cluster B:*** *Apply and extend previous understandings of multiplication and division to multiply and divide fractions.*

Standard: 5.NF.6

Solve real world problems involving multiplication of fractions and mixed numbers, *(e.g. by using visual fraction models or equations to represent the problem)* (See Table 2 to view situation types). **(5.NF.6)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 5.NF.3

Explanation and Examples:

This standard builds on all of the mathematics that students investigate in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving fractions. This standard could include fraction by a fraction, fraction by a mixed number, or mixed number by a mixed number.

Example:

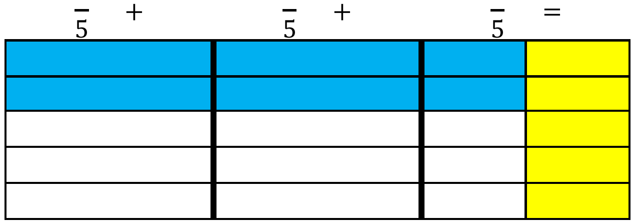
There are $2\frac{1}{2}$ busloads of students waiting in the parking lot. The students are preparing to go on a field trip. $\frac{2}{5}$ of the students are girls. How many busses would it take to carry ***only*** the girls?

Student 1 response

I drew 3 grids and 1 grid represents 1 bus. I cut the third grid in half and I marked out the right half of the third grid, leaving $2\frac{1}{2}$ grids. I then cut each grid into fifths, and shaded two-fifths of each grid to represent the number of girls.

When I added up the shaded pieces, $\frac{2}{5}$ of the 1st and 2nd bus were both shaded, and $\frac{1}{5}$ of the last bus was shaded.

$$\frac{2}{5} + \frac{2}{5} + \frac{1}{5} = \frac{5}{5} = 1\ whole\ bus$$



<u>*Student 2 response*</u>

$2\frac{1}{2} \times \frac{2}{5} =$

I split the $2\frac{1}{2}$ into 2 and $\frac{1}{2}$

$2 \times \frac{2}{5} = \frac{4}{5}$

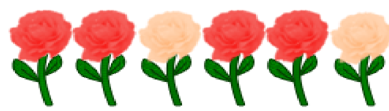
$\frac{1}{2} \times \frac{2}{5} = \frac{2}{10}$

I then added $\frac{4}{5}$ and $\frac{2}{10}$. That equals 1 whole bus load.

Example:

Evan bought 6 roses for his mother, $\frac{2}{3}$ of them were red. How many red roses were there?

- Using a visual, the student divides the 6 roses into 3 groups and counts the number in 2 of the 3 groups.

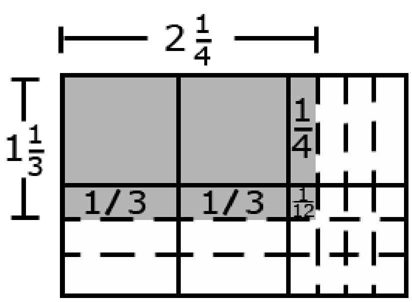


- A student can use an equation to solve
  $\frac{2}{3} \times 6 = \frac{12}{3} = 4\,red\ roses.$

Example:

Mary and Joe determined that the dimensions of their school flag needed to be $1\frac{1}{3}ft$ by $2\frac{1}{4}ft$. What will be the area of the school flag?

- A student could draw an array to find this product and use his or her understanding of decomposing numbers to explain the multiplication. (Thinking ahead a student may decide to multiply by $1\frac{1}{3}ft$ instead of $2\frac{1}{4}ft$.)



The explanation may include the following:

First, I am going to multiply $2\frac{1}{4}$ by 1 and then by $\frac{1}{3}$.
When I multiply $2\frac{1}{4}$ by 1, it equals $2\frac{1}{4}$.
Now I have to multiply $2\frac{1}{4}$ by $\frac{1}{3}$.
$\frac{1}{3}$ times 2 is $\frac{2}{3}$.
$\frac{1}{3}$ times $\frac{1}{4}$ is $\frac{1}{12}$.
So the answer is $2\frac{1}{4} + \frac{2}{3} + \frac{1}{12}$ or $2\frac{3}{12} + \frac{8}{12} + \frac{1}{12} = 2\frac{12}{12} = 3$

Instructional Strategies: See 5.NF.3

Resource/Tools

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.NF.B.6
    - Running to School
    - Drinking Juice
    - Half of a Recipe
    - Making Cookies
    - To Multiply or Not to Multiply?
    - To Multiply or Not to Multiply – Variation 2
    - Comparing Heights of Buildings
    - New Park

Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.NF.6 to access resources specifically for this standard.

Nrich Mathematics
Peaches Today, Peaches Tomorrow

Common Misconceptions: See 5.NF.3

Domain: Number and Operations – Fractions (NF)

▶ ***Cluster B:*** *Apply and extend previous understandings of multiplication and division to multiply and divide fractions.*

Standard: 5.NF.7

Apply and extend previous understandings of division (3.OA.2, 3.OA.5), to divide unit fractions by whole numbers and whole numbers by unit fractions. Division of a fraction by a fraction is not a requirement at this grade.

5.NF.7a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for* $\frac{1}{3} \div 4$*, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $\frac{1}{3} \div 4 = \frac{1}{12}$ *because* $\frac{1}{12} \cdot 4 = \frac{1}{3}$. **(5.NF.7a)**

5.NF.7b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for* $4 \div \frac{1}{5}$*, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $4 \div \frac{1}{5} = 20$ *because* $20 \cdot \frac{1}{5} = 4$. **(5.NF.7b)**

5.NF.7c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions*, e.g. by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{1}{3}$ *cup servings are in 2 cups of raisins?* **(5.NF.7c)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See 5.NF.3

Explanation and Examples:

This standard is the beginning of division with fractions. In fourth grade students divided whole numbers; but now in fifth grade, students experience division problems with whole number divisors and unit fraction dividends (fractions with a numerator of 1) AND with unit fraction divisors and whole number dividends.

For example, the fraction $\frac{3}{5}$ is three copies of the *unit fraction* $\frac{1}{5}$. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$ *which is* $\frac{1}{5} \times 3$ *or* $3 \times \frac{1}{5}$.
Students extend their understanding of the meaning of fractions, how many unit fractions are in a whole, and their understanding of multiplication and division as involving equal groups or shares and the number of objects in each

group/share. *In sixth grade, they will use this foundational understanding to divide into and by more complex fractions and develop abstract methods of dividing by fractions*.

5.NF.7a

This standard expects students to work with story contexts where a *unit fraction* is divided by a *non-zero whole number*. Students should use various fraction models and reasoning about fractions.

Example:

You have $\frac{1}{8}$ of a bag of pens and you need to share them among 3 people. How much of the bag does each person get?

5.NF.7b

This standard expects students to create story contexts and visual fraction models for division situations where a *whole number* is being divided by a *unit fraction*.

Example:

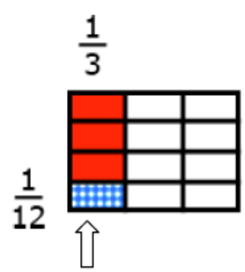
Create a story context for $5 \div \frac{1}{6}$. Then find your answer and draw a picture to prove your answer using multiplication to reason about whether your answer makes sense. *You have 5 pies. How many slices of size $\frac{1}{6}$ can you get from the pies?*

Example:

Knowing the number of groups/shares and finding how many/much in each group/share.

Four students sitting at a table were given $\frac{1}{3}$ of a pan of brownies to share. How much of a pan will each student get if they share the pan of brownies equally?

The diagram shows the $\frac{1}{3}$ pan divided into 4 equal shares with each share equaling $\frac{1}{12}$ of the pan.



5.NF.7c

Extends students' work from other the sub-standards in 5.NF.7. Students should continue to use visual fraction models and reasoning to solve these real-world problems.

Example:

How many $\frac{1}{3}$-cup servings are in 2 cups of raisins?

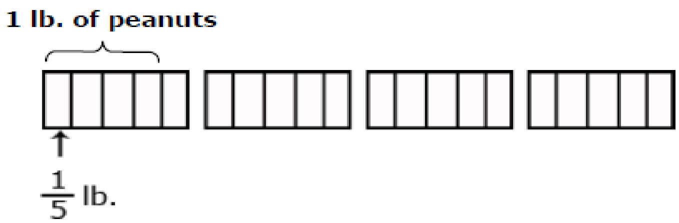
<u>*Student response*</u>

I know that there are three $\frac{1}{3}$ cup servings in 1 cup of raisins. So, there are 6 of those $\frac{1}{3}$ cup servings in 2 cups of raisins. I can also show this since 2 divided by $\frac{1}{3}$ means "How many one-thirds can I get out of 2 full cups?".

Example:

Angelo has 4 lbs. of peanuts. He wants to give each of his friends $\frac{1}{5}\,lb$. How many friends can receive $\frac{1}{5}\,lb$ of peanuts?

A diagram for $4 \div \frac{1}{5}$ is shown below. Students explain that since there are five-fifths in one whole, there must be 20 fifths in 4 lbs.



Example:

How much rice will each person get if three people share $\frac{1}{2}\,lb$ of rice equally?

$$\frac{1}{2} \div 3 = \frac{3}{6} \div 3 = \frac{1}{6}$$

- A student may think or draw ½ and cut it into 3 equal groups then determine that each of those part is $\frac{1}{6}$.
- A student may think of $\frac{1}{2}$ as equivalent to $\frac{3}{6}$. $\frac{3}{6}$ divided by 3 is $\frac{1}{6}$.

It's important that students *represent* the problems they are solving, have a visual image of the "why" behind the algorithm, and can explain their reasoning.

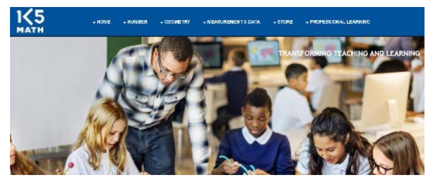
Instructional Strategies: See 5.NF.3

Resources/Tools

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.NF.B.7
  - Dividing by One-Half
  - How many servings of oatmeal?
  - Banana Pudding
- 5.NF.B.7.a
  - Painting a room
- 5.NF.B.7.b
  - How many marbles?
  - Origami Stars
- 5.NF.B.7.c
  - How many marbles?
  - Salad Dressing
  - Standing in Line

Visit K-5 Math Teaching Resources click on **Number**, then on **5th Grade**. Scroll down to 5.NF.7 to access resources specifically for this standard.



Common Misconceptions: See 5.NF.3

Domain: Measurement and Data (MD)

◆ ***Cluster A:*** *Convert like measurement units within a given measurement system.*

Standard: 5.MD.1

Convert among different-sized standard measurement units within a given measurement system *(e.g. convert 5 cm to 0.05 m)*, and use these conversions in solving multi-step, real world problems. **(5.MD.1)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.

Connections:

This cluster is connected to:

- Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations. (5th grade)
- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit (4.MD.2)

Explanation and Examples:

This standard expects students to convert measurements within the same system of measurement in the context of multi-step, real-world problems. Both **customary** and **metric** measurement systems are included. Students worked with both types of units using length in second grade. In third grade, students work with metric units of mass and liquid volume. In fourth grade, students work with both systems and begin conversions within systems in length, mass and volume.

Fifth graders build on their prior knowledge of related measurement units to determine equivalent measurements. Prior to making actual conversions, they examine the units to be converted, determine if the converted amount will be more or less than the original unit, and explain their reasoning. They use several strategies to convert measurements. When converting metric measurement, students apply their understanding of place value and decimals.

Students' work with conversions within the metric system (5.MD.1) provides opportunities for practical applications of place value understanding and supports major work at the grade in the cluster "Understand the place value system". (5.NBT.1)

Instructional Strategies:

Students should gain ease in converting units of measures in equivalent forms within the same system. To convert from one unit to another unit, the **relationship** between the units must be known. In order for students to have a better understanding of the relationships between units, they need to use measuring tools in class. The number of units must relate to the size of the unit.

For example, students should have discovered that there are 12 inches in 1 foot and 3 feet in 1 yard. This understanding is needed to convert inches to yards. Using 12-inch rulers and yardsticks, students can see that three of the 12-inch rulers are equivalent to one yardstick (3×12 inches = 36 inches; 36 inches = 1 yard). Using this knowledge, students can decide whether to multiply or divide when making conversions.

Once students have an understanding of the relationships between units and how to do conversions, they are ready to solve multi-step problems that require conversions within the same system. Allow students to discuss methods used in solving the problems. Begin with problems that allow for renaming the units to represent the solution before using problems that require renaming to find the solution.

Instructional Resources/Tools

- Yardsticks(meter sticks) and rulers (marked with customary and metric units)
- Teaspoons and tablespoons
- Graduated measuring cups (marked with customary and metric units)
- Analog and digital clocks

For detailed information see: Learning Progressions on Measurement and Data

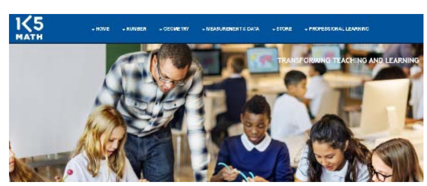
Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.MF.A.1
    - Converting Fractions of a Unit into a Smaller Unit
    - Minutes and Days

NCTM Illuminations – NCTM has many great resources available to educators, some of these resources (i.e. interactives) are open to any educator while others (i.e. lessons) require an individual or institutional membership. If you find that a resource requires membership access check with your school/district to see if they have an institutional membership which would grant you access all NCTM documents. If they do not have a membership this would be a valuable resource to request.

- "Discovering Gallon Man" NCTM.org, Illuminations. Students experiment with units of liquid measure used in the customary system of measurement. They practice making volume conversions in the customary system.
- "Do You Measure Up?" NCTM.org, Illuminations. Students learn the basics of the metric system. They identify which units of measurement are used to measure specific objects, and they learn to convert between units within the same system.

Visit K-5 Math Teaching Resources click on **Measurement and Data**, then on **5th Grade**. Scroll down to 5.MD.1 to access resources specifically for this standard.



Common Misconceptions:

When solving problems that require renaming units, students use their knowledge of renaming the numbers as with whole numbers. Students need to pay attention to the unit of measurement which dictates the renaming and the number to use. The same procedures used in renaming whole numbers should not be taught when solving problems involving measurement conversions. For example, when subtracting 5 inches from 2 feet, students may take one foot from the 2 feet and use it as 10 inches. Since there were no inches with the 2 feet, they put 1 with 0 inches and make it 10 inches.

Domain: Measurement and Data (MD)

◆ ***Cluster B:*** *Represent and interpret data.*

Standard: 5.MD.2

Make a data display (line plot, bar graph, pictograph) to show a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). Use operations (add, subtract, multiply) on fractions for this grade to solve problems involving information presented in the data display. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. After lunch everyone measured how much milk they had left in their containers. Make a line plot showing data to the nearest $\frac{1}{4}$ cup. Which value has the greatest amount? What is the total?* **(5.MD.2)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections:

This cluster is connected to:

- Use equivalent fractions as a strategy to add and subtract fractions (5.NF.1 and 5.NF.2).
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions (5.NF.4 and 5.NF.7).

Explanation and Examples:

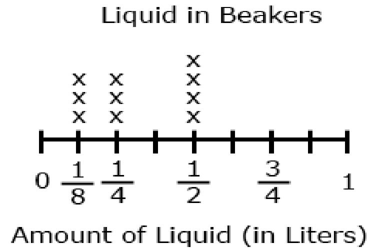
This standard provides a context for students to work with fractions by measuring objects down to one-eighth of a unit. This includes length, mass, and liquid volume. Students are making data displays of this data and then adding and subtracting fractions based on that data.

Example:

Students measured objects in their desk to the nearest $\frac{1}{2}, \frac{1}{4}, or\ \frac{1}{8}$ of an inch then displayed data collected on a line plot. How many object measured $\frac{1}{4}$? $\frac{1}{2}$? If you put all the objects together end to end what would be the total length of all the objects?

Example:

Ten beakers, measured in liters, are filled with a liquid.



Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in all of the beakers.

Instructional Strategies:

Using data displays to solve problems involving operations with unit fractions includes addition, subtraction, multiplication, and division. Revisit using a number line to solve multiplication and division problems with whole numbers. In addition to knowing how to use a number line to solve problems, students also need to know which operation to use to solve problems.

Use the tables for common addition and subtraction, and multiplication and division situations (Table 1 and Table 2 in the appendix) as a guide to the types of problems students need to solve without specifying the type of problem. Allow students to share methods used to solve the problems. Also have students create problems to show their understanding of the meaning of each operation.

Resources/Tools:

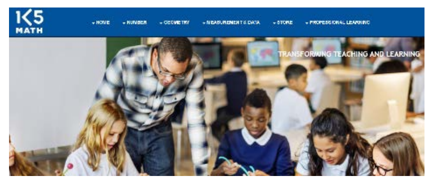
NCTM Illuminations – NCTM has many great resources available to educators, some of these resources (i.e. interactives) are open to any educator while others (i.e. lessons) require an individual or institutional membership. If you find that a resource requires membership access check with your school/district to see if they have an institutional membership which would grant you access all NCTM documents. If they do not have a membership this would be a valuable resource to request.

- "Fractions in Every Day Life", NCTM.org, Illuminations". This activity enables students to apply their knowledge about fractions to a real-life situation. It also provides a good way for teachers to assess students' working knowledge of fraction multiplication and division. Students should have prior knowledge of adding, subtracting, multiplying, and dividing fractions before participating in this activity. This will help students to think about how they use fractions in their lives, sometimes without even realizing it. The basic idea behind this activity is to use a recipe and alter it to serve larger or smaller portions.

http://mathforum.org/paths/fractions/frac.recipe.html

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.MD.B.2
  - Fractions on a Line Plot



Visit K-5 Math Teaching Resources click on **Measurement and Data**, then on **5th Grade**. Scroll down to 5.MD.2 to access resources specifically for this standard.

Common Misconceptions:

Some students will need additional help in understanding that it is possible to graph with fractions. This may be their first experience.

Students may confuse the various parts of the graph. Have graphs that are wrong displayed and discuss why they are wrong. Have the students find the errors.

Domain: Measurement and Data (MD)

▶ ***Cluster C:*** *Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.*

Standard: 5.MD.3 through 5.MD.5

5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

- 5.MD.3a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. **(5.MD.3a)**
- 5.MD.3b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **(5.MD.3b)**

5.MD.4 Measure volumes by counting unit cubes such as cubic cm, cubic in, cubic ft. or non-standard cubic units. **(5.MD.4)**

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

- 5.MD.5a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent three-dimensional whole-number products as volumes, *(e.g. to represent the associative property of multiplication.)* **(5.MD.5a)**
- 5.MD.5b. Apply the formulas $V = l \cdot w \cdot h$ and $V = B \cdot h$ (*B represents the area of the base)* for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. **(5.MD.5b)**
- 5.MD.5c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. **(5.MD.5c)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: 5.MD.3 through 5.MD.5

This cluster is connected to:

- Use place value understanding and properties of operations to perform multi-digit arithmetic (4.NBT.5).
- Solving measurement problems (4.MD.A).

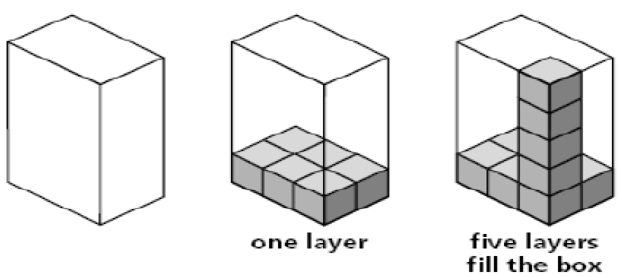
Explanation and Examples: (5.MD.3 through 5.MD.5)

These standards represent the first time that students begin exploring the concept of volume. Their prior experiences with volume were restricted to liquid volume (also called capacity). In third grade, students began working with area and covering spaces. The concept of volume should be extended from the understanding of area with the idea that an (such as the bottom of cube) can be built up with a layer of unit cubes and then adding layers of more unit cubes on top of the bottom layer *(see example picture below).*

Students should have ample experiences with concrete manipulatives (such as filling containers with unit cubes) before moving to pictorial representations. As students develop their understanding of volume they recognize that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit. This cubic unit is written with an exponent of 3 (e.g., in^3, m^3). Students connect this notation to their understanding of powers of 10 in our place value system.

In third grade, students measured and estimated liquid volume and worked with area measurement. In grade five, the concept of volume can be extended from area by relating earlier work covering an area to the bottom of cube with a layer of unit cubes and then adding layers of unit cubes on top of bottom layer. Models of cubic inches, centimeters, cubic feet, etc. are helpful in developing an image of a cubic unit. *For example: Student's estimate how many cubic yards would be needed to fill the classroom or how many cubic centimeters would be needed to fill a pencil box.*

Example:



(3 x 2) represented by first layer
(3 x 2) x 5 represented by number of
3 x 2 layers
(3 x 2) + (3 x 2) + (3 x 2) + (3 x 2)+ (3 x 2) =
6 + 6 + 6 + 6 + 6 + 6 = 30
6 representing the size/area of one layer

5.MD.4

Students understand that same sized cubic units are used to measure volume. They select appropriate units to measure volume. For example, they make a distinction between which units are more appropriate for measuring the volume of a gym and the volume of a box of books.

They can also improvise a cubic unit using any unit as a length (e.g., the length of their pencil). Students can apply these ideas by filling containers with cubic units (wooden cubes) to find the volume. They may also use drawings or interactive computer software to simulate the same filling process.

5.MD.5 a-b

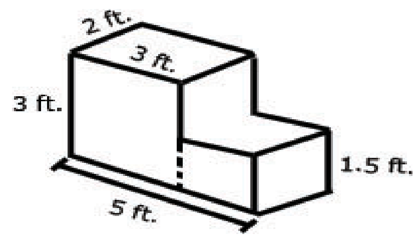
These standards involve finding the volume of right rectangular prisms *(as shown in picture on previous page)*. Students should have experiences to describe and reason about **why** the formula is true. Specifically, that they are covering the bottom of a right rectangular prism (length x width) with multiple layers (height). Therefore, the formula (length x width x height) is an extension of the formula for the area of a rectangle.

Examples:

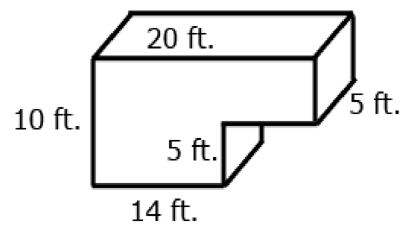
- When given 24 cubes, students make as many rectangular prisms as possible with a volume of 24 cubic units. Students build the prisms and record possible dimensions.

Length	Width	Height
1	2	12
2	2	6
4	2	3
8	3	1

- Students determine the volume of concrete needed to build the steps in the diagram below.



- A homeowner is building a swimming pool and needs to calculate the volume of water needed to fill the pool. The design of the pool is shown in the illustration below.



5.MD.5c

This standard expects students to extend their work with the area of composite figures from previous grades into the context of volume in fifth grade. Students should be given concrete experiences of breaking apart (decomposing) 3-dimensional figures into right rectangular prisms in order to find the volume of the entire 3-dimensional figure.

Students need multiple opportunities to measure volume by filling rectangular prisms with cubes and looking at the **relationship** between the total volume and the area of the base and the height. They derive the volume formula (volume equals the area of the base times the height) and explore how this idea would apply to other prisms. Students use the associative property of multiplication and decomposition of numbers using factors to investigate rectangular prisms with a given number of cubic units.

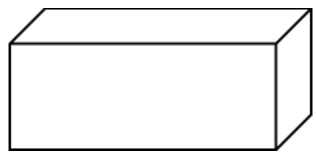
Instructional Strategies: **5.MD.3 through 5.MD.5:**

Volume refers to the amount of space that an object takes up and is measured in cubic units, such as cubic inches or cubic centimeters.

Provide students with opportunities to find the volume of rectangular prisms by counting unit cubes, in metric and standard units of measure, before the formula is generated. The students should generate this formula through their work with cubes and rectangular prisms. Multiple opportunities are needed for students to develop the formula for the volume of a rectangular prism with activities similar to the one described below.

Give students one block (a 1- or 2- cubic centimeter or cubic-inch cube), a ruler with the appropriate measure based on the type of cube, and a small rectangular box. Ask students to determine the number of cubes needed to fill the box. Have students share their strategies with the class using words, drawings or numbers. Allow them to confirm the volume of the box by filling the box with cubes of the same size.

By stacking geometric solids with cubic units in layers, students can begin understanding the concept of how *addition plays a part in finding volume*. This will lead to an understanding of the formula for the volume of a right rectangular prism, b x h, where b is the area of the base. A right rectangular prism has three pairs of parallel faces that are all rectangles.



Have students build a prism in layers. Then, have students determine the number of cubes in the bottom layer and share their strategies. Students should use multiplication based on their knowledge of arrays and its use in multiplying two whole numbers.

Ask what strategies can be used to determine the volume of the prism based on the number of cubes in the bottom layer. Expect responses such as "adding the same number of cubes in each layer as were on the bottom layer" or multiply the number of cubes in one layer times the number of layers.

Instructional Resources/Tools

See engageNY Module 5 Fifth Grade

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.MD.C.5
  - You Can Multiply Three Numbers in Any Order
- 5.MD.C.5.a
  - Using Volume to Understand the Associative Property of Multiplication
- 5.MD.C.5.b
  - Cari's Aquarium

See "Varying Volumes", NCSM, Great Tasks for Mathematics K-5, (2013)

NCTM Illuminations – NCTM has many great resources available to educators, some of these resources (i.e. interactives) are open to any educator while others (i.e. lessons) require an individual or institutional membership. If you find that a resource requires membership access check with your school/district to see if they have an institutional membership which would grant you access all NCTM documents. If they do not have a membership this would be a valuable resource to request.

- Cubes
- Isometric Drawing Tool

Learning Progressions on Measurement and Data

What is Volume video by Math Antics

Common Misconceptions:

Students are unsure as to which units to use to measure volume because they are not sure what they are measuring. Also, they may confuse the need to find volume with area.

Domain: Geometry (G)

● ***Cluster A:*** *Graph points on the coordinate plane to solve real-world and mathematical problems.*

Standard: 5.G.1

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond *(e.g. x-axis and x-coordinate, y-axis and y-coordinate)*. **(5.G.1)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: 5.G.1 through 5.G.2.

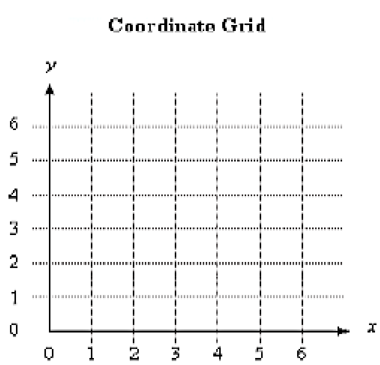
This cluster addresses Modeling numerical relationships with the coordinate plane.

Explanation and Examples:

5.G.1 and 5.G.2

These standards deal with only the first quadrant (positive numbers in the coordinate plane).

5.G.1 Examples:



Example:

Connect these points in order on the coordinate grid below:
(2, 2) (2, 4) (2, 6) (2, 8) (4, 5) (6, 8) (6, 6) (6, 4) and (6, 2).

What letter is formed on the grid?

Solution: "M" is formed.

Example:
Plot these points on a coordinate grid.

- Point A: (2,6)
- Point B: (4,6)
- Point C: (6,3)
- Point D: (2,3)

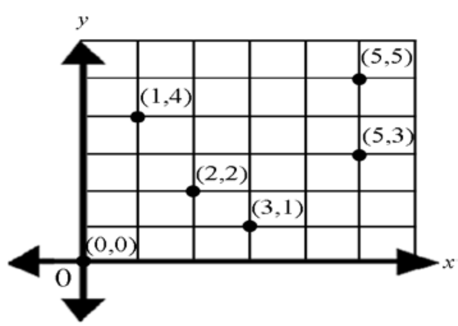
Connect the points in order. Make sure to connect Point D back to Point A.

1. What geometric figure is formed? What attributes did you use to identify it?
2. What line segments in this figure are parallel?
3. What line segments in this figure are perpendicular?

Solutions: 1. Trapezoid (student identifies appropriate attributes), 2. line segments AB and DC are parallel, 3. segments AD and DC are perpendicular

Example:
Emanuel draws a line segment from (1, 3) to (8, 10). He then draws a line segment from (0, 2) to (7, 9). If he wants to draw another line segment that is parallel to those two segments which points could he use?

Examples:
Students can use a classroom size coordinate system to physically locate the coordinate point (5, 3) by starting at the origin point (0,0), walking 5 units along the x axis to find the first number in the pair (5), and then walking up 3 units for the second number in the pair (3). The ordered pair names a point in the plane.



Instructional Strategies:
Students need to understand the underlying structure of the coordinate system and how axes make it possible to locate points anywhere on a coordinate plane. This is the first time students are working with coordinate planes, and only in the first quadrant. It is important that students create the coordinate grid themselves. This can be related to two number lines and have the students make connections to previous experiences with moving along a number line.

Coordinate grids are fun! Multiple experiences with plotting points are needed. Provide points plotted on a grid and have students name and write the ordered pair. Have students **describe** how to get to the location. Encourage students to articulate directions, attending to precision as they plot points.

Present real-world and mathematical problems and have students graph points in the first quadrant of the coordinate plane. Gathering and graphing data is a valuable experience for students. It helps them to develop an understanding of coordinates and what the overall graph represents. Students also need to analyze the graph by interpreting the coordinate values in the context of the situation.

Instructional Resources/Tools

For detailed information see, Learning Progressions for Geometry

http://www.learner.org/series/modules/express/videos/video_clips.html?type=1&subject=math&practice=structure

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.G.A.1
    - Battle Ship Using Grid Paper

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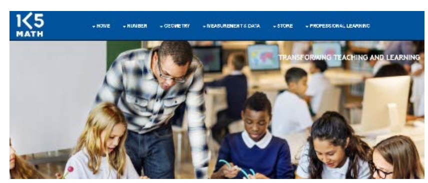
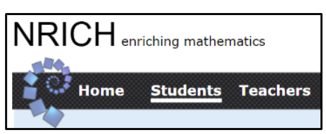
- *"Finding Your Way Around"*, NCTM, Illuminations. Students explore two-dimensional space via an activity in which they navigate the coordinate plane.
- "Describe the Graph"' NCTM.org, Illuminations. In this lesson, students will review plotting points and labeling axes. Students generate a set of random points all located in the first quadrant.

Visit K-5 Math Teaching Resources click on **Geometry**, then on **5th Grade**. Scroll down to 5.G.1 to access resources specifically for this standard.



Nrich Mathematics

- Coordinate Challenge



Common Misconceptions: 5.G.1 through 5.G.2

When playing games with coordinates or looking at maps, students may think the order in plotting a coordinate point is not important. Have students plot points so that the position of the coordinates is switched. For example, have students plot (3, 4) and (4, 3) and discuss the order used to plot the points. Have students create directions for others to follow so that they become aware of the importance of direction and distance.

Domain: Geometry (G)

● ***Cluster A***: *Graph points on the coordinate plane and solve real-world and mathematical problems.*

Standard: 5.G.2

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. *(e.g. plotting the relationship between two positive quantities such as maps, coordinate grid games (such as Battleship), time/temperature, time/distance, cost/quantity, etc.).* **(5.G.2)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

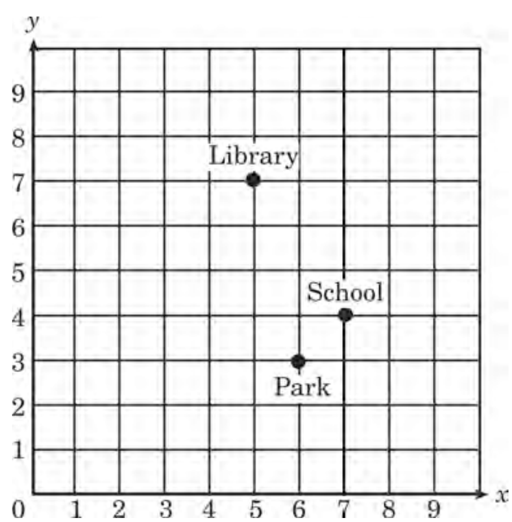
Connections: See 5.G.1

Explanation and Examples:

This standard references real-world and mathematical problems, including the traveling from one point to another and identifying the coordinates of missing points in geometric figures, such as squares, rectangles, and parallelograms.

Example:

Using the coordinate grid, which an ordered pair represents the location of the School? Explain a possible path from the school to the library.

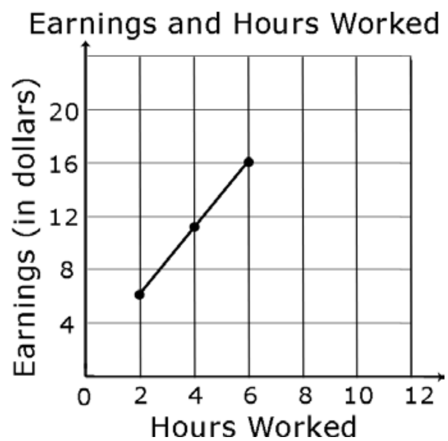


Examples:

Barb has saved $20. She earns $8 for each hour she works.

- If Barb saves all of her money, how much will she have after working 3 hours? 5 hours? 10 hours?
- Create a graph that shows the relationship between the hours Barb worked and the amount of money she has saved.
- What other information do you know from analyzing the graph?

Use the graph below to determine how much money Barb makes after working exactly 9



Instructional Strategies: See 5.G.1

Resources/Tools:

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.G.A.2
  - Meerkat Coordinate Plane Task

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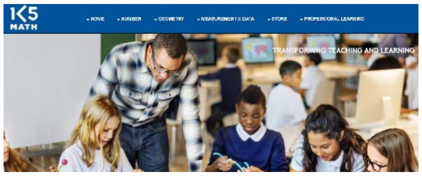
- Plotter the Penguin

Visit K-5 Math Teaching Resources click on **Geometry**, then on **5th Grade**. Scroll down to 5.G.2 to access resources specifically for this standard.



Common Misconceptions: See 5.G.1

Domain: Geometry (G)

● ***Cluster B:*** *Classify two-dimensional figures into categories based on their properties.*

Standard: 5.G 3

Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* **(5.G.3)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: 5.G.3-4

This cluster is connected to:

- Reason with shapes and their attributes (3.G.A).
- Draw and identify lines and angles, and classify shapes by properties of their lines and angles (4.G.A).

Explanation and Examples:

This standard expects students to reason about the attributes (properties) of shapes. Student should have many experiences discussing the properties of shapes and explaining their reasoning.

Geometric properties include sides (parallel, perpendicular, congruent), angles (type, measurement, congruent), and properties of symmetry (point and line).

Example:

Examine whether all quadrilaterals have right angles. Give examples and non-examples.

Example:

If the opposite sides on a parallelogram are parallel and congruent, then rectangles are parallelograms.
A sample of questions that might be posed to students include:

- A parallelogram has 4 sides with both sets of opposite sides parallel. What types of quadrilaterals are parallelograms? Explain.
- Regular polygons have all of their sides and angles congruent. Name or draw some regular polygons. Explain your drawings.
- All rectangles have 4 right angles. Squares have 4 right angles so they are also rectangles. True or False? Explain your reasoning.
- A trapezoid has 2 sides parallel so it must be a parallelogram. True or False? Explain your reasoning.

Instructional Strategies: 5.G.3 through 5.G.4

This cluster builds from Grade 3 when students described, analyzed and compared properties of two-dimensional shapes. They compared and classified shapes by their sides and angles, and connected these with definitions of shapes.

In Grade 4 students built, drew and analyzed two-dimensional shapes to deepen their understanding of the properties of two-dimensional shapes. They looked at the presence or absence of parallel and perpendicular lines or the presence or absence of angles of a specified size to classify two-dimensional shapes.

Now, students classify two-dimensional shapes in a hierarchy based on properties. Details learned in earlier grades need to be used in the descriptions of the attributes of shapes. The more ways that students can classify and discriminate shapes, the better they can understand them. Shapes are not limited to quadrilaterals.

Students can use graphic organizers, such as flow charts or T-charts, to compare and contrast the attributes of geometric figures. Have students create a T-chart with a shape on each side. Have them list attributes of the shapes, such as number of side, number of angles, types of lines, etc. They need to then discuss and determine what is alike or different about the two shapes to determine the categories for the shapes and be able to explain their properties.

Pose questions such as, “Why is a square always a rectangle?” and “Why is a rectangle not always a square?” Expect students to use precision in justifying and explaining their reasoning.

Resources/Tools
See engageNY Module 5

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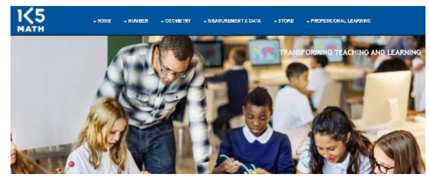
- "Geometric Solids", NCTM.org, Illuminations has a is a tool that allows the student to learn about various geometric solids and their properties. The shapes can be manipulated, colored to explore the number of faces, edges, and vertices and they can be to investigate question such as: what is the relationship between the number of faces, vertices and edges?
- Exploring the Properties of Rectangles and Parallelograms



Visit K-5 Math Teaching Resources click on **Geometry**, then on **5th Grade**. Scroll down to 5.G.3 to access resources specifically for this standard.

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.G.B.3
    - Always, Sometimes, Never

Common Misconceptions: 5.G.3 through 5.G.4
Students think that when describing geometric shapes and placing them in subcategories, the last category is the only classification that can be used.

Domain: Geometry (G)

● ***Cluster B:*** *Classify two-dimensional figures into categories based on their properties.*

Standard: 5.G.4

Classify two-dimensional figures in a hierarchy based on properties. **(5.G.4)**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See 5.G.3

Explanation and Examples:

This standard builds on what was done in 4th grade. Figures from previous grades: **polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle**

Properties of figure may include:

- sides—parallel, perpendicular, congruent, number of sides
- angles—types of angles, congruent

Examples:

- A right triangle can be both scalene and isosceles, but not equilateral.
- A scalene triangle can be right, acute and obtuse.

Triangles can be classified by:

- Angles
    - Right: The triangle has one angle that measures 90º.
    - Acute: The triangle has exactly three angles that measure between 0º and 90º.
    - Obtuse: The triangle has exactly one angle that measures greater than 90º and less than 180º.
- Sides
    - Equilateral: All sides of the triangle are the same length.
    - Isosceles: At least two sides of the triangle are the same length.
    - Scalene: No sides of the triangle are the same length.

Example:
Create a Hierarchy Diagram using the following terms:

polygons – a closed plane figure formed from line segments that meet only at their endpoints.

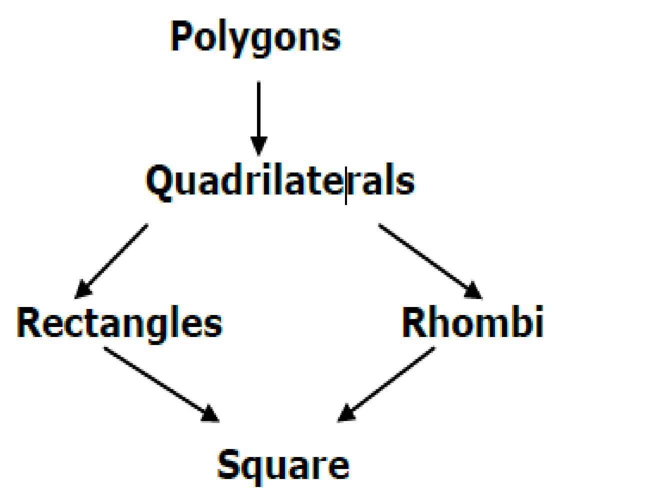
quadrilaterals - a four-sided polygon.

rectangles - a quadrilateral with two pairs of congruent parallel sides and four right angles.

rhombi – a parallelogram with all four sides equal in length.

square – a parallelogram with four congruent sides and four right angles.

Possible student solutions:



quadrilaterals - a four-sided polygon.

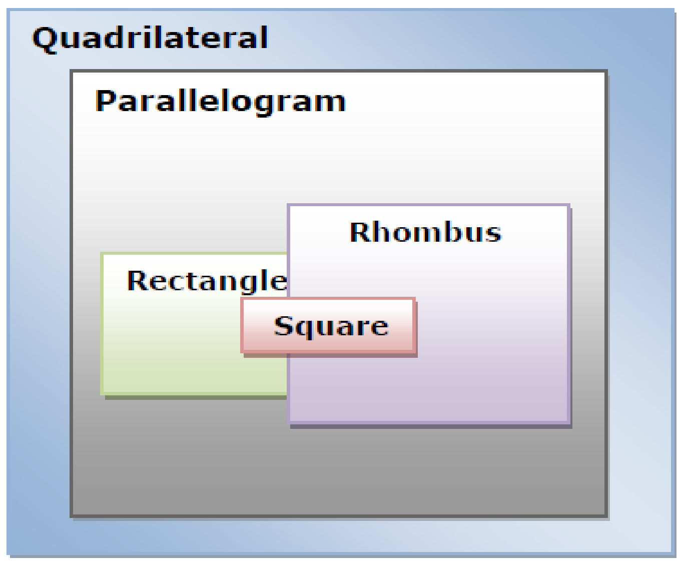
parallelogram: a quadrilateral with two pairs of parallel and congruent sides.

rectangles - a quadrilateral with two pairs of congruent parallel sides and four right angles.

rhombus – a parallelogram with all four sides equal in length.

square – a parallelogram with four congruent sides and four right angles.

Possible student solution:



Instructional Strategies: See 5.G.3

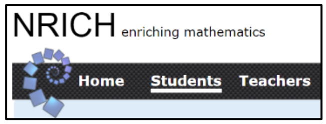
Resources/Tools

Illustrative Mathematics Grade 5 tasks: **Scroll to the appropriate section to find named tasks.**

- 5.G.B.4
  - What is a Trapezoid? (Part 2)
  - What Do These Shapes Have in Common?

Nrich Mathematics

- Quadrilaterals



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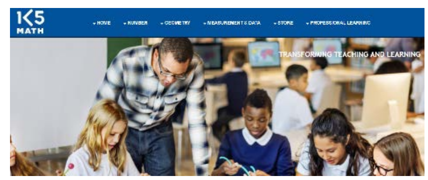
Properties of Triangles and Quadrilaterals

Visit K-5 Math Teaching Resources click on **Geometry**, then on **5th Grade**. Scroll down to 5.G.4 to access resources specifically for this standard.



Common Misconceptions: See 5.G.3

APPENDIX: TABLE 1. Common Addition and Subtraction Situations

Shading taken from OA progression

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Taken from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and variants (blue and green). Yellow indicates problems that are the difficult four problem subtypes or variants that students in Grade 1 work with but do not need to master until Grade 2.

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

TABLE 2. Common Multiplication and Division Situations

Grade level identification of introduction of problem situations taken from OA progression

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 x 6 = ?	3 x ? = 18; 18 ÷ 3 = ?	? x 6 = 18; 18 ÷ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[4], Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	a × b = ?	a × ? = p, and p ÷ a = ?	? × b = p, and p ÷ b = ?

Multiplicative compare problems appear first in Grade 4 (green), with whole number values and with the "times as much" language from the table. In Grade 5, unit fractions language such as "one third as much" may be used. Multiplying and unit language change the subject of the comparing sentence ("A red hat costs n times as much as the blue hat" results in the same comparison as "A blue hat is 1/n times as much as the red hat" but has a different subject.)

TABLE 3. The Properties of Operations

Name of Property	Representation of Property	Example of Property, Using Real Numbers
Properties of Addition		
Associative	$(a+b)+c=a+(b+c)$	$(78+25)+75=78+(25+75)$
Commutative	$a+b=b+a$	$2+98=98+2$
Additive Identity	$a+0=a$ *and* $0+a=a$	$9875+0=9875$
Additive Inverse	For every real number a, there is a real number $-a$ such that $a+-a=-a+a=0$	$-47+47=0$
Properties of Multiplication		
Associative	$(a\times b)\times c=a\times(b\times c)$	$(32\times 5)\times 2=32\times(5\times 2)$
Commutative	$a\times b=b\times a$	$10\times 38=38\times 10$
Multiplicative Identity	$a\times 1=a$ *and* $1\times a=a$	$387\times 1=387$
Multiplicative Inverse	*For every real number* a, $a\neq 0$, *there is a real number* $\frac{1}{a}$ *such that* $a\times\frac{1}{a}=\frac{1}{a}\times a=1$	$\frac{8}{3}\times\frac{3}{8}=1$
Distributive Property of Multiplication over Addition		
Distributive	$a\times(b+c)=a\times b+a\times c$	$7\times(50+2)=7\times 50+7\times 2$

(Variables a, b, and c represent real numbers.)

Excerpt from NCTM's ***Developing Essential Understanding of Algebraic Thinking****, grades 3-5 p. 16-17*

TABLE 4. The Properties of Equality

Name of Property	Representation of Property	Example of property
Reflexive Property of Equality	$a = a$	$3{,}245 = 3{,}245$
Symmetric Property of Equality	$If\ a = b, then\ b = a$	$2 + 98 = 90 + 10, then\ 90 + 10 = 2 + 98$
Transitive Property of Equality	$If\ a = b\ and\ b = c, then\ a = c$	$If\ 2 + 98 = 90 + 10\ and\ 90 + 10 = 52 + 48$ *then* $2 + 98 = 52 + 48$
Addition Property of Equality	$If\ a = b, then\ a + c = b + c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} + \frac{3}{5} = \frac{2}{4} + \frac{3}{5}$
Subtraction Property of Equality	$If\ a = b, then\ a - c = b - c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} - \frac{1}{5} = \frac{2}{4} - \frac{1}{5}$
Multiplication Property of Equality	$If\ a = b, then\ a \times c = b \times c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} \times \frac{1}{5} = \frac{2}{4} \times \frac{1}{5}$
Division Property of Equality	$If\ a = b\ and\ c \neq 0, then\ a \div c = b \div c$	$If\ \frac{1}{2} = \frac{2}{4}, then\ \frac{1}{2} \div \frac{1}{5} = \frac{2}{4} \div \frac{1}{5}$
Substitution Property of Equality	If $a = b$, then b may be substituted for a in any expression containing a.	$If\ 20 = 10 + 10$ *then* $90 + 20 = 90 + (10 + 10)$

(Variables a, b, and c can represent any number in the rational, real, or complex number systems.)

TABLE 5. The Properties of Inequality

Exactly one of the following is true: $a < b, a = b, a > b$.

$If\ a > b\ and\ b > c\ then\ a > c.$

$If\ a > b, then\ b < a.$

$If\ a > b, then\ -a < -b.$

$If\ a > b, then\ a \pm c > b \pm c.$

$If\ a > b\ and\ c > 0, then\ a \times c > b \times c.$

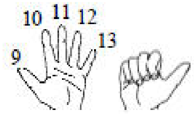
$If\ a > b\ and\ c < 0, then\ a \times c < b \times c.$

$If\ a > b\ and\ c > 0, then\ a \div c > b \div c.$

$If\ a > b\ and\ c < 0, then\ a \div c < b \div c.$

Here a, b, and c stand for arbitrary numbers in the rational or real number systems.

TABLE 6. Development of Counting in K-2 Children

Levels	8 + 6 = 14	14 – 8 = 6
Level 1: Count all	Count All a: 1 2 3 4 5 6 7 8 / b: 1 2 3 4 5 6 1 2 3 4 5 6 7 8 9 10 11 12 13 14 (c)	Take Away a: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 b: 1 2 3 4 5 6 7 8 / c: 1 2 3 4 5 6
Level 2: Count on	Count On 8 → 8 9 10 11 12 13 14	To solve 14 – 8 I count on 8 + ? = 14 I took away 8  9 10 11 12 13 14 8 to 14 is 6 so 14 – 8 = 6
Level 3: Recompose Make a ten (general): one addend breaks apart to make 10 with the other addend Make a ten (from 5's within each addend)	Recompose: Make a Ten 10 + 4 10 + 4	14 – 8: I make a ten for 8 + ? = 14 8 + 2 + 4 (2 + 4 = 6) 8 + 6 = 14
Doubles ± *n*	6 + 8 = 6 + 6 + 2 = 12 + 2 = 14	

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

Beginning--A child can count very small collections (1-4) collection of items and understands that the last word tells "how many" even. Beyond on small numbers the number words may be said without the lack of numerical understanding. This is often referred to as rote counting.

Level 1—The child uses one to one correspondence to find the number of objects in two sets. Even if the quantity is known for the first set, the child will start with the first set and continue counting on into the second set. The child begins the count with one. This also connects to Piaget's Hierarchical Inclusion – that in order to have 7 – you have to have 6, 5, 4, etc.

Level 2 – At this level the student begins the counting, starting with the known quantity of the first set and "counts on" from that point in the sequence to establish how many. This method is used to find the total in two sets. This counting is not rote. This level of counting requires several connections between cardinality and counting meanings of the number words.

Level 3 - At this level the student begins using known facts to solve for unknown facts. For example, the student uses "make a ten" where one addend breaks apart to make 10 with another addend OR a doubles plus/minus one strategy. Students begin to implicitly use the properties of operations.

Table 7. Cognitive Rigor Matrix/Depth of Knowledge (DOK)

Kansas Math Standards require high-level cognitive demand asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target the depth(s) of knowledge (DOK) that the student needs to bring to the item/task will be identified, using the Cognitive Rigor Matrix shown below.

Depth of Thinking (Webb)+ Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	• Recall conversions, terms, facts			
Understand	• Evaluate an expression • Locate points on a grid or number on number line • Solve a one-step problem • Represent math relationships in words, pictures, or symbols	• Specify, explain relationships • Make basic inferences or logical predictions from data/observations • Use models/diagrams to explain concepts • Make and explain estimates	• Use concepts to solve non-routine problems • Use supporting evidence to justify conjectures, generalize, or connect ideas • Explain reasoning when more than one response is possible • Explain phenomena in terms of concepts	• Relate mathematical concepts to other content areas, other domains • Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	• Follow simple procedures • Calculate, measure, apply a rule (e.g., rounding) • Apply algorithm or formula • Solve linear equations • Make conversions	• Select a procedure and perform it • Solve routine problem applying multiple concepts or decision points • Retrieve information to solve a problem • Translate between representations	• Design investigation for a specific purpose or research question • Use reasoning, planning, and supporting evidence • Translate between problem & symbolic notation when not a direct translation	• Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	• Retrieve information from a table or graph to answer a question • Identify a pattern/trend	• Categorize data, figures • Organize, order data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern	• Compare information within or across data sets or texts • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph	• Analyze multiple sources of evidence or data sets
Evaluate			• Cite evidence and develop a logical argument • Compare/contrast solution methods • Verify reasonableness	• Apply understanding in a novel way, provide argument or justification for the new application
Create	• Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	• Generate conjectures or hypotheses based on observations or prior knowledge and experience	• Develop an alternative solution • Synthesize information within one data set	• Synthesize information across multiple sources or data sets • Design a model to inform and solve a practical or abstract situation

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